

T73S04 (R5V2/3) – Session 38A: Ratcheting
Relates to Knowledge & Skills items 1.47, 1.48
Last Update 4/6/2015

What are the two types of ratcheting? Material ratcheting versus structural ratcheting; How are the two types of ratcheting taken into account in R5V2/3?; Demonstration of avoidance of ratcheting beyond the R5 global shakedown criterion; Use of cyclic FEA with plastic-creep models (FRSV, LeMaitre-Chaboche, etc); Relevance of cyclic constitutive relations; A simple four-bar model of the structural ratchet mechanism; Existing exact ratcheting solutions (Bree, etc.); Acceptable ratcheting limits

Is it “ratcheting” or “ratchetting”? I have opted for the former but opinions differ
This session addresses only elastic-plastic cycling, with no creep.

Qu.: Revision – What are shakedown and ratcheting?

- If, after some initial plasticity and after the first few cycles, the structure cycles elastically it is said to have achieved Strict Shakedown.
- If a structure accumulates successively higher levels of plastic strain on every load cycle it is said to be “ratcheting”. The increment of strain between corresponding points on successive cycles is the ratchet strain per cycle.
- Between these two cases is the case of stable elastic-plastic cycling, in which some parts of the section undergo plastic hysteresis cycling but with zero ratchet strain.

When a structure is subject to stable hysteresis cycles (elastic or elastic-plastic) the methods of R5V2/3 can be used to assess creep-fatigue crack initiation.

One possible test of whether cycling is stable is provided by the R5 definition of Global Shakedown (which, roughly speaking, permits only 80% of the section to be cycling elastically).

Qu.: What if a structure fails the R5 global shakedown criterion?

Your best attempt to find a residual stress field which puts $\geq 80\%$ of the section within $\bar{\sigma} < K_S S_y$ at all times may not succeed. Does the structure actually really ratchet? Not necessarily.

Qu.: What’s the purpose of this session?

The precise boundary between stable hysteresis cycling and ratcheting is tricky to calculate. The R5 Global Shakedown rule is a sufficient but not a necessary criterion, i.e., it is safe but may be overly restrictive. Moreover, even if the structure is subject to limited ratcheting, might it still be fit for purpose? The objectives of this session are,

- (i) To clarify what causes ratcheting;
- (ii) To discuss more sophisticated ways of investigating whether a structure actually ratchets or not;
- (iii) If ratcheting does occur, to discuss ways of calculating the ratchet strain and assessing acceptability.

Qu.: Can (ii) and (iii) be done analytically?

In most cases one hopes that the shakedown criteria of R5V2/3 or a design code will be sufficient to convince the assessor that ratcheting does not happen. Bear in mind that ratcheting is a rather extreme condition in practice, so most well designed structures are likely to have a clear and easily demonstrable shakedown margin.

Exact, closed form, algebraic solutions of ratcheting problems can be found in only a few cases involving a very simple geometry and idealised loading. Some of the exact solutions are summarised below.

However, in most cases of practical interest, and where the simple R5/design code methods have failed to show a margin, ratcheting must be addressed by finite element analysis. So this session makes some observations regarding what FE facilities are available.

Qu: What are the two types of ratcheting?

- 1) Material ratcheting, and,
- 2) Structural ratcheting

In both cases the signature (or definition) of ratcheting is the persistent movement of the hysteresis cycle to higher strains, cycle on cycle.

However, there are two completely different mechanisms by which this can occur: due to the material constitutive behaviour or due to a structural mechanism.

Qu.: What is “material ratcheting”?

Ratcheting can occur in a purely uniaxial, load-controlled test under uniform (membrane) cyclic stressing. This is “material ratcheting” because, under these conditions, there is no structural mechanism which could cause the ratcheting.

- At a given temperature (held constant during the load cycling), and,
- for a given R ratio (minimum:maximum stress ratio),

material ratcheting will occur only above a certain maximum stress.

Qu.: What is “structural ratcheting”?

If a structure exhibits ratcheting at a maximum stress below the material ratchet limit (for the relevant temperature and R-ratio) then this must be structural ratcheting.

Structural ratcheting can occur under uniaxial loading. The basic mechanism is as follows,

- (i) At one point of the load cycle (say, at the “top” of the hysteresis cycle) a certain fraction (f) of the section undergoes positive plastic straining, whilst the rest of the section ($1 - f$) remains elastic or in plastic compression;
- (ii) At another point in the load cycle (say, at the “bottom” of the hysteresis cycle) the whole of the previously elastic region, together with any previously compressive plastic regions, ($1 - f$), undergoes positive plastic straining.

Over the whole cycle, therefore, the entire section increases its positive plastic strain, i.e., it ratchets. Roughly speaking, in step (i) the region f acquires plastic strain because it ‘pivots’ about the elastic region, $1 - f$. In step (ii) the reverse is the case.

This is analogous to the operation of a ratchet in which the elastic parts of the structure prevent the plastic strains reversing so as to close the hysteresis loop.

Structural ratcheting generally involves at least two different loads. Often one load is secondary and the other primary. Typically it is the large magnitude of the secondary load which is primarily responsible for causing the ratcheting – but some primary load is almost always necessary in practice, though there are subtleties...

Qu.: Can structural ratcheting occur with no primary load?

Some references (e.g., Ref.[1]) will say that structural ratcheting can occur due to a single cycling secondary load alone.

There is an interesting paradox about this. Assuming that we identify “secondary” with “strain controlled”, then structural ratcheting under just one such cycling load is clearly impossible. If the only loading is taking place between fixed applied strains (i.e., fixed applied displacements), then obviously there cannot be any build-up of unbounded ratchet strains.

However, usually the problem would be defined in terms of some cycling temperatures. This is not actually the same thing as specifying the total strain at the two ends of the cycle (i.e., the thermal strain is not the total strain).

However, there is a second paradox. If there is no primary load to give the ratcheting a sense of direction, why should the ratcheting be in the positive sense (tensile straining) rather than the negative sense (compressive straining)?

We will see below, using a simple four-bar model, how ratcheting *can* occur due to temperature cycling when the primary stress is arbitrarily small but non-zero (see Figure 10) but not if the primary stress is exactly zero (Figure 11). The latter case agrees with the classical Bree problem, which does not produce ratcheting in the limit of zero primary stress, however large the cyclic thermal loading (see Figure 12).

Qu.: Can material ratcheting and structural ratcheting happen together?

Yes.

Qu.: How is material ratcheting accounted for in R5V2/3?

Material ratcheting is taken into account in R5V2/3 via the shakedown factor, K_S .

Qu.: How is structural ratcheting avoided?

If a material is elastic-perfectly plastic with a yield stress σ_y , including under cyclic loading, then strict shakedown is assured (and hence ratcheting avoided) if $\bar{\sigma} < \sigma_y$ at all points of the structure and at all times during the load cycle, where $\bar{\sigma}$ is the actual, elastic-plastic, Mises stress. This is related to the elastic stress by the addition of a “shakedown residual stress field” generated by the cyclic plasticity, i.e.,

$\sigma_{ij} = \sigma_{ij}^{elastic} + \sigma_{ij}^{residual}$. If any self-equilibrated residual stress field, $\sigma_{ij,j}^{residual} = 0$, exists such that $\bar{\sigma} < \sigma_y$ at all points of the structure and at all times during the load cycle, then strict shakedown is assured.

Qu.: How is structural ratcheting avoided in R5V2/3?

To accommodate the possibility of stable elastic-plastic cycling beyond the strict shakedown limit, R5 weakens the “ $\bar{\sigma} < \sigma_y$ everywhere” criterion to the global shakedown 80% rule, i.e., “ $\bar{\sigma} < \sigma_y$ over at least 80% of every section”.

Qu.: How are material and structural ratcheting avoided in R5V2/3?

To accommodate the possibility of material ratcheting within the structural ratcheting criterion, the perfectly plastic yield strength, σ_y , is replaced with $K_S \sigma_y$. This finally gives us the R5V2/3 criterion (global shakedown) to avoid ratcheting:-

If at least 80% of every section of the structure has a Mises stress which is less than $K_S \sigma_y$ at all times during the load cycle, then the structure is said to be in “global shakedown” and this ensures the avoidance of ratcheting.

Qu.: Why is the 0.2% proof stress, S_y , used in $K_S \sigma_y$?

This is merely a matter of convention. Ratchet test data is analysed to provide the shakedown factor, K_S , assuming the 0.2% proof stress, S_y , as a normalising quantity.

Qu.: How is the shakedown factor, K_S , measured?

There is no testing standard for K_S .

An example of how K_S tests are carried out is Ref.[6] – for CMV up to 550°C. These are uniaxial tests under load control with $R = -0.9$. Structural ratcheting cannot occur in these specimens, loaded by membrane stressing alone and isothermal during cycling. The stress amplitude is gradually increased (with R fixed at -0.9) until ratcheting occurs.

How large a ratchet strain is large enough to be classed as ratcheting? In Ref.[6] a definition of 10^{-6} per cycle was used (i.e., a ratchet strain per cycle of 0.0001%).

Figure 1a shows the room temperature behaviour at a maximum stress of 320 MPa. This was found to be the stress at the onset of ratcheting. As can be seen from Figure 1a, the cycling is virtually stable for ~300 cycles, but becomes ratcheting by about cycle 400. Ratcheting did not occur at a maximum stress of 310 MPa, whereas Figure 1b shows how much more pronounced the ratcheting becomes at a maximum stress of 330 MPa. This illustrates the sensitivity of the ratcheting behaviour to stress.

Figures 2a,b are the equivalent experimental results at 150°C.

Normalising the onset of ratchet stress by the 0.2% proof stress gives K_S . The early draft results from Ref.[6] are shown as Figure 3. These results, in final form, were used as the basis of the advice on K_S for ferritics in the Revision 2 of R5V2/3 issue 3, 2014. It was little different from the earlier advice.

Figure 1a CMV Ratchet Test, 21°C, 320 MPa – from Ref.[6]

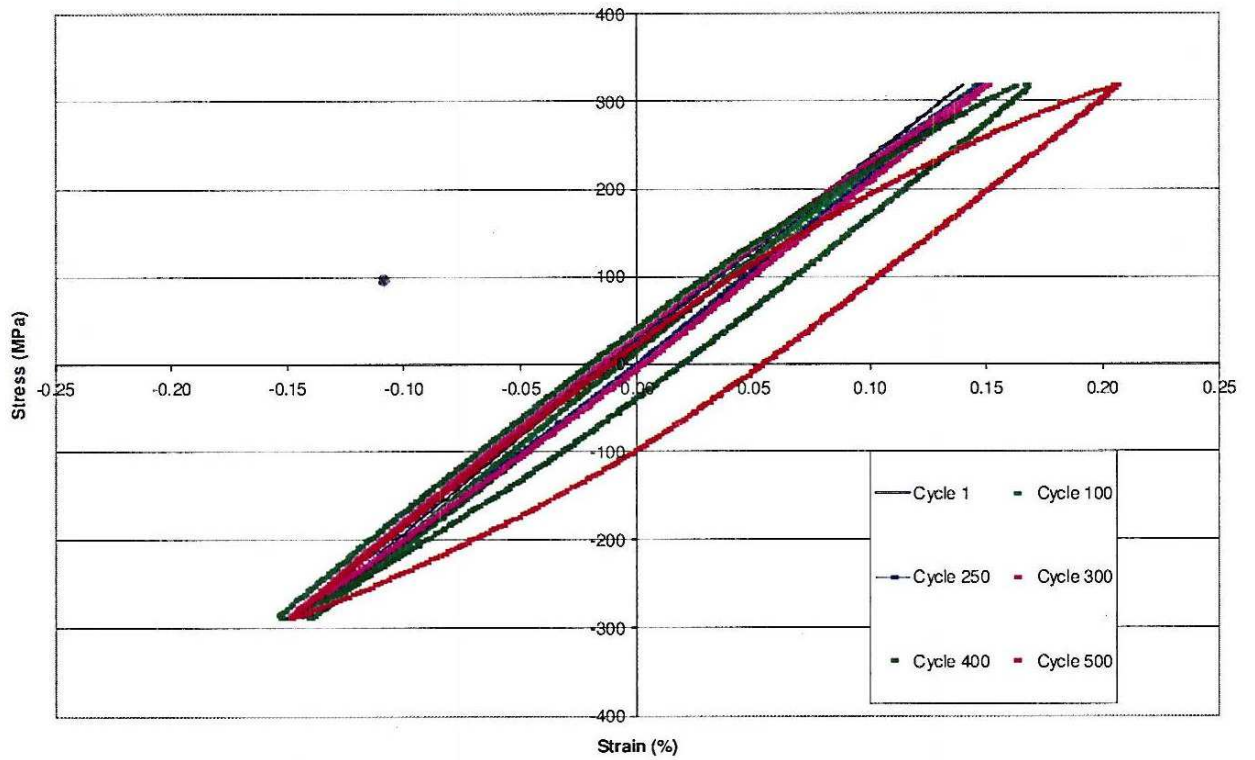


Figure 1b CMV Ratchet Test, 21°C, 330 MPa – from Ref.[6]

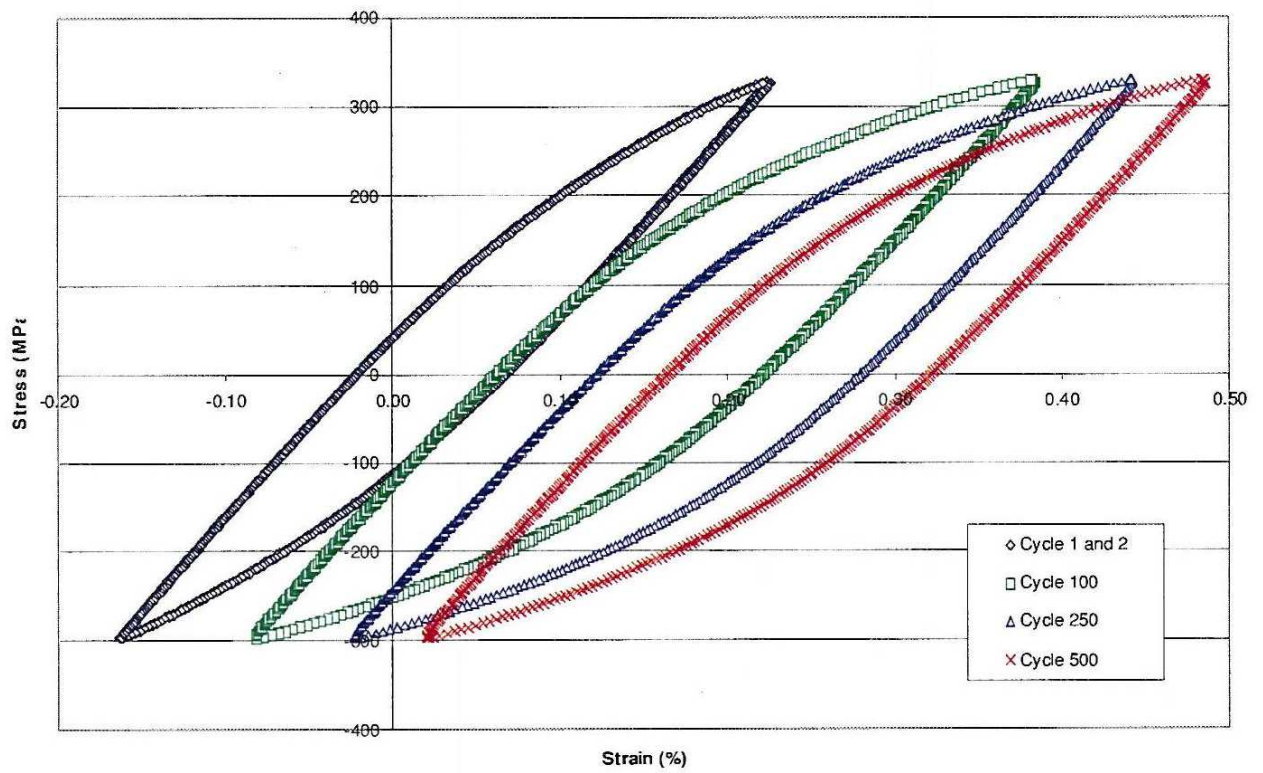


Figure 2a CMV Ratchet Test, 150°C, 280 MPa – from Ref.[6]

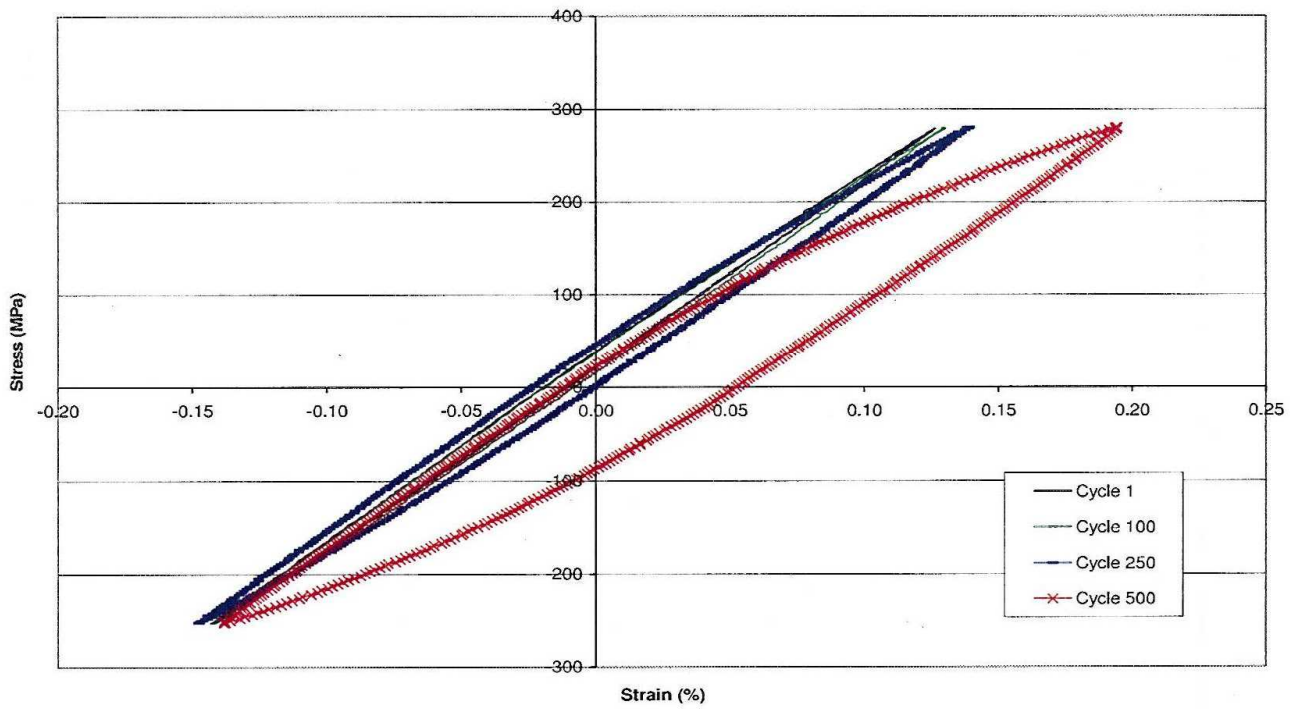
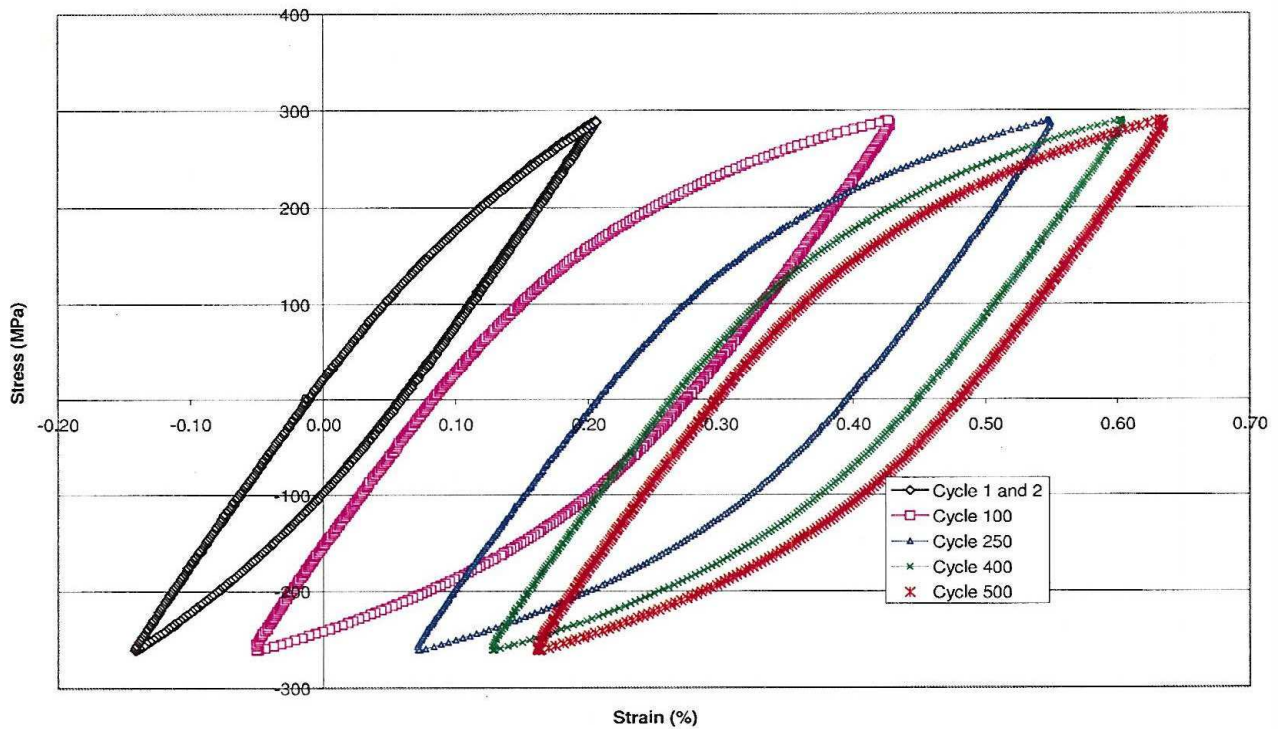


Figure 2b CMV Ratchet Test, 150°C, 290 MPa – from Ref.[6]



**Figure 3 Shakedown Factor, K_s , for CMV from Ref.[6]
DRAFT RESULTS ONLY slightly different from what ultimately
appeared in R5V2/3 Issue 3 Rev.002 Fig.5.2**

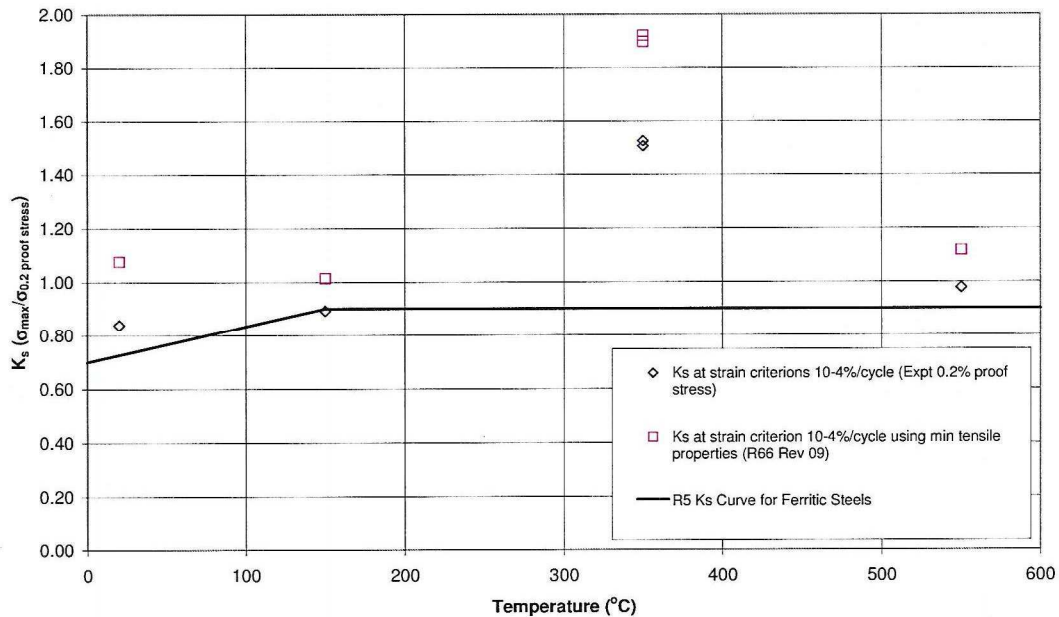


Figure 32: K_s factors for $\frac{1}{2}\text{Cr}\frac{1}{2}\text{Mo}\frac{1}{4}\text{V}$ steel.

Qu.: Help! I don't understand what "structural ratcheting" is

Consider the humble worm...

There is an analogy between ratcheting and the way in which worms and caterpillars and the like manage to move. They do so by cyclically expanding and contracting and exploiting friction.

A ratchet is something which permits forward motion but prevents backward motion. Hence the crucial feature of a ratchet is that it is non-linear (if it were linear it would be reversible).

In structural ratcheting the non-linearity is due to plasticity. No plasticity, no ratcheting.

In the worm analogy it is friction which plays the part of plasticity. No friction, no net movement.

A structural ratchet mechanism involves the global response of the structure. A worm makes net progress because, during one part of its cycle some parts of its body are anchored and the rest is pushed forward, whilst at other times in the cycle the reverse is true. The key is that the behaviour of one part of the structure (the bit that's currently moving/plastic) depends upon the anchorage provided by the rest of the structure (or the elastic parts). Net progress (ratchet strain) depends upon alternating the role of the two parts. This means that you cannot understand structural ratcheting from what is happening at a single point in the structure alone. It is a global mechanism.

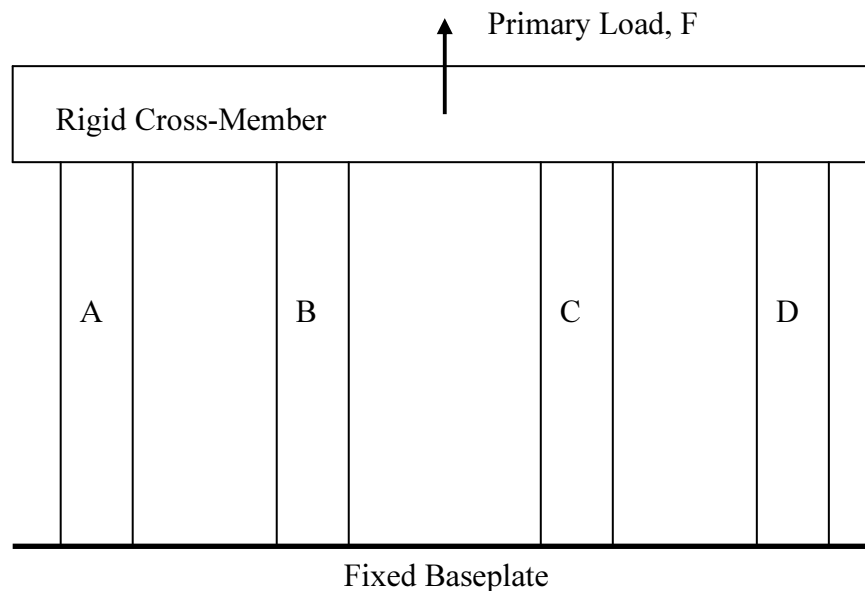
You cannot predict ratcheting by considering only a single point without consideration of what is happening in the rest of the section/structure.

Qu.: Is there a simple example of ratcheting?

I think I have managed to dream up a fairly simple example of ratcheting. Let me know if you can think of a simpler one. My model is...

The Four-Bar Model

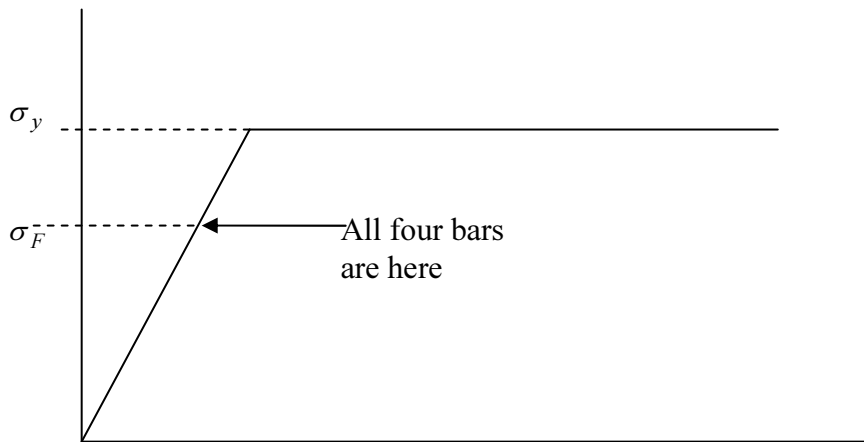
Figure 4



- All four bars are identical
- The bars are constrained to always have exactly the same length due to the rigid cross-member and the baseplate.
- Bars A and D are at an equal, constant temperature.
- Bars B and C are always at the same temperature as each other, but this temperature varies cyclically.
- If the bars were all elastic and at the same temperature, they would each carry one-quarter of the primary load, F .
- The bars are elastic-perfectly-plastic, with Young's modulus E .
- (NB: I have used four bars only so that their lengths are constrained to be the same, i.e., so that rotation of the cross-member cannot occur. It would have been the same to use just bars A and B and to stipulate that the cross-member could not rotate).

(1) Initially the bars are all at the same temperature. If the primary load does not cause yielding, the situation on a stress-strain plot is,

Figure 5

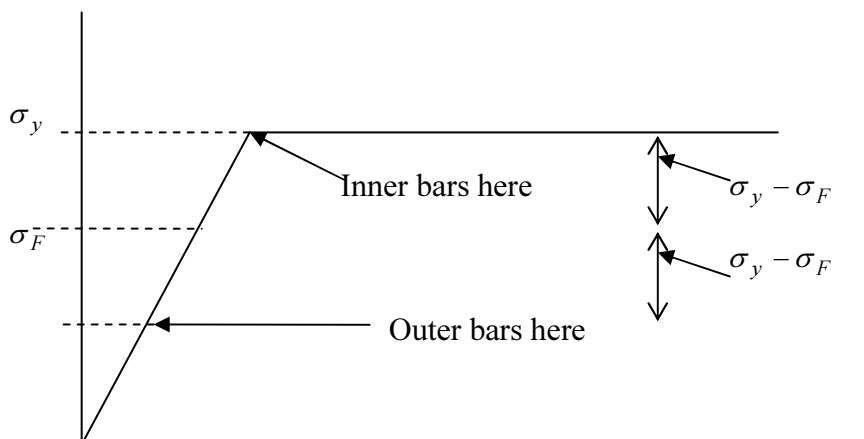


where $\sigma_F = \frac{F}{4A}$ is the primary stress in the bars, where A is the cross-section of each bar.

Note that in all the stress-strain graphs (and invariably in such plots in any references) the x-axis does not include the thermal strain, i.e., it is the elastic-plastic strain only.

(2) Now reduce the temperature of bars B and C. This increases the tension in bars B and C. But the average stress in the four bars is fixed at σ_F by equilibrium with the applied load. The behaviour of all bars remains elastic so long as the temperature decrease is $|\Delta T| \leq 2(\sigma_y - \sigma_F)/E\alpha$. At exactly $\Delta T = \Delta T_e = -2(\sigma_y - \sigma_F)/E\alpha$ the situation is,

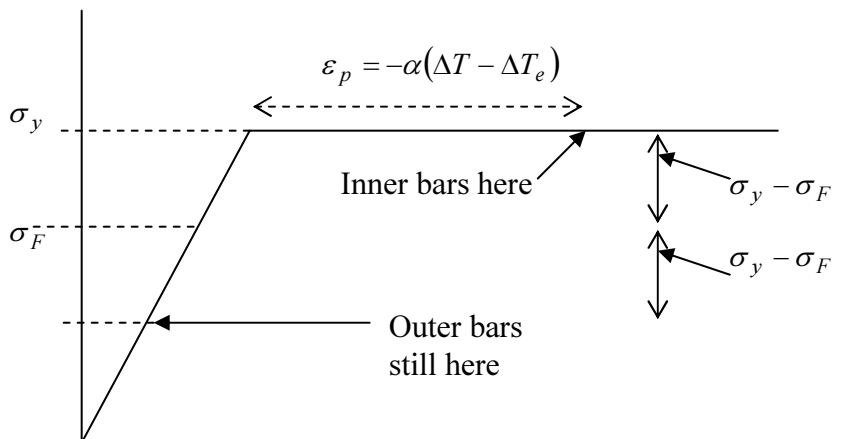
Figure 6



(3) Reduce the temperature of the inner bars further so as to cause plasticity.

Because we are assuming perfect plasticity, the stress carried by the inner bars cannot increase above σ_y . Consequently the stress in the outer bars cannot change either because this is fixed by equilibrium with the applied load. So, as the temperature of the inner bars is reduced further, all stresses remain unchanged. How is this possible? It can only mean that the *additional* thermal strain in the inner bars is exactly cancelled by the plastic strain: $\varepsilon_p = -\alpha(\Delta T - \Delta T_e)$. This means that the length of the inner bars does not change during yielding, and hence is compatible with the outer bars whose length also cannot change since their stress is constant and they remain elastic. The situation is now,

Figure 7



(4) Return inner bars to same temperature as outer bars. The inner bars now unload elastically, at least initially, and the outer bars pick up additional stress – initially elastically. There are three situations,

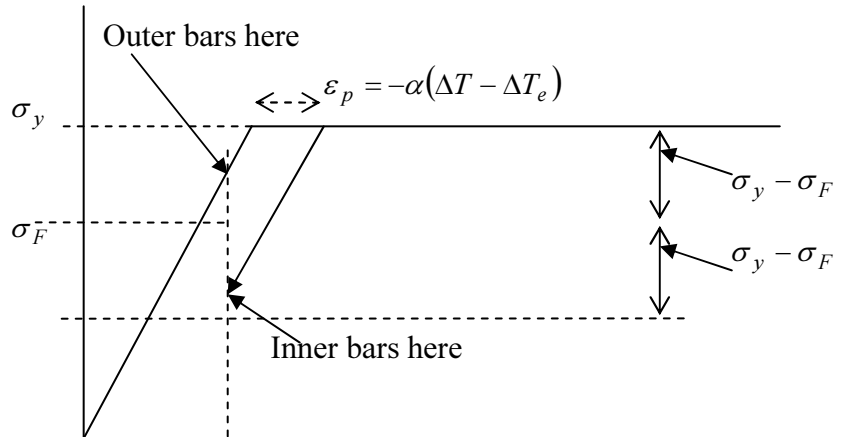
- Both bars behave elastically during this step;
- The inner bars behave elastically but the outer bars yield in tension;
- The outer bars yield in tension and the inner bars yield in compression. This can only happen if the primary load is zero, $\sigma_F = 0$.

These situations are considered in turn.

> (4a) Both bars behave elastically in Step (4)

At the end of step (4) the bars are at the same temperature and, as always, at the same length. Consequently they must have the same elastic-plastic strain. Moreover, the average of their stresses must be, as always, σ_F . This defines the final position as follows,

Figure 8



The final stresses are,

$$\sigma_{inner} = \sigma_y - \frac{E\alpha|\Delta T|}{2} \quad \sigma_{outer} = 2\sigma_F - \sigma_y + \frac{E\alpha\Delta T}{2}$$

Note that the average of these stresses is σ_F as it should be.

The inner bars have picked up a plastic strain of $\epsilon_p = \alpha|\Delta T| - 2(\sigma_y - \sigma_F)/E$ but there is no plastic strain in the outer bars.

The structure now contains residual stresses, compressive in the inner bars and tensile in the outer bars, of magnitude $\sigma_{res} = \pm[E\alpha|\Delta T|/2 - (\sigma_y - \sigma_F)]$.

The requirement that the inner bars yield but the outer bars do not means that this solution applies in the regime,

$$(\sigma_y - \sigma_F) \leq \frac{E\alpha|\Delta T|}{2} \equiv \sigma_{elastic}^{thermal} \leq 2(\sigma_y - \sigma_F)$$

Most importantly, because the final step is elastic, it is clear than if the cycle were repeated by reducing the temperature of the inner bars by ΔT again, we would simply revert to the position shown in Figure 7. Cycling is elastic from this point onwards.

Hence,

$$\frac{E\alpha|\Delta T|}{2} \equiv \sigma_{elastic}^{thermal} \leq 2(\sigma_y - \sigma_F) \text{ produces strict shakedown}$$

> (4b) Inner bars elastic but outer bars yield in step (4)

Figure 9

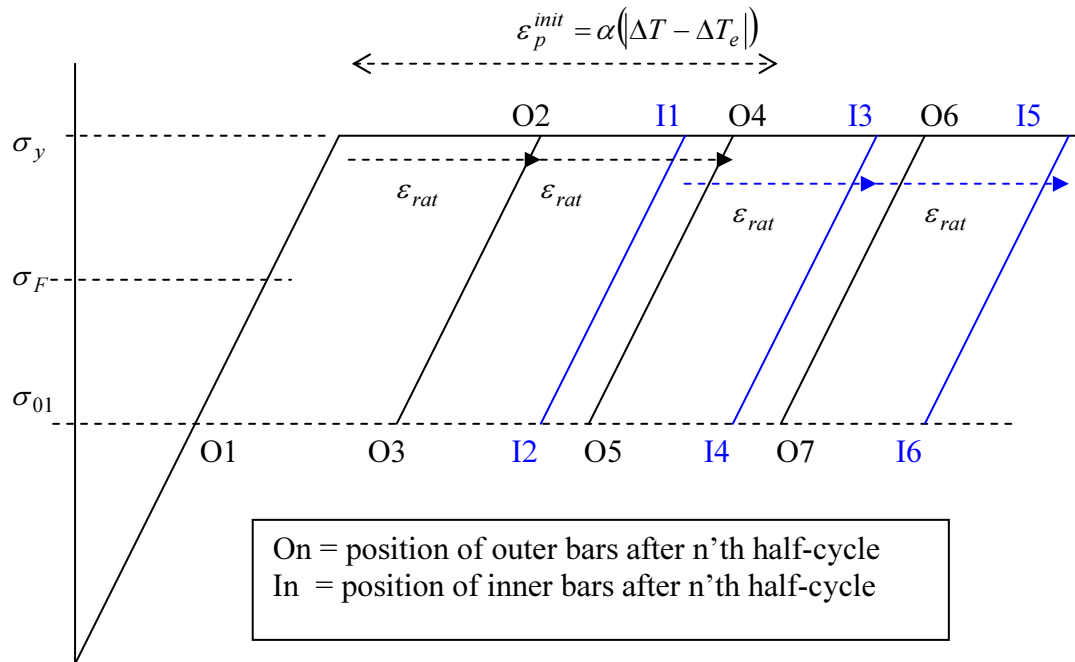


Figure 9 shows the positions of the inner and outer bars after each half-cycle. When the inner bars are first cooled down they are at positions O1 and I1 respectively. This is as per Figure 7.

When the inner bars are increased in temperature so as to be at the same temperature as the outer bars again, the positions shift to O2 and I2. This differs from Figure 8 because the outer bars have yielded. The inner bars stop unloading when they reach the stress $\sigma_{O1} = 2\sigma_F - \sigma_y$. This is because the requirement that the average of the inner and outer bar stresses be σ_F at all times means that the outer bars have reached the yield stress, σ_y , by that point. This occurs when the change in temperature wrt the end of the previous half-cycle is just $2|\Delta T_e| = \frac{4(\sigma_y - \sigma_F)}{E\alpha}$. This follows because ΔT_e is defined as the temperature change resulting in an elastic stress change of $\sigma_y - \sigma_F$. There is thus a further temperature change of $\Delta T - 2\Delta T_e$ still to be accommodated. This occurs at constant stress in all bars. The inner bars expand freely because the outer bars are now presenting no further resistance, i.e., they cannot be stressed beyond σ_y . The plastic strain developed in the outer bars must be compatible with the free thermal expansion of the inner bars, and hence equals $\varepsilon_{rat} = \alpha(|\Delta T - 2\Delta T_e|)$.

The elastic stress due to the inner bar temperature change of ΔT is

$\sigma_{elastic}^{thermal} = E\alpha|\Delta T|/2$. Hence the plastic strain in the outer bars after the first full cycle, i.e., at point O2, is,

$$\varepsilon_{rat} = 2\left[\sigma_{elastic}^{thermal} - 2(\sigma_y - \sigma_F)\right]/E$$

This is applicable only if $\sigma_{elastic}^{thermal} > 2(\sigma_y - \sigma_F)$.

(5) Reduce temperature of inner bars by ΔT again, then return to isothermal

The analysis is now simple because we see from Figure 9 that the inner bars at I2 are equivalent to where the outer bars were before the previous half-cycle, i.e., O1, and vice-versa, i.e., the outer bars at O2 are equivalent to where the inner bars were previously (I1). It follows that the outer bars will elastically unload to O3 whilst the inner bars initially elastically reload from I2 to I1 and then acquire additional plastic strain, of an amount equal to ϵ_{rat} , ending at position I3 as shown in Figure 9.

On increasing the temperature of the inner bars to equal the temperature of the outer bars again, the inner bars move from I3 to I4, and the outer bars from O3 to O4. It is again time for the outer bars to acquire a plastic strain increment of ϵ_{rat} .

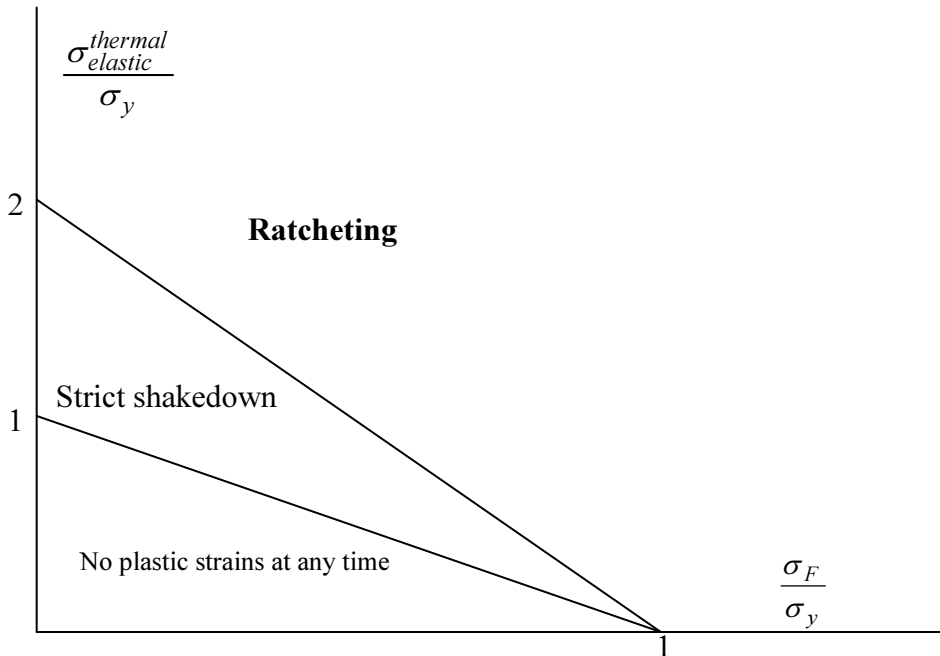
Thus every alternate half-cycle either the inner or the outer bars accumulate an additional plastic strain of ϵ_{rat} . Hence, both bars acquire this extra plastic strain every full cycle. Hence ϵ_{rat} is a ratchet strain (justifying the notation). In terms of the applied load the ratchet strain has therefore been calculated to be as given above, i.e.,

$\epsilon_{rat} = 2[\sigma_{elastic}^{thermal} - 2(\sigma_y - \sigma_F)] / E$. Ratcheting occurs only when this is positive and hence for,

$$\frac{\sigma_{elastic}^{thermal}}{\sigma_y} > 2 \left(1 - \frac{\sigma_F}{\sigma_y} \right)$$

Qu.: What is the Shakedown/Ratchet Diagram for the Four-Bar Model?

Figure 10



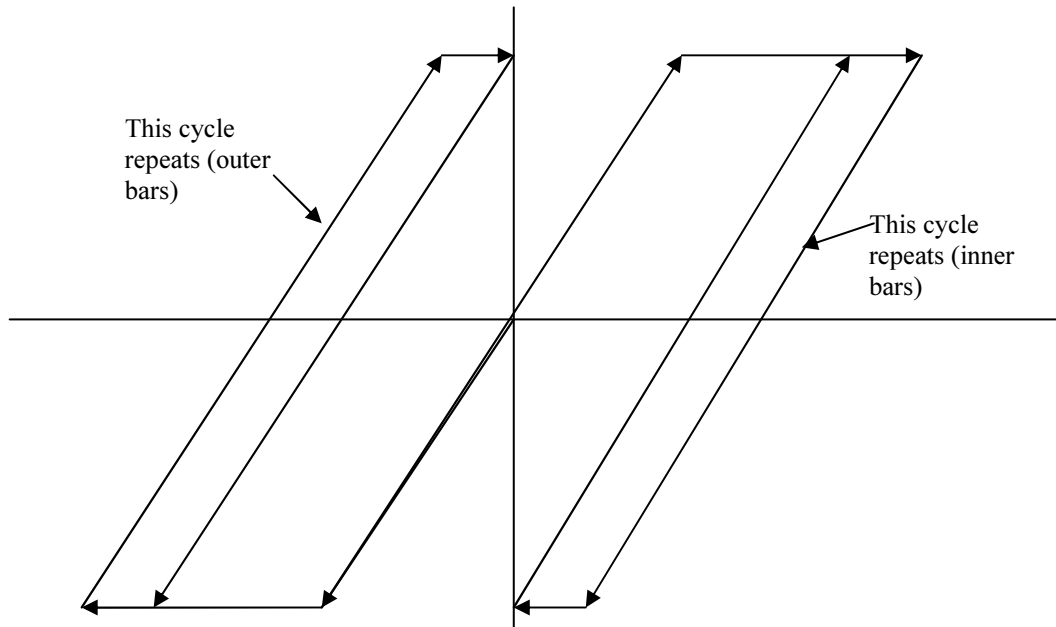
Unlike the Bree analysis, there is no region showing stable hysteresis cycle behaviour in the four bar model.

However...

Four Bar Model With No Primary Load

The case of no primary load at all is pathological, i.e., it does not conform to Figure 10. The stress-strain plot looks like this...

Figure 11



Consequently *in theory* there is no ratcheting if the primary stress is exactly zero. This is not surprising because it is the primary stress which determines the *direction* of the ratchet strain. However, if $\sigma_{elastic}^{thermal} > 2\sigma_y$ (so that Figure 11 applies) then there is a stable elastic-plastic hysteresis cycle (a potential creep-fatigue issue).

Consequently the stable elastic-plastic cycle region in the Bree diagram (P) has become squeezed into an infinitely narrow line at exactly $\sigma_F = 0$ in the four bar model.

BUT in practice the idealised situation represented by Figure 11 may not be realisable, even if primary loading is carefully excluded. This is because the material will, in reality, behave differently in tension and compression. This will tend to give the structure a 'sense of direction' which can lead to ratcheting even in the absence of primary loading – as is claimed in many references based on test work. So Figure 10 is probably a good guide – since this does indeed imply ratcheting even for vanishingly small primary load.

According to Figure 10, ratcheting can occur even for vanishingly small primary loading in some situations (e.g., the four-bar model). Tests imply that this is indeed true, i.e., ratcheting can occur for sufficiently severe thermal loading with no primary load.

Qu.: What other exact ratcheting solutions are available?

I won't attempt a definitive list. Some examples follow.

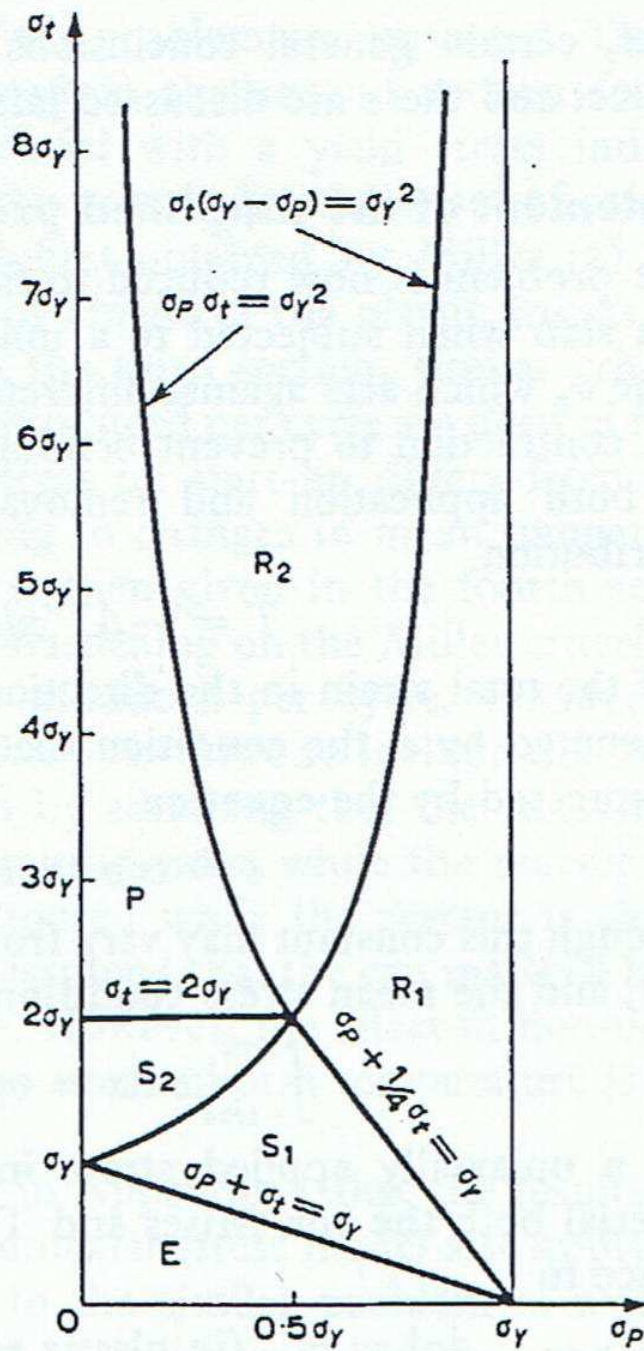
Qu.: What is Bree's Problem/Solution?

Without doubt the most important exact ratcheting solution is Bree's, Ref.[7]. The Bree problem is,

- Uniaxial stressing over a rectangular cross-section;
- Constant (i.e., non-cycling) primary membrane stress;
- Linear temperature distribution through-wall, ΔT cycling from zero to some maximum;
- Elastic-perfectly plastic material.

The Bree ratchet diagram is reproduced (again!) below, for comparison.

Figure 12 The Bree Diagram



Stress régime	Can behaviour
R_1 and R_2	Ratchetting
S_1 and S_2	Shakedown after first half-cycle
P	Plastic cycling
E	Elastic

Note that the four bar model corresponds to the point at which the regions S1, S2, P, R1 and R2 all meet being pushed over to the position (0, 2). The region P then disappears (or, more accurately, becomes an infinitely thin vertical line corresponding to the mechanism shown in Figure 11).

Qu.: What if the primary load cycles in the Bree problem?

In-Phase

The ratchet diagram for the case of a primary membrane load cycling in-phase with the secondary wall bending stress was derived in Ref.[8] and is reproduced as Figure 13 below, and compared with the original Bree problem in Figure 14. See Ref.[8] for the algebraic expressions for the ratchet strain. “In-phase” cycling means that the thermal and the primary load are at their maxima at the same time, and are also zero at the same time.

180° Out-of-Phase

If the primary membrane and thermal bending loads are cycling perfectly out-of-phase then the ratchet boundaries, the ratchet strains and the shakedown boundaries are identical to those for in-phase cycling. “180° out-of-phase” means that the thermal load is at its maximum when the primary load is zero and vice-versa.

Arbitrary Phase Difference

I have recently solved the Bree-type problem with the primary membrane load cycling with an arbitrary phase difference from the thermal bending load. For this purpose it is necessary to assume some specific form of temporal variation. My solution is for loads which vary as 'square waves', see Figure 15. This simplifies the problem in that each load is either on or off, but never of intermediate magnitude.

Referring to Fig.15, a phase difference is defined as $\phi = \frac{2\Delta\tau}{\tau_0}$ so that $\phi = 0$ is in-phase and

$\phi = 1$ is perfectly out-of-phase. It turns out that the ratchet boundaries and ratchet strains, and the shakedown boundaries, are the same for a phase of $(1 - \phi)$ as for a phase of ϕ . It turns out that $\phi = 0.5$ (i.e., 90-degrees out-of-phase) is the least onerous case, whilst the in-phase (or perfectly out-of-phase) cases are the most onerous. For all phases, the ratchet boundaries with cycling primary load are substantially higher than for Bree loading (constant primary) - though this is not true of the shakedown boundaries.

Curiously the ratchet boundaries are independent of phase for $X < 2/3$. Since this coincides with the usual design code limit on primary membrane stress, this is a rather interesting observation.

Figure 13: The Shakedown Diagram for Bree but with the Primary Load Cycling In-Phase with the Thermal Bending Load

- Region E never yields
- Regions S1, S2 produce strict shakedown to elastic cycling after the first half-cycle
- Regions P1, P2, P3 produce stable plastic cycling
- Regions R1, R2 produce ratcheting

The stress and strain plots corresponding to the various regions are as follows (for the odd and even half-cycles respectively),

S1 Fig.2a	R2 Fig.3a
R1 Fig.2b	P2 Fig.3b
P1 Fig.2c	P3 Fig.3c
	S2 Fig.3d

The equations for the various boundary lines are,

ak	$X + Y = 1$	cg	$1 - X = (Y - 1)(1 - 1/Y)/4$
hbg	$X + Y = 2$	hc	$Y(2 - X) = 4$
abcd	$Y(1 - X) = 1$	fc	$XY = 2$

Point c is $X = 0.667, Y = 3$

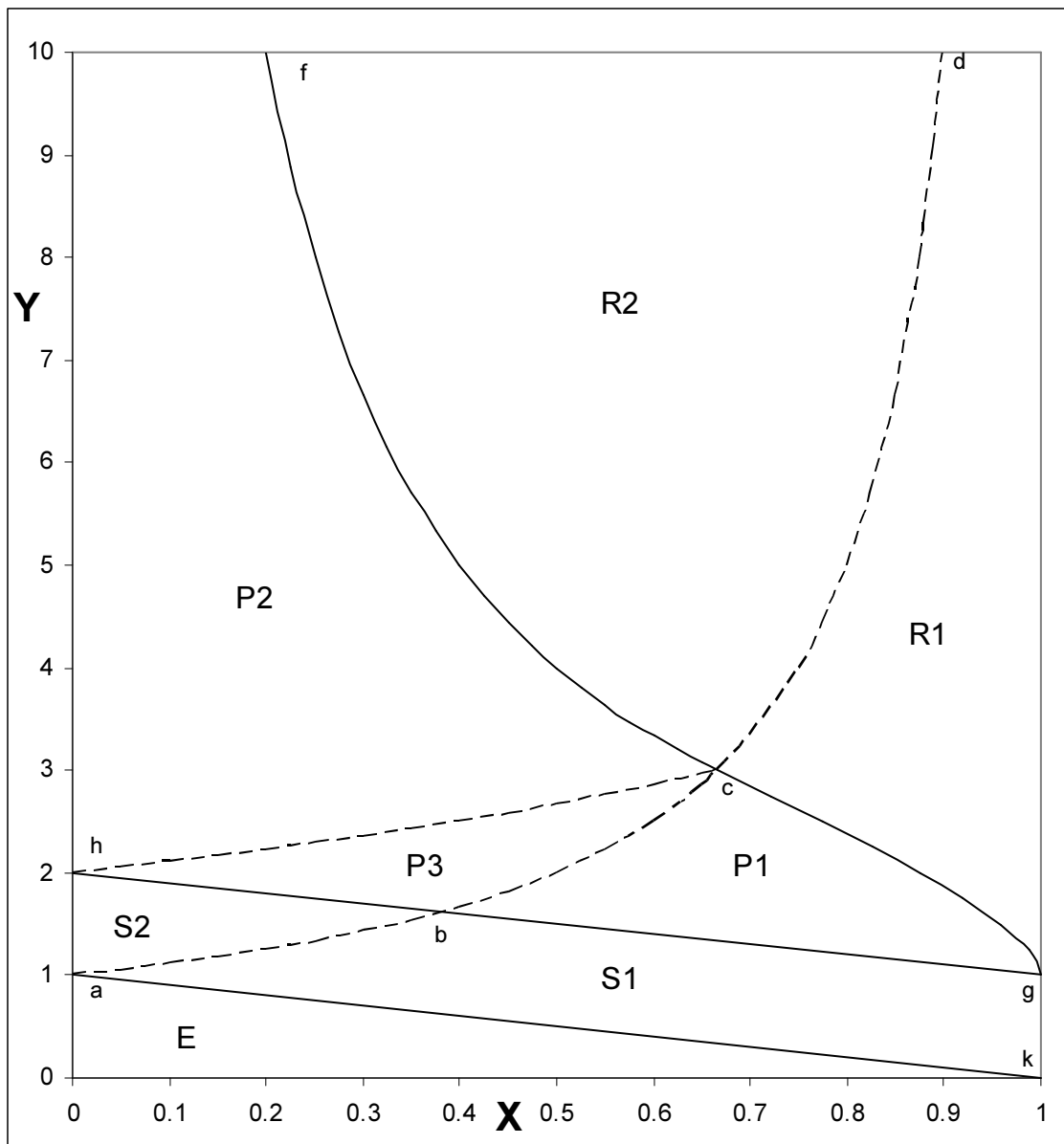


Figure 14 Shakedown and Ratcheting Regions for In-Phase Cycling Primary Load Compared with those of the Original Bree Problem

Black lines = in-phase cycling primary load

Red lines = Bree, Ref.[1]

For both problems, the upper region is ratcheting, the lower region is elastic or strict shakedown to elastic cycling, and the middle region is stable plastic cycling.

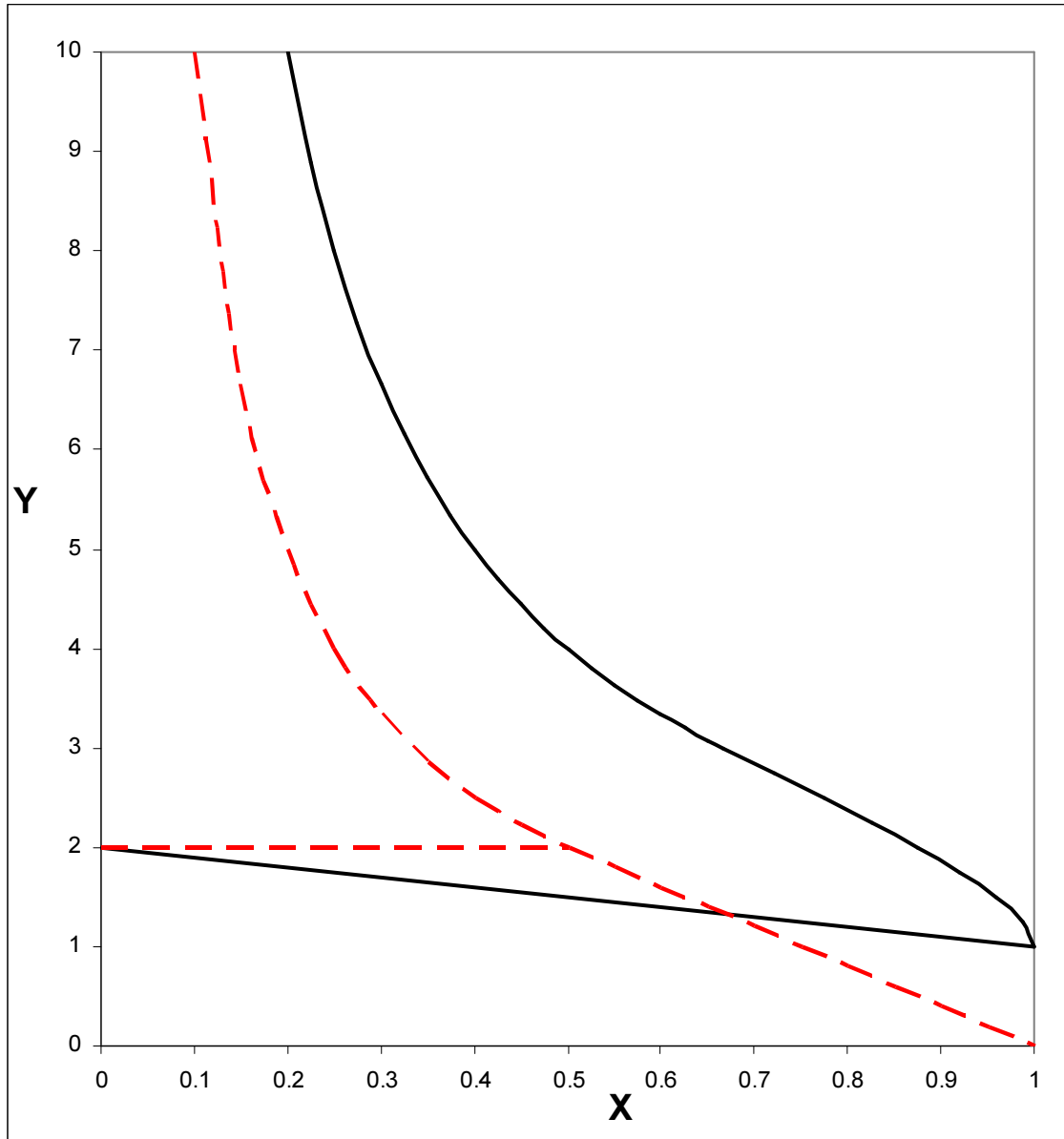


Figure 15: Bree-Type Problem with the Primary Membrane Load Cycling Out-of-Phase with the Thermal Load Cycles

Fig.15(a): Positive-Phase Load Sequence

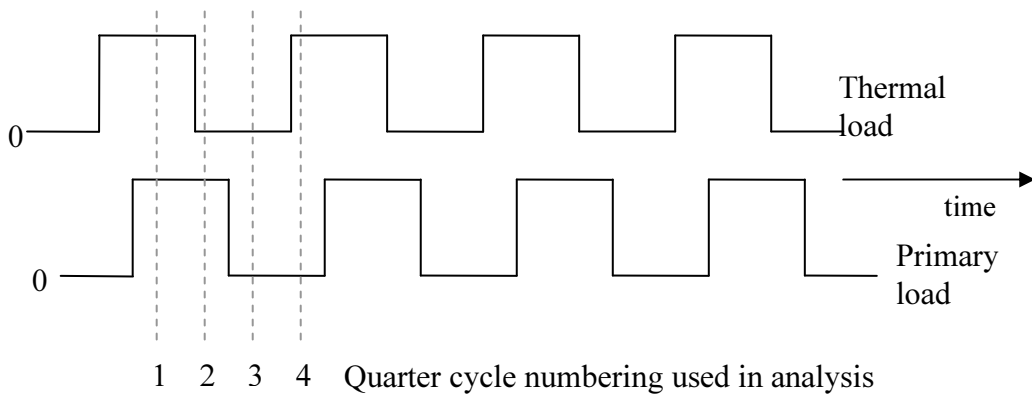


Fig.15(b): Negative-Phase Load Sequence

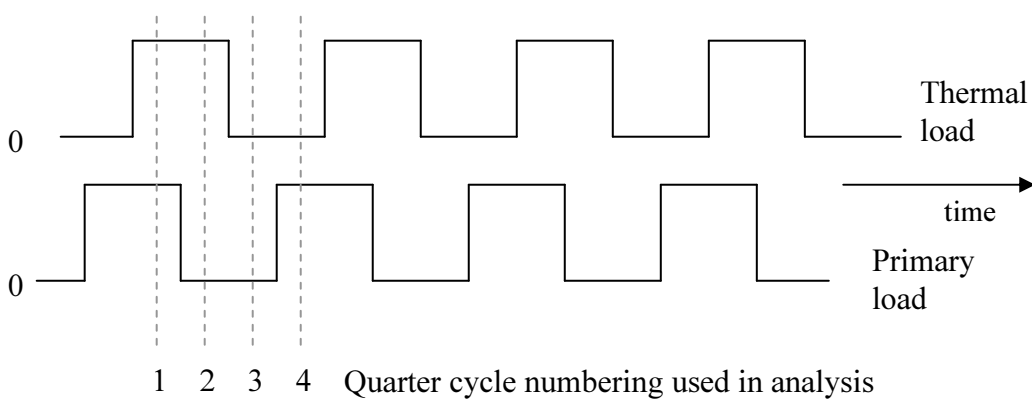
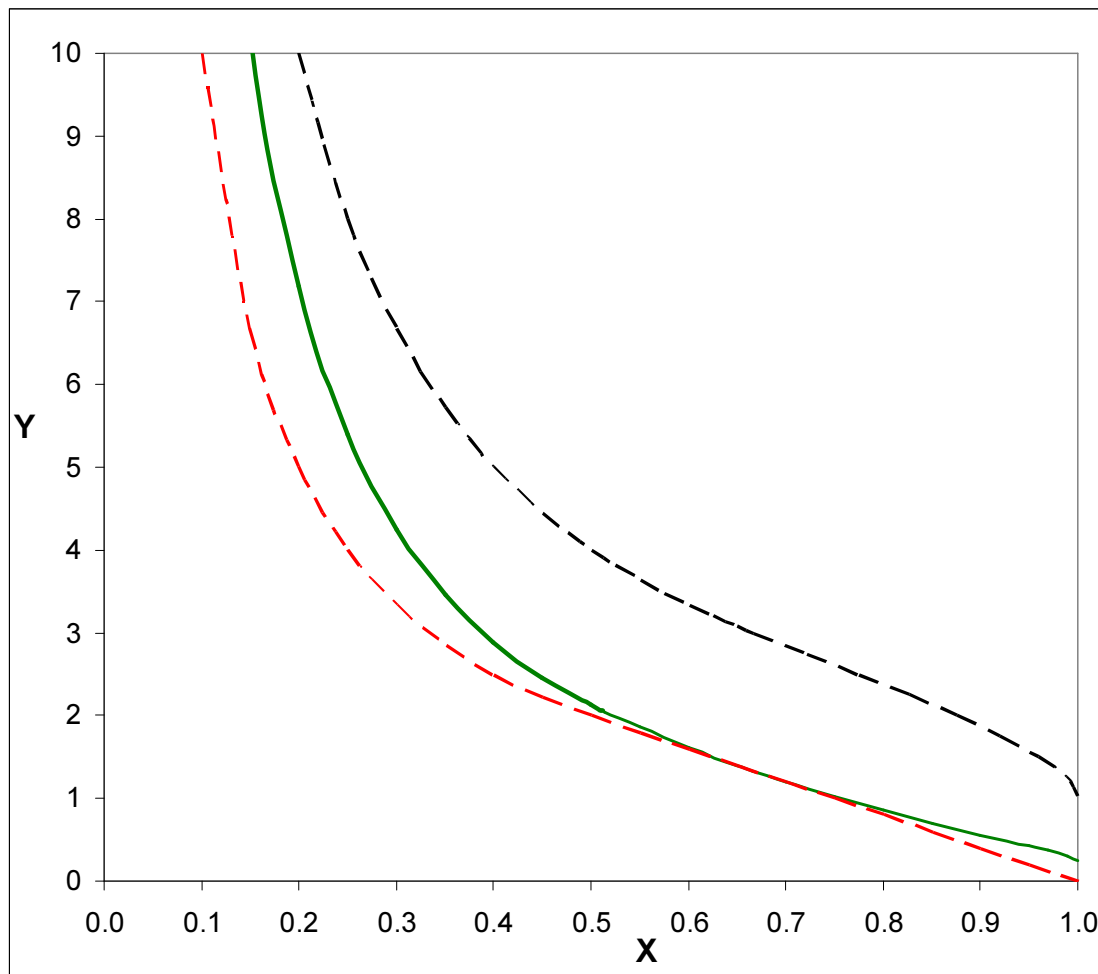


Figure 16: Ratchet Boundary for the Problem of Fig.15 (Positive Phase Case) Compared with the Bree and In-Phase Ratchet Boundaries

Red dashed = Bree; Black dashed = In-Phase; Green = Positive Out-of-Phase



Qu.: What other analytic ratchet/shakedown solutions are there?

The Bree problem, including the in-phase case, has been solved for different yields stresses hot and cold, Ref.[9].

The first analytic solution I am aware of for a problem which is biaxially stressed is that of Ref.[10] which considers the Bree problem with an additional primary membrane load applied orthogonally to the other Bree loadings.

A solution which has been submitted to IJPVP, Ref.11], addresses a pressurised thin cylinder with an additional axial load and a global secondary bending moment. Like Ref.[10] this is also a biaxial problem.

Qu.: How are the ratchet boundaries derived for more complicated problems?

FEA, of course.

Qu.: What are the limitations of cyclic elastic-plastic FE analyses?

The FE formulation will have no difficulty with imposing equilibrium and compatibility. The only thing left which enters the problem is the material constitutive behaviour. (I'm ignoring any problems purely with the numerical solution algorithm, e.g., convergence issues).

Qu.: But we understand plastic hardening, don't we?

No.

If we really understood plastic hardening we could predict the cyclic stress-strain relation using the monotonic tensile stress-strain curve as the empirical input. But this cannot usually be done (to the best of my knowledge – though there may be instances in the literature where researchers have managed to do it for specific materials and models). That's why R66 contains separate advice on monotonic tensile data and the cyclic stress-strain curve.

Qu.: What's the problem with plastic hardening?

The problem is the behaviour under reversed stressing. If a material is work hardened under tensile loading, will it subsequently display equal hardening under compressive loading? For that matter, what hardening will be displayed if we subsequently load the item in shear – or any other direction in “stress space”?

Moreover, plain-vanilla hardening laws will not predict material ratcheting.

It is perfectly possible to reproduce ratcheting behaviour using a non-ratcheting material model – but that addresses only the structural ratcheting, not the material ratcheting.

Qu.: What Constitutive Models are available?

ORNL (Oakridge National Laboratory)

FSRV (Fast Reactor State Variable – now called R5SV)

Various Chaboche models, including LeMaitre-Chaboche

CRIEPI model

LMM (Linear Matching Method)

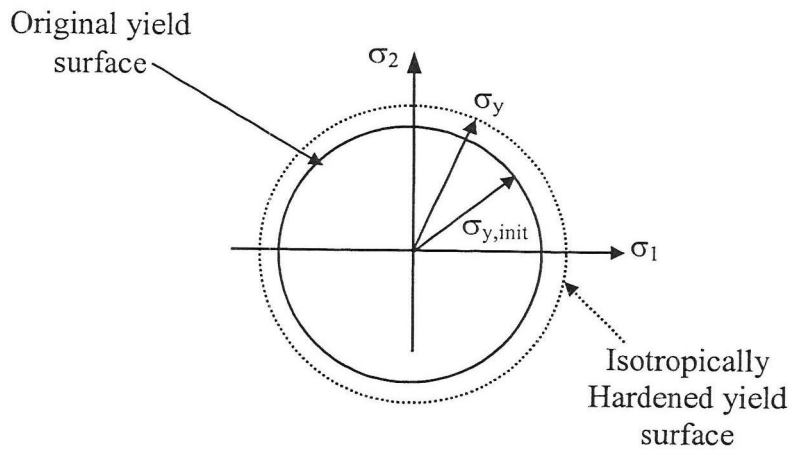
Qu.: What are their strengths & weaknesses?

In an earlier version of these notes I attempted to summarise strengths and weaknesses of the models. However I have now deleted this because I'm not happy that I got it right. Sources of advice are R5V2/3 Appendix A12 (Table A12.1 is particularly helpful) and Manus O'Donnell's report, Ref.[2].

Qu.: What is isotropic hardening?

Isotropic hardening keeps the shape and centre of the yield surface fixed and requires the whole yield surface to dilate uniformly in all directions (isotropically) as hardening takes place, thus,

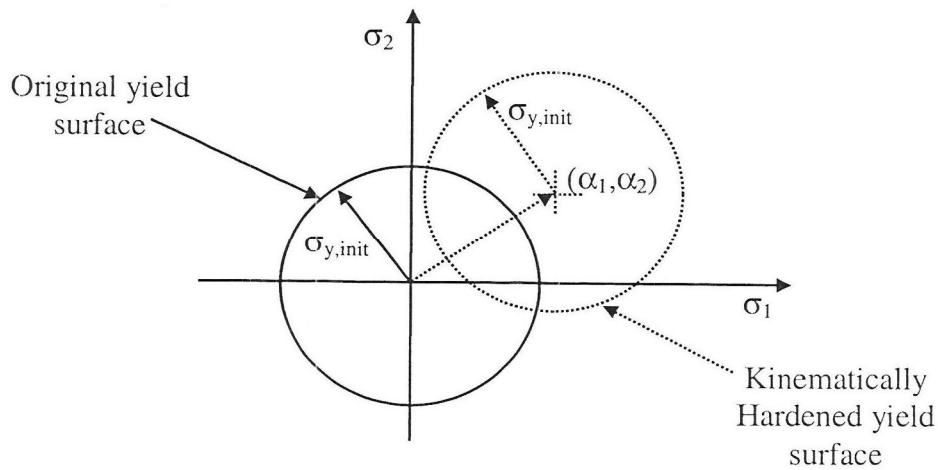
Figure 17: Isotropic Hardening



Qu.: What is kinematic hardening?

In kinematic hardening the size and shape of the yield surface is fixed and hardening takes place only by moving the yield surface in stress space.

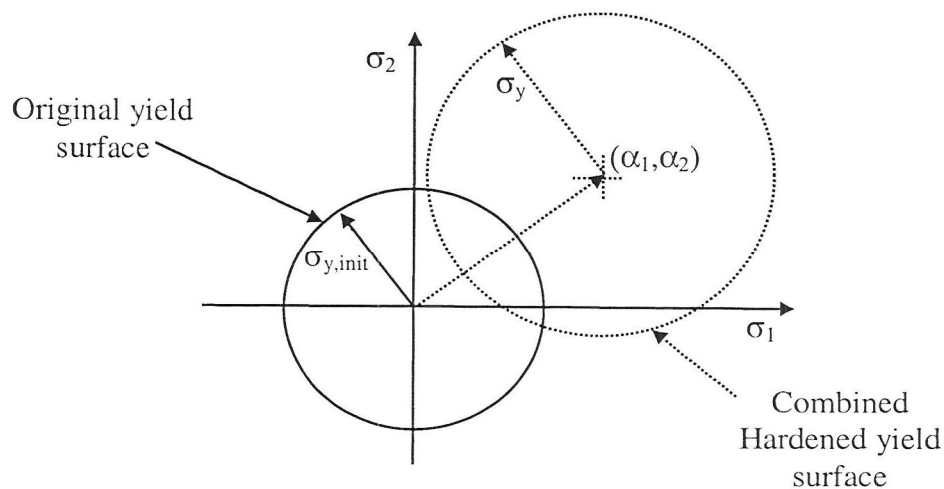
Figure 18: Kinematic Hardening



Qu.: What is meant by “mixed hardening”?

Mixed hardening involves both the size and the position of the centre of the yield surface changing due to plastic straining. It is generally taken to mean a mixture of isotropic and kinematic, which means that the shape of the yield surface is constant.

Figure 19: Mixed Hardening



Qu.: What is “unlimited” hardening

This is a term I am introducing myself for convenience. It refers to the situation where the tensile curve is assumed to rise indefinitely. For example, if we assume power law hardening, $\epsilon_p \propto \sigma^n$, this is an example of “unlimited” hardening. So is a bilinear hardening law in which, for $\sigma > \sigma_y$, the plastic strain increases linearly with stress, i.e., $\epsilon_p = (\sigma - \sigma_y) / E_p$, for some constant plastic modulus, E_p .

Qu.: Will FEA with unlimited isotropic hardening predict ratcheting?

No.

Unlimited isotropic hardening shakes-down to elastic cycling (see Appendix).

Corollary,

If you are attempting to demonstrate shakedown, do not ever assume unlimited isotropic hardening. You will get shakedown alright, but only because you have effectively constrained that to be the only possible behaviour by virtue of the hardening law adopted.

Qu.: Will FEA with unlimited kinematic hardening predict ratcheting?

I think not, no.

I suspect that kinematic hardening will always tend to a stable plastic cycle (see Appendix).

Qu.: But analyses using kinematic hardening do seem to ratchet - ?

Some people have obtained results with FEA which apparently show a ratchet strain on each cycle using unlimited kinematic hardening. However, I suspect that this ‘ratchet strain’ is actually further hardening occurring cycle on cycle, i.e., the limit cycle has not been achieved but ultimately there would be a stable limit cycle. The signature of this is that the apparent ‘ratchet strain’ decreases cycle on cycle.

However, it is a moot point as to whether this is mere semantics. If the plastic strain accumulated prior to reaching the ultimate limit cycle is excessive then it does not matter what you call it!

On the other hand, if the FEA takes the maximum stress above $K_S S_y$ then it is not valid since a plain-vanilla kinematic hardening model fails to account for material ratcheting – which will occur above a stress of $K_S S_y$.

Corollaries,

Any FE model which takes the maximum stress above $K_S S_y$ is suspect since it has probably failed to account for material ratcheting.

The apparent ratchet strain resulting from a model assuming unlimited kinematic hardening is probably not really ratchet strain but hardening strain.

Qu.: So how should you do a ratcheting FEA?

This is probably contentious, and you should seek advice from others who know more about such things, but my advice would be...

Use elastic-perfectly plastic material behaviour with the ‘yield’ stress set to $K_S S_y$. This ensures that you will address material ratcheting as well as being the most secure (i.e., conservative) means of addressing structural ratcheting.

However, the LMM has now matured into a working tool, in conjunction with Abaqus, and is recommended.

Qu.: Is there an assessment procedure for ratcheting?

Yes. It is given in Ref.[2], though it is not part of the formal R5 suite. The key parts are,

- The ratchet strain per cycle should not be increasing cycle on cycle;
- Creep damage can be calculated in the normal way (I think – but check the procedure);
- Fatigue damage can be calculated in the normal way except that it should be based on the total strain range in each cycle including the ratchet increment.
- For 316H material, the total strain, including the ratchet strain, is subject to the following limits,
 - the local plastic strain (at the assessment point) is less than 5% for parent and 2.5% for weld; and,
 - the average equivalent plastic strain over the section of interest is less than 2% for parent and 1% for weld; and,
 - the total accumulated displacement must be sufficiently small so as not to unduly influence the duty of the component.

These strain limits may also be applicable to other materials (see Ref.[2]).

Manus tells me that in the context of the above ratcheting limits, the strain is to be interpreted as the algebraic sum of the signed equivalent strain increments (and so be careful what ABAQUS output variable you use in this context). On this definition, a stable cycle will not accumulate any ratchet strain.

Qu.: What about the effect of creep on shakedown/ratcheting?

The modelling of the creep-plasticity interaction is poor in all models. For example, I am not aware of any model which claims to be able to predict whether primary creep will be regenerated by the plastic straining in the hysteresis loop. This is a pity since this is a burning question.

At a far simpler level, we know that the creep strain can create hysteresis where none would otherwise occur. This is because a stress-strain loop with creep strain cannot be closed unless plastic strain also occurs. It seems likely, therefore, that creep could act synergistically with plasticity to cause ratcheting. However I am unconvinced that any of the models really get this right – though some are better than others.

Note that R5 addresses this point by having separate assessment criteria for,

- Shakedown ignoring creep, and,
- Cyclically enhanced creep.

This is fine when those criteria are met. But when outside global shakedown one cannot rely upon the R5 cyclically enhanced creep assessment criterion. It is necessary to include both plasticity and creep in your shakedown/ratcheting assessment. But the tools we have to do so are very limited in reliability.

References

- [1] R5V2/3 Appendix A12
- [2] Manus O'Donnell's report on assessment of limited ratcheting, E/REP/BDBB/0026/AGR/03
- [3] Tipping D.J., R5 State Variable Model: A Guide to the ABAQUS UMAT, E/REP/BDBB/0088/GEN/05
- [4] Tipping D.J. and Parry M.R., Enhancements to the R5 State Variable UMAT Version 1.0, BEGL Report E/REP/BDBB/0087/GEN/05 Revision 000, (2005).
- [5] **CRIEPI reference? Not identified. Takahashi, probably.**
- [6] Colin Austin, "Ks Values for CMV Steels", AMEC report AMEC12416/R003 (Issue 2, 2014 but my Fig.3 from the May 2013 draft)
- [7] J.Bree, "Elastic-Plastic Behaviour of Thin Tubes Subject to Internal Pressure and Intermittent High-Heat Fluxes with Application to Fast Nuclear Reactor Fuel Elements", *Journal of Strain Analysis* (1967) **2**, 226-238.
- [8] R.A.W.Bradford, "The Bree Problem with Primary Load Cycling In-Phase with the Secondary Load", E/EAN/BBAB/0027/GEN/12 (May 2012). See also R.A.W.Bradford, "The Bree problem with primary load cycling in-phase with the secondary load", *International Journal of Pressure Vessels and Piping* 99-100 (2012) 44-50. See also H.W.Ng and D.N.Moreton, *Journal of Strain Analysis for Engineering Design* 21 (1986) 1-6, and, H.W.Ng and D.N.Moreton, *Journal of Strain Analysis for Engineering Design* 22 (1987) 107-113.

- [9] R.A.W.Bradford, J.Ure and H.F.Chen, "The Bree Problem with Different Yield Stresses On-Load and Off-Load and Application to Creep Ratcheting", International Journal of Pressure Vessels and Piping Vol.113C (2014) 32-39.
- [10] R.A.W.Bradford, "Solution of the ratchet-shakedown Bree problem with an extra orthogonal primary load", International Journal of Pressure Vessels and Piping 129-130 (2015) 32-42.
- [11] R.A.W.Bradford and D.Tipping, "The Ratchet-Shakedown Diagram for a Thin Pressurised Pipe Subject to Additional Axial Load and Cyclic Secondary Global Bending", IJPVP to appear.

Appendix

Behaviour of Unlimited Isotropic and Kinematic Hardening for Uniaxial Membrane Load

For uniform, uniaxial stressing, the behaviour of these simplest of hardening models after a sufficient number of cycles, assuming that the hardening is unlimited, will be,

- Isotropic hardening will shakedown to elastic cycling,
- Kinematic hardening will result in stable plastic cycling (or elastic cycling).

This is demonstrated below.

Isotropic Hardening

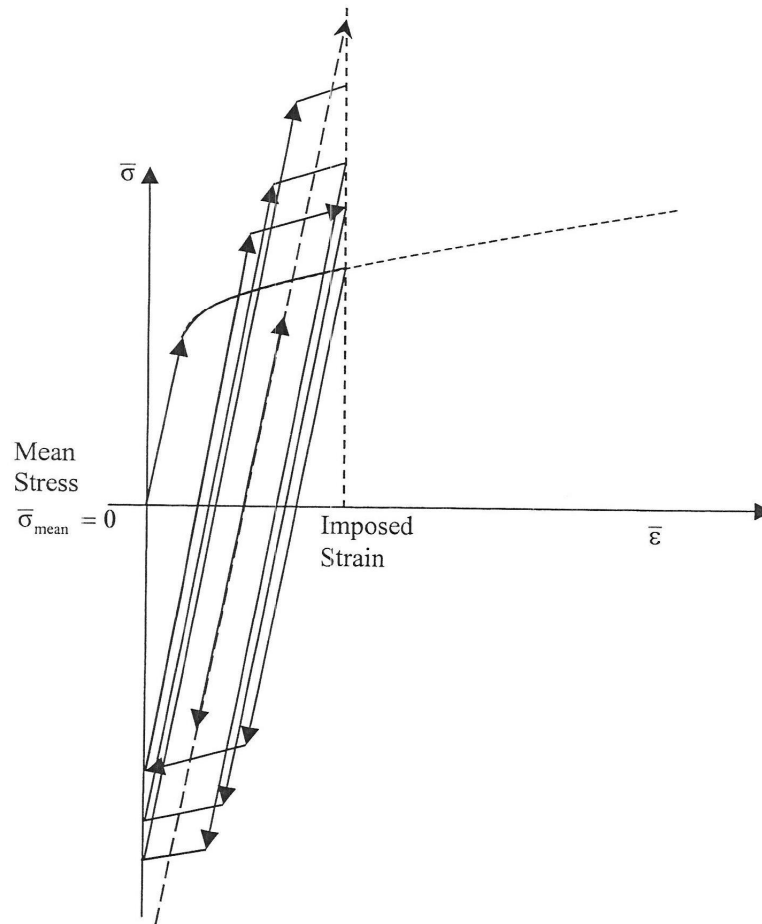
Stress Control

This is trivial. Loading to a stress σ_a in the tensile sense will mean that no yielding occurs on subsequent reverse loading to $-\sigma_a$ simply by virtue of the isotropic hardening law. For the same reason, no yielding occurs when the tensile stress σ_a is re-applied. QED.

Strain Control

This is not so obvious, but Figure 7 demonstrates what happens.

Figure 20 Isotropic Hardening with Strain Controlled Cycling



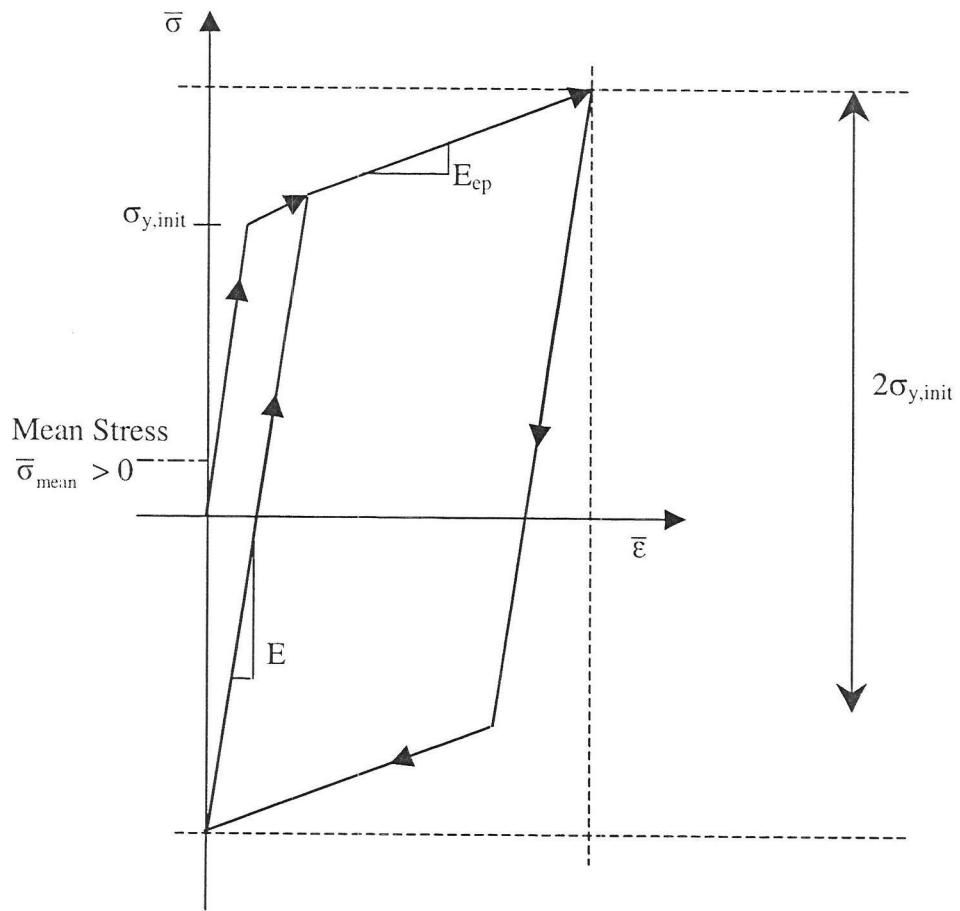
Essentially incremental hardening takes place on successive cycles and the assumed unlimited capacity for hardening leads eventually to elastic cycling. You may find this obvious from Figure 7. If not, one of the homework questions for this session is to prove it for bilinear hardening.

Kinematic Hardening

The case of unlimited kinematic hardening under strain control is illustrated in Figure 21. For cycling between zero strain and ϵ_a , shakedown to elastic behaviour occurs for $\epsilon_a < 2\sigma_y / E$. The hysteresis cycle is obviously stable after the first cycle for strain

control. The stress range of the stable cycle is $\Delta\sigma = 2\left(1 - \frac{E_{ep}}{E}\right)\sigma_y + E_{ep}\epsilon_a$.

Figure 21: Kinematic Hardening: Strain Controlled Cycling



The case of unlimited kinematic hardening under stress control is illustrated in Figure 22. For cycling between $+\sigma_a$ and $-\sigma_a$, shakedown to elastic behaviour occurs for $\sigma_a < \sigma_y$. For stress control a stable cycle exists because it is always possible to fit between the ‘tram lines’ at $\pm\sigma_a$ a parallelogram with the slopes E and E_{ep} , and for which the vertical extent of the steeper (elastic) side is $2\sigma_y$ (Figure 22). The total strain range for the stable cycle is $\Delta\varepsilon = 2\left[\frac{\sigma_y}{E} + \frac{\sigma_a - \sigma_y}{E_{ep}}\right]$.

Figure 22: Kinematic Hardening: Stress Controlled Cycling

