Equivalent Follow-Up Factor for Combined Primary and Secondary Load

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Elastic follow-up factors, Z, are most often required when the applied load is intermediate in character between primary and secondary. An example is a displacement applied at a finite distance from the location of interest. However, another situation in which Z may be required is when two loads are applied to the structure, one being a pure primary load (which alone would have $Z \to \infty$) and a pure secondary load (which alone would have Z = 1). What is the effective Z for both loads applied together?

Of course, there is no unique answer. If we wait long enough for the secondary stress to have fully relaxed, there will only be the primary stress left and hence $Z \to \infty$ as $t \to \infty$. On the other hand, at the start of the relaxation Z will be little bigger than unity. The relaxation curve starts off dropping almost vertically and turns to become asymptotic to the primary stress. Thus, Z starts off little bigger than 1 and increases without limit. So how is our question meaningful? The question can be made meaningful by specifying that we want Z at the time when the secondary stress is ~99% relaxed. (NB: This can happen quite quickly if the primary stress is large enough). This Z will be denoted $Z_{99\%}$.

It is argued here, by way of an illustrative model, that primary plus secondary loading is approximately equivalent to a load with a follow-up factor of $Z_{99\%} = \dfrac{1}{1-\xi}$ where $\xi = \left(\dfrac{\sigma_P}{\sigma_P + \sigma_S}\right)^2$ and σ_P and σ_S are characteristic elastic stresses at the point of interest for the primary and secondary loads applied alone.

The illustration considered is the two-bar problem for elastic-creep. Two parallel bars, of equal length and width (w) but with thicknesses t_1 and t_2, are constrained together so as to ensure they always have exactly the same length. A tensile secondary load is applied to bar No.2 by increasing the temperature of bar No.1. By choosing $t_1 >> t_2$, the tensile thermal load on bar No.2 is ensured to be pure secondary, i.e. having Z ~ 1 when considered alone. For our illustration we have used $t_1/t_2 = 100$.

A primary tensile load F is also applied to the composite pair of bars. The individual bars bear loads of F_1 and F_2 respectively, such that $F = F_1 + F_2$. The values of F_1 and F_2 are determined by the requirement that the bars remain of the same length.

The creep deformation of the material is assumed to follow a Norton law, $\dot{\varepsilon}^c = A\sigma^n$. In obvious notation the equations defining the problem are;

$$Ew(t_1\varepsilon_1^e + t_2\varepsilon_2^e) = F$$

$$\alpha\Delta T + \varepsilon_1^e + \varepsilon_1^c = \varepsilon_2^e + \varepsilon_2^c$$

$$\dot{\varepsilon}_1^c = \tilde{A}\left(\varepsilon_1^e\right)^n$$

$$\dot{\varepsilon}_2^c = \tilde{A}\left(\varepsilon_2^e\right)^n$$

$$\tilde{A} = AE^n$$

These are simply solved numerically by stepping forward incrementally in time, updating the creep strains using the strain rate equations and then solving for the elastic strains from the first pair of equations.

For our illustration we have used the following specific data,

F = 101 kN	w = 1 mm	E = 160 MPa
t_1 = 1000 mm	A = 10^{-23} $hr^{-1}MPa^{-n}$	
t_2 = 10 mm	n = 8.2	

which give an initial primary tensile stress in bar No.2 of 100 MPa. Four levels of secondary loading were considered, defined by,

αΔT	Initial Secondary Stress,	$\xi = \left(\frac{\sigma_P}{\sigma_P + \sigma_S}\right)^2$	Z = 1 / (1 - ξ)
0.2 x 10^{-3}	31.7	0.58	2.36
0.3 x 10^{-3}	47.5	0.46	1.85
0.5 x 10^{-3}	79.2	0.31	1.45
1.0 x 10^{-3}	158.4	0.15	1.18

The results are shown in Figures 1, 2 and 3. Fig.1 shows the relaxation of stress against total strain (elastic plus creep). Fig.2 shows the relaxation of stress against time. Both graphs show full relaxation of the secondary stress, i.e. the total stress reduces to just the primary stress of 100 MPa. The effective follow-up factor increases almost linearly with time (Fig.2), starting from a low value close to unity and increasing to a value close to that given by the simple estimation formula, $Z_{99\%} = \frac{1}{1-\xi}$, when the secondary stress is virtually fully relaxed. This is illustrated by Fig.3 which plots the effective Z against stress, with time increasing to the left. On the Z-axis of Fig.3 the values given by the simple formula are plotted as crosses for comparison.

It can be seen from Figs.1, 2 or 3 that for longer times Z will continue to increase beyond these values (inevitably). Nevertheless, the simple formula is conservative up to the time when the secondary stress is 98.7% relaxed. This can be seen in more detail from the following Table which gives the follow-up factor, Z, for a range of different relaxed stresses closely approaching the primary stress,

Relaxed Total Stress, MPa	**Initial Secondary Stress**			
	31.7 MPa	**47.5 MPa**	**79.2 MPa**	**158.4 MPa**
102	1.80	1.55	1.34	1.17
101	2.04	1.71	1.44	
100.6	2.21	1.83		
100.4	2.36			
Simple Formula Result	**2.36**	**1.85**	**1.45**	**1.18**

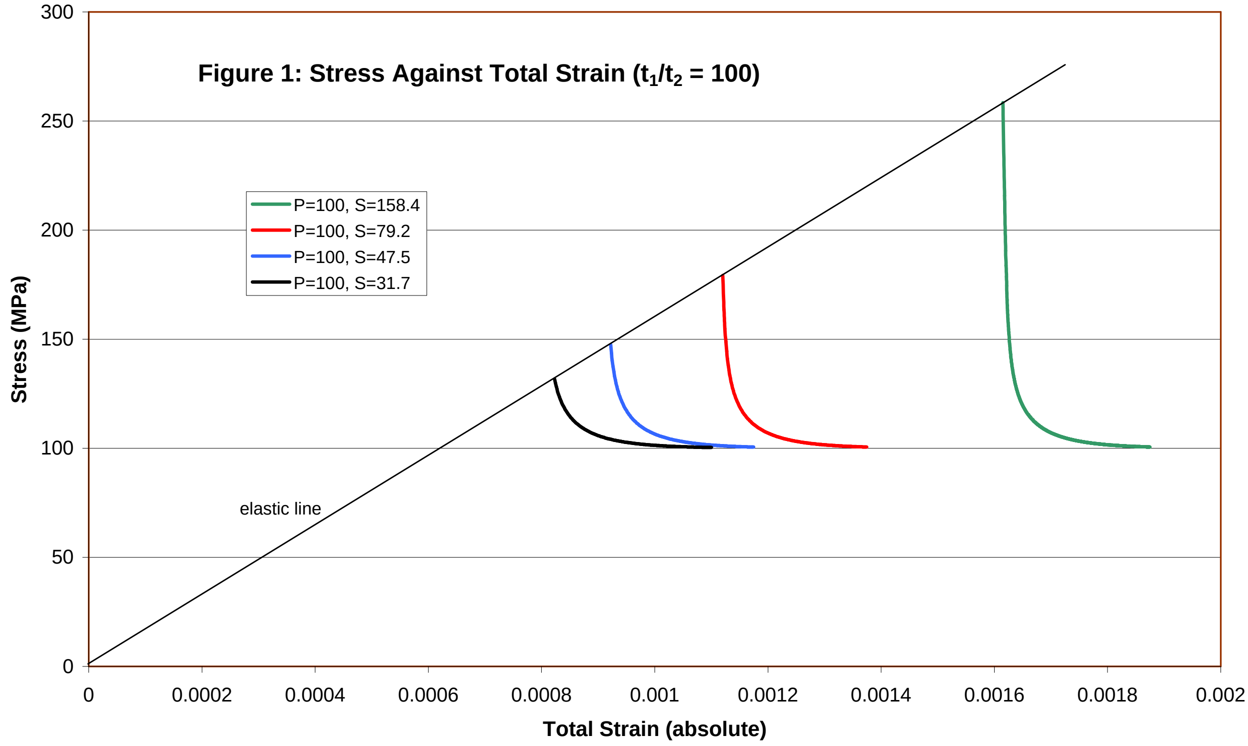

Figure 1: Stress Against Total Strain (t_1/t_2 = 100)

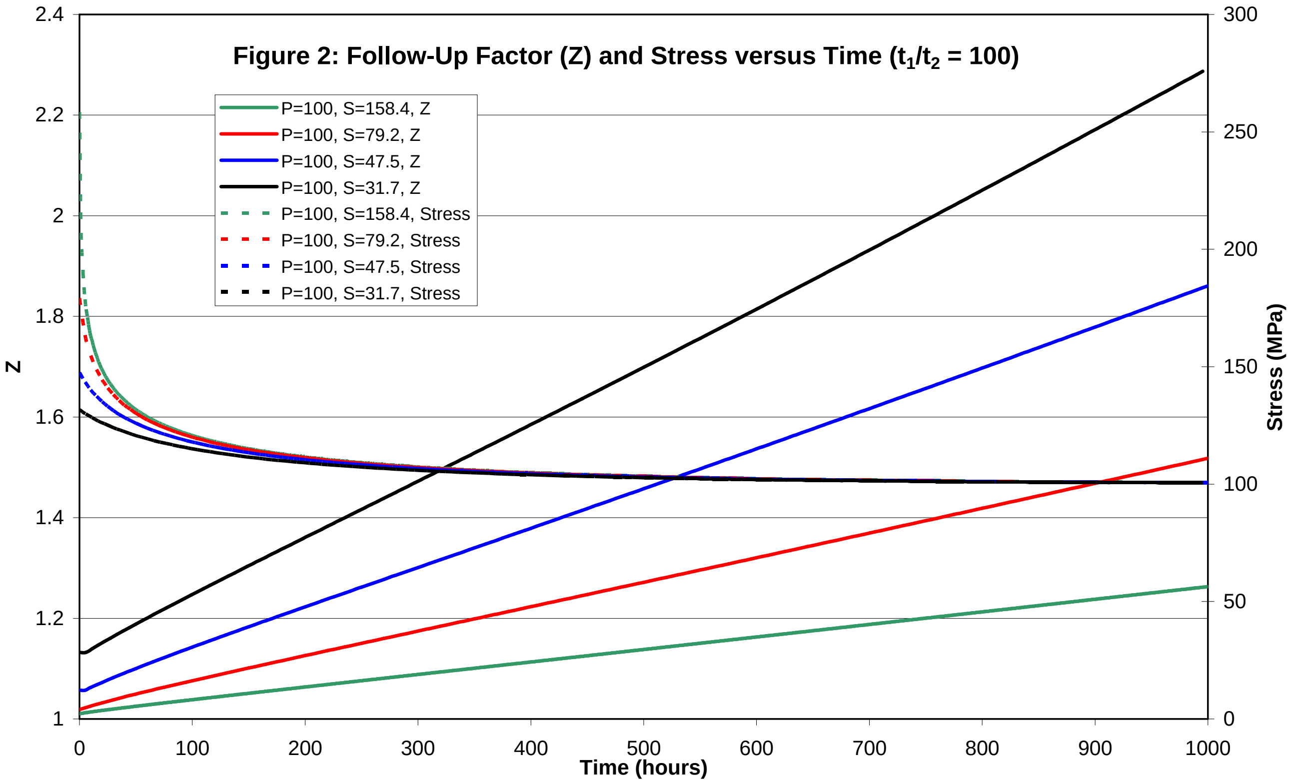

Figure 2: Follow-Up Factor (Z) and Stress versus Time (t_1/t_2 = 100)

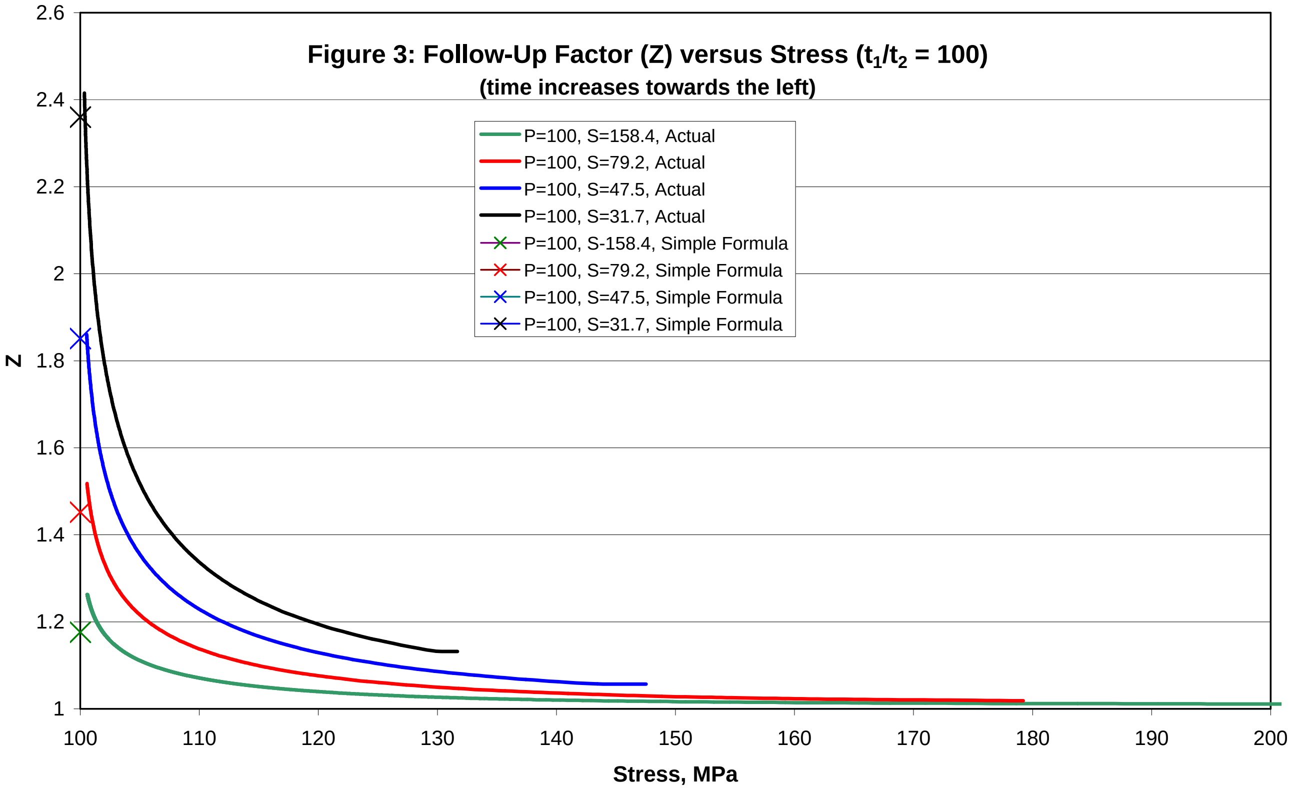

Figure 3: Follow-Up Factor (Z) versus Stress (t_1/t_2 = 100)
(time increases towards the left)
P=100, S=158.4, Actual
P=100, S=79.2, Actual
P=100, S=47.5, Actual
P=100, S=31.7, Actual
P=100, S-158.4, Simple Formula
P=100, S=79.2, Simple Formula
P=100, S=47.5, Simple Formula
P=100, S=31.7, Simple Formula
Z
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
100
110
120
130
140
150
160
170
180
190
200
Stress, MPa