Why Is There A Fatigue Crack Growth Threshold ΔK_{th}?

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When fatigue crack growth is governed by a Paris law, $\frac{da}{dN} = C\Delta K^m$, this applies[1] only above a certain threshold, $\Delta K > \Delta K_{th}$. Below this threshold, $\Delta K < \Delta K_{th}$, fatigue crack growth does not occur. This is particularly important in high-cycle vibration fatigue. For example, vibration at 100Hz will accumulate 3×10^9 cycles per year. Even the slightest amount of growth per cycle would rapidly lead to failure. Engineering structures, which all contain flaws at some level, survive vibration because of the fatigue crack growth threshold. They survive only because crack growth per cycle is exactly zero below this threshold. But why should this be?

The answer is, "because materials are made of atoms/molecules". There is a minimum quantum of growth. A crack cannot advance by less than one atomic spacing (or, at least, a dimension of roughly that size). Suppose our structure were experiencing load cycles leading to a crack advance of just one atomic spacing per cycle. This is the smallest non-zero growth rate possible. Atoms are roughly a couple of Angstroms (2×10^{-10} m) in size. For our example, vibration at 100Hz, the growth in one year would thus be $3 \times 10^9 \times 2 \times 10^{-10}$m = 0.6m = 600mm. For most mechanical engineering structures, crack growth at this rate would lead to failure in a few weeks or months at most. If this is true at the smallest non-zero fatigue crack growth rate, it follows that engineering structures survive vibration only because the growth per cycle is actually zero at the quantum level.

The fatigue crack growth threshold, ΔK_{th}, is therefore the stress intensity factor range at which the growth rate drops to exactly zero. For a SIF range just very slightly greater than the threshold, the growth rate must be one atomic spacing per cycle. (Because otherwise the growth rate would have to be exactly zero, which, by *reductio ad absurdum*, conflicts with ΔK_{th} being the threshold).

Now growth rate laws like $\frac{da}{dN} = C\Delta K^m$ will not be accurate close to the threshold (see footnote). However, we can get a rough estimate for the size of ΔK_{th} by assuming the Paris law does apply and setting the growth per cycle to two Angstroms (2×10^{-10} m). Lower bound thresholds derived in this way, based on upper bound fatigue growth laws from R66 Section 10 (units: powers of MPa√m), are:-

Material	**C**	**m**	$\mathbf{\Delta K_{th}}$
CMn	1.5×10^{-11}	3	2.4
CMn (ST)	4×10^{-12}	4	2.7
CMV	4.1×10^{-11}	3.5	1.6
316 parent	1.4×10^{-10}	3	1.1
316 weld	9.1×10^{-10}	3	0.6

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[1] There are, of course, formulations like $\frac{da}{dN} = C(\Delta K - \Delta K_{th})^m$, applicable in some limited range like $\Delta K_{th} < \Delta K < 2\Delta K_{th}$ which avoid a discontinuous start of growth at $\Delta K = \Delta K_{th}$. But growth is still exactly zero for $\Delta K < \Delta K_{th}$.

Gratifyingly, these estimates of ΔK_{th} are pretty good, though they should never be claimed in any safety critical arguments! Threshold data from fatigue tests generally support $\Delta K_{th} \geq 2MPa\sqrt{m}$, with thresholds as low as 2 MPa√m tending to apply only where there is a fairly large mean stress (see R66 Section 10). This supports the contention that the fatigue crack growth threshold has a quantum (atomic) origin.

R66 notes that in corrosive environments the effective ΔK_{th} can be much lower, values as low as 0.1 MPa√m, or even zero, being recommended in some circumstances. R66 is rightly cautious. Corrosion-fatigue can be a virulent mechanism for rapid crack growth. However, how can such low thresholds be possible? The above argument purports to show that such low thresholds are impossible.

I do not know the answer for sure. But the likely explanation is that these very low values of ΔK_{th} are not true fatigue thresholds. Corrosion fatigue tests will involve both load cycling and also dwells in the corrosive environment. The duration of the dwells will affect the amount of crack growth observed. To isolate the purely cyclic contribution to the growth, as opposed from corrosion, the limit of continuous cycling would be required. It is not clear that the tests will have been interpreted in this way. Moreover, corrosion will probably drive crack growth even without any cycling. In other words, there will be growth even for $\Delta K = 0$, so clearly there is no threshold in this sense. But the mechanism is then purely corrosion, not fatigue. The resolution of the paradox therefore lies in the interpretation of the tests rather than anything fundamental (probably).

A couple of further notes of caution are needed lest the reader be left with the wrong impression. Firstly, the threshold ΔK_{th} is quite strongly dependent upon mean stress (or mean SIF). This is analogous to the effect of mean stress in reducing fatigue endurance (for example as in the Goodman diagram, where the reduction in fatigue endurance is proportional to the mean stress as a fraction of the UTS). However, this is not in conflict with the above arguments. The minimum ΔK_{th} values derived above are broadly consistent with the minimum values pertaining to high mean stress levels.

The other issue which the reader should be aware of is that the naïve depiction of vibration loading given above, as a ‘pure tone’ of fixed frequency and constant amplitude, is not at all typical of real structural vibration. Whilst vibration may often be dominated by a particular frequency, or a small number of discrete frequencies, the amplitude will rarely be constant. More typically, vibration consists of narrow band random excitation in which the amplitude varies randomly. Typically a Rayleigh distribution may apply. Unless the structure in question is in desperate trouble, the usual expectation is that the vast majority of the vibration cycles will lie below threshold. Only a small number of occasional cycles will be above threshold and contribute to growth. An analogy is with coastal erosion, where normal waves do very little and virtually all the damage is done by storm surges. For more on vibration fatigue consult R2.