Tidal Energy

Data

Earth mass, M = 5.977 x 10^{24} kg G = 6.67 x 10^{-11}
Earth radius, R = 6,371 km
Moon mass, m = 7.35 x 10^{22} kg
Radius of moon's orbit around Earth, s = 3.84 x 10^{8} m

Earth's rotation is slowing down at a rate 2.3 millisec per day per century.
The Moon's orbital radius is increasing by 4m per century.
The height of the lunar tide is 0.54m (solar tide adds 0.25m). Conducive land formations can increase local tidal heights to 5m or more, but 0.54m is the free-ocean peak height.

Energy in Earth's Rotation

Moment of inertia, I = $0.4MR^2$ = 9.7 x 10^{37} kg.m^2
Angular speed, $\omega = 2\pi / (24 \times 3600)$ = 7.27 x 10^{-5} /sec
Kinetic energy of rotation = ½ $I\omega^2$ = **2.57 x 10^{29} Joules**

Human Energy Consumption

Peak power use per person in the UK is ~1kW. Average use is about 1/3 kW. If everyone on the planet were as profligate as in the West the total power usage would be 6 billion times 1/3 kW = **2 x 10^{12} J/sec**. So the Earth-flywheel would keep us going for 4 billion years.

Energy in the Tides

Assume each of the two the tidal bulges occupies about ¼ of the Earth's surface, i.e. a surface area of πR^2 = 1.275 x 10^{14} m^2 each. Approximate the average height of water in the bulge (above mean sea level) to be half of the peak, ½ x 0.54 = 0.27 m. The volume of water in the two bulges together is thus 6.88 x 10^{13} m^3, or a mass of 6.88 x 10^{16} kg. Again approximating the mean height of this volume of water to be 0.27m gives a potential energy (mgh) of **1.82 x 10^{17} Joules**. This is available daily, so if it were all used for human purposes, with 100% efficiency, the available power would be **2.1 x 10^{12}J/sec**.

Remarkably, this is only just enough power to supply all human consumption – and capturing ***all*** this energy is obviously grossly unrealistic.

Rate of Loss of Earth's Rotational Energy

$\delta(\text{K.E.}) = I\omega \cdot \delta\omega$, $\omega = \frac{2\pi}{T}$ so $\delta\omega = \frac{2\pi}{T^2}\delta T$ where T = 24 hours, and δT = 2.3 millisec per century. So δ(K.E.) = 1.365 x 10^{22} Joules/century = **4.33 x 10^{12} Joules/sec**.

Hence, it is *possible* that the energy in the tides could be fully dissipated daily, since the Earth is slowing down at a rate which can support this energy loss rate, i.e. the tidal loss accounts for about half the total loss of the Earth's rotational energy. [The other half is presumably lost in driving the weather, including losses from the wind to the sea in raising waves, and possibly also losses to tide-like cyclic strains in the solid structures of the Earth].

Is There Enough Energy Left To Lift The Moon?

$$\frac{mv^2}{s} = \frac{GMm}{s^2} \quad \text{so} \quad K.E. = \frac{1}{2}mv^2 = \frac{1}{2}\cdot\frac{GMm}{s} = -\frac{1}{2}P.E.$$

So the Moon's total energy = K.E. + P.E. = ½ P.E. = -K.E. = $-\frac{1}{2}\cdot\frac{GMm}{s}$

If the Moon's orbit increases from s to s + δs then the Moon's energy increases (decreases in magnitude) by $\delta E = \frac{1}{2}\cdot\frac{GMm}{s^2}\cdot\delta s$. For δs = 4m (in a century) the Moon's energy increases by 3.97 x 10^{20} Joules per century = **<u>1.26 x 10^{11} Joules/sec</u>**. This is only ~3% of the rate at which the Earth is loosing rotational energy, so there is plenty of energy left over to lift the Moon.

Solar Power?

Assuming about ¼ of the Earth's surface is illuminated at any given time with a flux of $1kW/m^2$, the total radiant power is of order 10^{14} kW = **<u>10^{17} Joules/sec</u>**. This is 50,000 times larger than the available tidal power, but it is even more ridiculous to suppose it could all be captured, requiring as it does that 100% of the Earth's surface be covered with solar panels.

0.002% of the Earth's surface (63 x 63 miles) would need to be covered with solar panels to supply all the power needs of a planet with global energy greed comparable with the UK at present.

RAWB, 13/7/07.