

Tutorial Session 33 (T73S04): Detailed Hysteresis Cycle Construction

Last Update: 15/3/15

Relates to Knowledge & Skills items 1.18, 1.20, 1.21, 1.22, 1.23, 1.24, 1.25, 1.26, 1.27
*Detailed methodology for hysteresis cycle construction (intermediate dwell, parent material);
The Neuber construction; Different A and K_S around cycle; Cycle positioning along stress
axis; Finding the dwell stress; Is the cycle closed? Definition of total strain range: elastic,
plastic, creep, volumetric; Interacting hysteresis cycles*

Qu.: Where is the hysteresis cycle construction methodology defined?

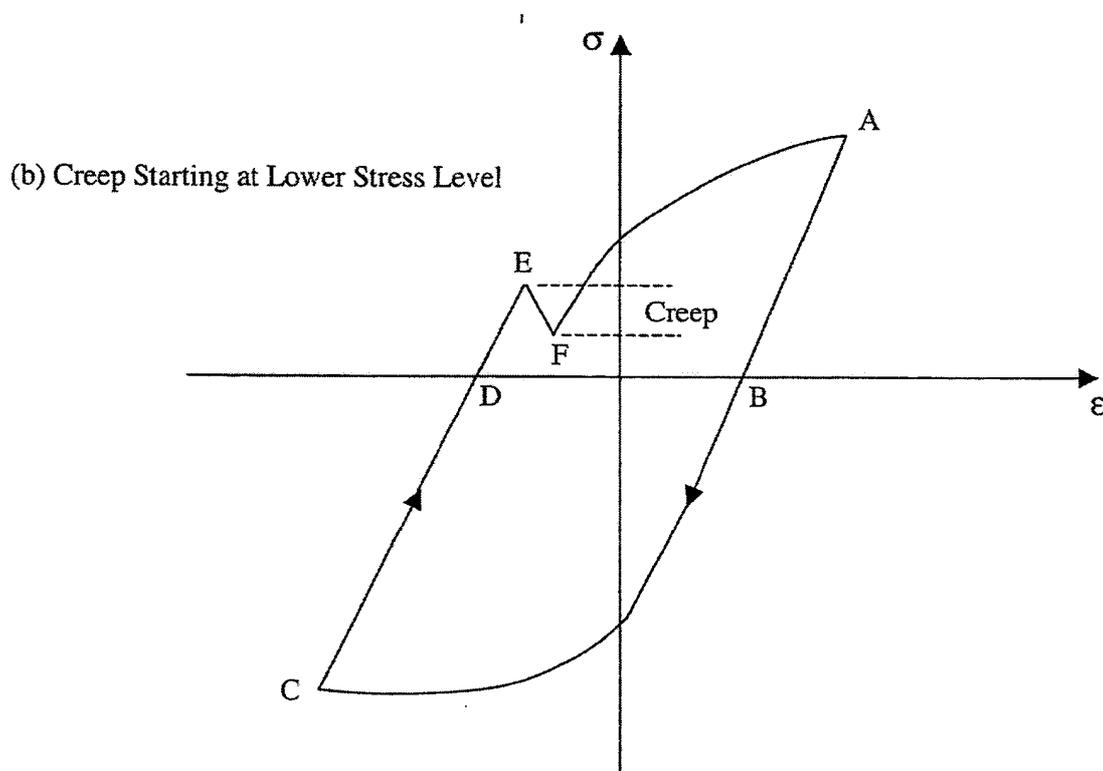
R5V2/3 Appendix A7.

The full details are spelled out in these notes. This has largely been copied from Appendix A of my report E/REP/BBAB/0008/AGR/08 (Ref.1). This is the level of detail needed to write code to carry out an assessment. It should be fully consistent with R5V2/3 Appendix A7.

Qu.: What is the procedure?

Start by identifying the qualitative nature of the hysteresis cycle (for the major cycle if there is more than one cycle type). Here we shall assume Figure 1 below.

Figure 1: Cycle Type Assumed in these Notes



This is the generic cycle type which involves hysteresis with plasticity at both ends of the cycle and a creep dwell on the left-hand side.. If the dwell is actually at the peak of the cycle, omit part FA. Point A will be referred to as the “hot” end of the cycle, and point C as the “cold” end.

Qu.: The steady cyclic state versus transient cycling

The procedure deals with finding the steady cyclic state. In later sessions we shall see how to calculate the creep and fatigue damage corresponding to the derived hysteresis cycle. Typically the R5V2/3 assessments are based upon the assumption that the steady cyclic state is attained quickly and damage is evaluated only for this cycle.

In truth the steady cyclic state is established only after a certain number of load cycles – which may typically be a few tens of cycles. Transient cycles will be discussed in a later session. But this complication is commonly ignored in assessments, preferring the fiction that all cycles are the steady state cycle.

Qu.: Are all cycles the same?

On plant every cycle is different. Even cycles which are classed as the same, say reactor cycles to hot standby conditions, can differ markedly. In deterministic assessments it is usual to bound the plant cycling using just one or two load cycles repeated the appropriate number of times. In this session we will make a few observations about how to handle two different cycle types. A future session will discuss the more realistic case when every cycle is different.

Qu. Is the damage per cycle constant if all cycles are the same and at steady state?

Not necessarily.

Even if all hysteresis cycles are approximated by the steady cycle, and all load cycles and dwells are the same, the damage per cycle will not necessarily be constant. The effects of creep hardening (strain accumulation) between cycles may lead to the first few cycles being far more damaging than later cycles. Also, if a tertiary correction to the creep rate is included, this can lead to increasing creep damage in later cycles (in principle, though this is rarely an issue in practice). Consequently the damage-time graph will generally not be linear even for a steady rate of cycling. However, if the tertiary correct is small and creep relaxation is calculated assuming primary creep reset (of which more later) then the damage per cycle *will* be constant.

Qu.: Are we assessing a weldment?

No.

To keep things simple these notes refer to the procedure for a parent material feature. The weldments procedure will be dealt with in [Session 37](#).

Qu.: What are the key inputs?

It is assumed that the elastic stress components, or elastic stress ranges, at salient times are known. The hysteresis cycle construction is therefore a means of converting these elastic stresses into elastic-plastic stresses and strains. Other required inputs will become apparent below, e.g., the cyclic stress-strain curve.

Qu.: What's the hysteresis cycle construction procedure in outline?

The hysteresis cycle construction consists broadly of the following steps,

- (i) Half-cycle without creep (ABC on Figure 1);
- (ii) Cycle positioning: finding the reverse stress datum, σ_D (Point C on Figure 1);
- (iii) Partial half-cycle before dwell (CDE on Figure 1);
- (iv) Evaluating the creep relaxation and creep strain (EF on Figure 1);
- (v) Complete half-cycle with creep (CDEFA in Figure 1);

The methodology for each of these steps is defined in detail below

Qu.: What is the purpose of these various steps?

Half-cycle without creep: The purpose of this step is to convert the elastic stress range into an elastic-plastic stress and strain range – and hence to find the *relative* positions of points A and C on the stress-strain diagram.

Cycle positioning: finding the reverse stress datum, σ_D : The purpose of this step is to determine the absolute position of point C along the stress axis – and hence the absolute position of the whole hysteresis curve using point C as datum.

Partial half-cycle before dwell: The purpose of this step is to find the dwell stress at E. This is done by finding the stress change along CDE and using the reverse stress datum at C.

Evaluating the creep relaxation and creep strain: The purpose of this step is to calculate the creep strain and the stress drop. The creep strain is used to evaluate creep damage. The creep strain also contributes to the overall strain range, from which fatigue damage is calculated.

Complete half-cycle with creep: The purpose of this step is to provide a second estimate of the elastic-plastic strain range for the whole cycle, i.e., as an alternative to that derived in the first step. The elastic stress range is adjusted using the stress drop evaluated above before finding the strain range. The larger of the strain ranges calculated for the two half-cycles, after addition of a volumetric correction, is used in evaluating fatigue damage.

Qu.: What is the “Neuber construction”?

The Neuber construction is used extensively in the R5 hysteresis cycle construction. The generic problem is that we know the elastic stress range but we require the elastic-plastic stress and strain ranges. How can this be done? Of course, it can't really. There is no unique elastic-plastic stress or strain for a given elastic field. But the assumption of the Neuber construction is that we are likely to be assessing a local region of peak stress. The implicit assumption, therefore, is that we are assessing a notch-like stress concentration.

Neuber's relation for the stress and strain fields near a notch is $(\sigma\varepsilon)_{el} \approx (\sigma\varepsilon)_{el-pl}$. Since we know that $\varepsilon_{el} = \sigma_{el} / \bar{E}$, and that the elastic-plastic stress and strain are related by some appropriate stress-strain curve, say $\varepsilon_{el-pl} = f(\sigma_{el-pl})$, we have,

$$\frac{\sigma_{el}^2}{\bar{E}} = \sigma_{el-pl} f(\sigma_{el-pl}) \quad (1)$$

This relation can thus be inverted to find the elastic-plastic stress, σ_{el-pl} , as long as the elastic stress is known. In practice these stresses are replaced by stress ranges in what follows.

The R5 hysteresis cycle construction is crucially dependent upon the Neuber relation
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Qu.: What cyclic stress-strain behaviour is assumed?

In principle any form of cyclic stress-strain relation could be used. But here we will assume a Ramberg-Osgood relation between the cyclic stress and strain ranges as follows,

$$\Delta\varepsilon = \frac{\Delta\sigma}{\bar{E}} + \left(\frac{\Delta\sigma}{A}\right)^{1/\beta} \quad (2)$$

where the modified Young's modulus is $\bar{E} = \frac{3E}{2(1+\nu)}$ and the Ramberg-Osgood coefficients A and β can be taken from R66, Section 8. Recall that the cyclic stress-strain curve can be very different from the monotonic tensile stress-strain curve.

Qu.: How is the half-cycle without creep (ABC) constructed?

Neuber Construction

We follow the procedure of R5V2/3 Appendix A7, Section A.7.5.3.1. The relevant elastic Mises stress range is that between the cycle peaks, i.e., between A and C, denoted $\Delta\sigma_{el}^{AC}$. Recall that all Mises equivalent stress ranges are to be calculated from the Mises combination of the ranges of the components, as discussed in session 32. This is understood throughout.

The *unmodified* Ramberg-Osgood expression is used to represent the cyclic strain range in terms of the cyclic stress range. Hence the elastic-plastic strain and stress ranges are found from the Neuber construction as follows,

$$\Delta\sigma_{ep}^{ABC} \Delta\varepsilon_{ep}^{ABC} = \frac{(\Delta\sigma_{el}^{AC})^2}{\bar{E}}, \text{ where, } \Delta\varepsilon_{ep}^{ABC} = \frac{\Delta\sigma_{ep}^{ABC}}{\bar{E}} + \left(\frac{\Delta\sigma_{ep}^{ABC}}{A}\right)^{1/\beta} \quad (3)$$

This is consistent with R5V2/3 Appendix A7, Equations (A7.13) and (A7.14) in the case that $\sigma_D = 0$ and $\sigma_N = \Delta\sigma_{ep}^{ABC}$.

Volumetric Correction

A 'volumetric' correction is required to the derived elastic-plastic strain range, $\Delta\varepsilon_{vol}$, found by following R5V2/3 Section 7.4.2, Equations (7.13-16). For completeness the required equations are,

$$\Delta\bar{\varepsilon}_{vol} = (K_v - 1)\Delta\bar{\varepsilon}_{el,r} \quad (4a)$$

where

$$K_v = \left(\frac{1+\bar{\nu}}{1+\nu}\right)\left(\frac{1-\nu}{1-\bar{\nu}}\right) \quad (4b)$$

and

$$\bar{\nu} = \nu \frac{E_s}{E} + 0.5\left(1 - \frac{E_s}{E}\right) \quad (4c)$$

And the secant modulus, E_s , is obtained from the relationship,

$$E_s = \frac{\Delta\sigma_{ep}^{ABC}}{\Delta\varepsilon_{ep}^{ABC}} \quad (4d)$$

and the elastic strain term $\Delta\bar{\epsilon}_{el,r}$ can be identified with $\Delta\sigma_{el}^{AC} / \bar{E}$.

The overall elastic-plastic strain range is then estimated to be,

$$\Delta\epsilon_{ep}'^{ABC} = \Delta\epsilon_{ep}^{ABC} + \Delta\epsilon_{vol} \quad (4e)$$

Qu.: How is the reverse stress datum, σ_D , found?

This is the “**symmetrisation**” procedure alluded to in [Session 32](#).

The procedure follows R5V2/3 Appendix A7, Section A7.5.3.2. The values of $K_s S_y$ are evaluated at the hot and cold ends of the cycle (recalling that S_y is the *lower bound* 0.2% proof stress, and K_s is the shakedown factor).

The assumption of these notes is that the elastic-plastic stress range for half-cycle ABC, $\Delta\sigma_{ep}^{ABC}$, will exceed the $K_s S_y$ ‘range’, i.e., $\Delta\sigma_{ep}^{ABC} > (K_s S_y)_{hot} + (K_s S_y)_{cold}$. This is the condition for the cycle to be qualitatively like Figure 1. If $\Delta\sigma_{ep}^{ABC} < (K_s S_y)_{hot} + (K_s S_y)_{cold}$ the hysteresis cycle will be qualitatively different (and the hysteresis cycle construction algorithm given here will not be appropriate).

In the “symmetrisation” procedure the hysteresis cycle is positioned so as to extrude beyond $K_s S_y$ equally at each end of the cycle. In other words the reverse stress datum is defined by,

$$\sigma_D = \frac{\Delta\sigma_{ep}^{ABC}}{2} + \frac{(K_s S_y)_{cold} - (K_s S_y)_{hot}}{2} \quad (5)$$

Finally note that σ_D is defined here as a positive number, but it represents the compressive datum stress at point C, not point D!

Having established the absolute position of the cycle along the stress axis, the absolute stress at point A (the so-called ‘forward stress datum’) is then found as $\sigma_F = \Delta\sigma_{ep}^{ABC} - \sigma_D$ (6)

[*Aside: In passing we note that if $\Delta\sigma_{ep}^{ABC}$ were less than $(K_s S_y)_{hot} + (K_s S_y)_{cold}$ then the reverse stress datum would be defined either by the cycle bottom end reaching $\sigma_D = (K_s S_y)_{cold}$. However, in these notes we have assumed that is not the case, i.e., Figure 1 indicates a stress range in excess of $(K_s S_y)_{hot} + (K_s S_y)_{cold}$].*

Qu.: Isn't there some limit on the relaxed stress?

R5V2/3 App.A7 includes an instruction to shift the whole cycle upwards if the relaxed stress at point F is found to drop below the rupture reference stress, σ_{ref}^R . However, I will be advising you to use a modified relaxation equation which guarantees that this cannot happen, so you can forget about this stipulation.

Qu.: How is the partial half-cycle before dwell (CDE) constructed?

Neuber Construction

Here we follow the procedure of R5V2/3 Appendix A7, Section A.7.5.4. The relevant elastic Mises stress range is that between the ‘cold’ end of the cycle and normal operation, i.e., between point C and point E, denoted $\Delta\sigma_{el}^{CE}$. The **modified** Ramberg-Osgood expression is used, with A replaced by A/2, because the purpose here is to approximate the shape of the hysteresis loop itself (as opposed from the tip-to-tip relationship). Hence the partial elastic-

plastic strain range and the normal operating (dwell) stress, σ_0 , are estimated from the Neuber construction by solving the following for σ_0 ,

$$(\sigma_D + \sigma_0)\Delta\varepsilon_{ep}^{CE} = \frac{(\Delta\sigma_{el}^{CE})^2}{E}, \text{ where, } \Delta\varepsilon_{ep}^{CE} = \frac{\sigma_D + \sigma_0}{E} + \left(\frac{2\sigma_0}{A}\right)^{1/\beta} \quad (7)$$

This is consistent with R5V2/3 Appendix A7, Equations (A7.13) and (A7.14). Note that the reverse stress datum, σ_D , follows from (5), above.

Because our assessments are often creep dominated, the estimate of the dwell stress, σ_0 , provided by the solution to (7), is frequently the most important outcome of the hysteresis loop construction.

Note that when calculating creep damage, assessments may replace the start-of-dwell stress given by (7) with the rupture reference stress where the latter is larger (see session on creep damage).

Qu.: How are the creep relaxation and creep strain evaluated?

Integration of Forward Creep

This is treated in more detail in the session on creep damage. For now we note that the relaxation of the stress is most often found by integrating the forward creep rate. This requires knowledge of the rupture reference stress and the elastic follow-up factor, Z , as well as the starting stress (σ_0) and, of course, the temperature, T . The expression which requires integration is,

$$\frac{Z}{E} \cdot \frac{d\bar{\sigma}}{dt} = -(\dot{\bar{\varepsilon}}_c(\varepsilon_c, \bar{\sigma}, T) - \dot{\bar{\varepsilon}}_c(\varepsilon_c, \sigma_{ref}^R, T)) \quad (8)$$

The effect of subtracting the second term (the creep strain rate due to the primary stress alone) is to ensure that relaxation does not reduce the stress below the primary stress level – which clearly it should not do. Integrating (8) provides stress as a function of time during the dwell. The creep strain increment is then given by,

$$\Delta\bar{\varepsilon}_c = \Delta\bar{\varepsilon}_{c,ref}^R + Z \frac{\Delta\sigma_c}{E} \text{ where, } \Delta\sigma_c = \bar{\sigma}(t_1) - \bar{\sigma}(t_2) \quad (9)$$

where t_1, t_2 are the times at the start/end of the creep dwell in question. The creep strain increments $\Delta\bar{\varepsilon}_c$ and $\Delta\bar{\varepsilon}_{c,ref}^R$ are the result of integrating the two terms on the RHS of (8). [There are some subtleties involved in this which will be addressed in [Session 34](#)].

Note that since the 2014 Issue 3 of R5V2/3, Appendix A11 does now give Equ.(8) as an option (it did not previously). Equ.(8) is to be preferred to the version of the equation which omits the primary term. This will be discussed in detail in [Session 34](#). Note that R5 still has the equation $\Delta\bar{\varepsilon}_c = Z \frac{\Delta\sigma_c}{E}$ in parts, but this is incorrect if (8) is used because then equ.(9) results.

There are a number of advantages to using (8), one of which is that it works also in forward creep, when there is no relaxation.

Note that the effect of elastic follow-up, Z , is to reduce the stress relaxation but to increase the creep strain. Even if the primary term in (9) were omitted, so that $\Delta\bar{\varepsilon}_c = Z \frac{\Delta\sigma_c}{E}$, you

should not imagine this means that doubling Z would double the creep strain - because if Z is increased, $\Delta\sigma_c$ will decrease. So the creep strain is less sensitive to Z than it looks.

Qu.: How is the half-cycle including creep (CDEFA) constructed?

The purpose of this final step in the hysteresis loop construction is to provide a second estimate of the elastic-plastic strain range for the whole cycle (noting that we have one estimate of this already, i.e., $\Delta\varepsilon_{ep}^{ABC}$). The larger of the two estimates will be used for assessing fatigue damage. It also provides an alternative estimate of the elastic-plastic stress peak at point A (also referred to as the ‘forward stress datum’, σ_F) although this quantity is not actually used in the assessment.

Neuber Construction

Here we follow the procedure of R5V2/3 Appendix A7, Section A.7.5.6.2, noting that the creep dwell decreases the stress range (and increases the strain range), see Figure 1. The relevant elastic Mises stress range is that between the extreme ends of the cycle, i.e. between points C and A, less the stress drop during the creep dwell. Hence we use,

$$\Delta\sigma_{el}^{CA} = \Delta\sigma_{el}^{AC} - \Delta\sigma_c \quad (10)$$

where $\Delta\sigma_{el}^{AC}$ is the same elastic stress range as used in the construction of half-cycle ABC and the stress relaxation, $\Delta\sigma_c$, is found from Eqs.(8,9). The **modified** Ramberg-Osgood expression, with A replaced by A/2, is used to approximate the shape of hysteresis loop. Hence the elastic-plastic strain range and the peak trip stress are initially estimated from the Neuber construction for parent material by solving the following for the forward stress datum, σ'_F ,

$$(\sigma_D + \sigma'_F)\Delta\varepsilon_{ep}^{CEA} = \frac{(\Delta\sigma_{el}^{CA})^2}{E}, \quad \text{where,} \quad \Delta\varepsilon_{ep}^{CEA} = \frac{\sigma_D + \sigma'_F}{E} + \left(\frac{2\sigma'_F}{A}\right)^{1/\beta} \quad (11)$$

This is consistent with R5V2/3 Appendix A7, Equations (A7.13) and (A7.14) for $\sigma_N = \sigma'_F$. Note that the reverse stress datum, σ_D , follows from (5) above.

Volumetric and Creep Corrections

A ‘volumetric’ correction, $\Delta\tilde{\varepsilon}_{vol}$, is required to derived the total elastic-plastic strain range. It is found by following R5V2/3 Section 7.4.2, Equations (7.13-16) – reproduced above as Eqs.(4a-d). The stress range used is,

$$\Delta\bar{\sigma} = \sigma_D + \sigma'_F \quad (12)$$

The elastic strain range term $\Delta\bar{\varepsilon}_{el,r}$ is not simply $\Delta\sigma_{el}^{CA} / \bar{E}$ because the strain range is increased by the creep dwell, so instead put,

$$\Delta\bar{\varepsilon}_{el,r} = (\Delta\sigma_{el}^{CA} + \Delta\sigma_c) / \bar{E} = \Delta\sigma_{el}^{AC} / \bar{E} \quad (13)$$

The overall elastic-plastic strain range for the half-cycle with creep, CEA, is then estimated to be the sum of the elastic-plastic strain range, the creep strain and the volumetric correction,

$$\Delta\varepsilon_{ep}^{CEA} = \Delta\varepsilon_{ep}^{CEA} + \Delta\varepsilon_c + \Delta\tilde{\varepsilon}_{vol} \quad (14)$$

where the creep strain, $\Delta\varepsilon_c$, is found from (9).

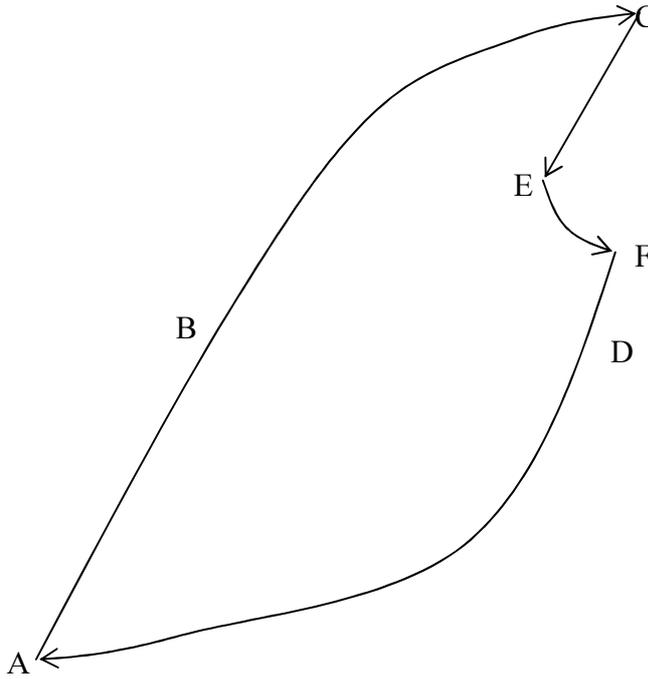
Qu.: What is the final strain range for fatigue damage estimation?

The strain range to be used for the assessment of fatigue damage is the larger of $\Delta\varepsilon_{ep}^{rABC}$ found in step 1 (Equ.4e) and $\Delta\varepsilon_{ep}^{rCEA}$ from (14).

Qu.: What if the creep dwell is on the right-hand side of the hysteresis loop?

The procedure is the same, of course. But the position of the dwell makes a difference as to how the stress drop and creep strain are taken into account in the cycle construction. Consider the cycle shown in Figure 2.

Figure 2 - Creep Dwell on the Right



The half-cycle ABC is that without creep, which would be constructed using the unmodified Ramberg-Osgood equation. The half-cycle CEFDA is that with creep and would be constructed using the modified Ramberg-Osgood curve. The difference from the treatment of Figure 1, though, is that in constructing CEFDA the adjusted stress range is not equ.(10) but,

$$\Delta\sigma_{el}^{CA} = \Delta\sigma_{el}^{AC} + \Delta\sigma_c \quad (10b)$$

because the stress drop now *increases* the stress range from C to A. However, the strain range for CEFDA is now calculated by subtracting the creep strain. This applies in the calculation of the volumetric strain as well as the total strain range, so eqs.(13) and (14) become,

$$\Delta\bar{\varepsilon}_{el,r} = (\Delta\sigma_{el}^{CA} - \Delta\sigma_c) / \bar{E} = \Delta\sigma_{el}^{AC} / \bar{E} \quad (13b)$$

$$\Delta\varepsilon_{ep}^{rCEA} = \Delta\varepsilon_{ep}^{rCEA} - \Delta\varepsilon_c + \Delta\tilde{\varepsilon}_{vol} \quad (14b)$$

Qu.: What about compressive creep dwells?

Again this has a bearing on the signs of the corrections due to stress drop and creep strain. For a compressive dwell on the left, eqs.(10b, 13b, 14b) are appropriate (because the stress range is increased by the dwell and the strain range decreased). For a compressive dwell on the right, use eqs.(10, 13, 14).

Qu.: Is the hysteresis loop closed?

Since the construction is intended to represent the steady cyclic state without ratcheting, the hysteresis loop really should be closed. The calculated cycle is closed only if the stress and strain ranges estimated for the right hand side, ABC, equal those estimated for the left hand side, CDEFA. In other words,

$$\text{Hysteresis loop is closed if: } \Delta \varepsilon'_{ep}{}^{ABC} = \Delta \varepsilon'_{ep}{}^{CEA} \quad \text{and} \quad \sigma_F = \sigma'_F \quad (15)$$

where σ_F and σ'_F are the two estimates of the forward stress datum (at point A) derived from the right and left hand side half-cycles, from Eqs.(6) and (11) respectively.

In practice the approximations inherent in the process will mean that the equalities in (15) will not be precisely obeyed. This is normal and should not be a cause of concern.

However, the differences between the two estimates of stress and strain range should not be large. If they are, then something has probably gone wrong in your calculations. So this is a useful check.

Qu.: Is integration of forward creep a good method for estimating relaxation?

Not really, no. At least, not with existing formulations of forward creep. The reason is, virtually by definition, the shortcomings in our knowledge of the state variable which controls creep hardening.

Qu.: Wouldn't it be better to use relaxation data?

The trouble with most available relaxation data (e.g., fits to a Feltham expression) is that its range of validity is so restricted as to be useless. Much relaxation data has been obtained from tests at very high temperature over a period of just a few days - or just one day!

In any case, relaxation data would not address the key issue, which is creep hardening and the influence of prior cycling on the relaxation and creep strain rate. And relaxation in cyclic creep-fatigue is not the same as monotonic relaxation.

So, at the present time, integration of forward creep is generally the best we have. The main issue here is whether hardening is continuous between cycles or primary creep is reset on each cycle – see Session 34.

Qu.: Why is the modified Young's modulus, \bar{E} , used in the Neuber constructions?

The Neuber constructions, Eqs.(3,7,10), use the Ramberg-Osgood equations involving the modified Young's modulus, \bar{E} . Recall that there is no definition of equivalent strain which works for both plastic strain and elastic strain. R5 adopts the definition appropriate for plastic strain. So the modification of Young's modulus is just to correct the 'error' in the elastic strain.

Qu.: Why is the modified Young's modulus, \bar{E} , used in Equ.(9) for creep strain?

Due to the above, the elastic strain decrement during the creep dwell is $\Delta\sigma_c / \bar{E}$, and Z is defined as the factor by which the (secondary part of the) creep strain exceeds the elastic strain decrement – so we get Equ.(9). You could use the ordinary E in (9) if the value of Z were found consistently (so the ratio Z/E would be the same). But this Z would not then be the factor by which the (secondary) creep strain exceeded the elastic strain decrement.

Qu.: Is the creep hardening law important to cyclic creep damage?

Yes.

The creep hardening law is very important indeed.

This is why (8) has been written with the dependence on creep strain explicit. Of course, if strain hardening is not the correct hardening law this would be replaced by some other state variable. But the spirit of Equ.(8) is that the creep hardening is to be understood as cumulative from one cycle to the next. This means that, when in primary creep, the calculated creep damage per cycle can reduce – often very rapidly. But is this right?

Qu.: Is the continuity of creep hardening between cycles certain?

No.

In fact it is not even recommended in some cases.

For example, for 316ss, Ref.[2] recommends continuity of hardening below 550°C, but not above 550°C. It follows that creep-fatigue initiation assessments of 316ss to R5V2/3 will be far more onerous above 550°C.

The difference is very emphatic. With no continuity of hardening, primary creep is reset to datum time zero at the start of every cycle. The result will generally be far, far greater predicted creep damage.

Because of the sensitivity of the assessments to whether hardening is continuous (i.e., whether primary creep is reset on each cycle), and because there is relatively little data to support current advice (even for 316ss, and probably nothing at all for other materials) Condition Report 548915 has been raised to obtain and analyse additional test data. Provisional indications are not favourable for current 316 assumptions, see Ref.[3].

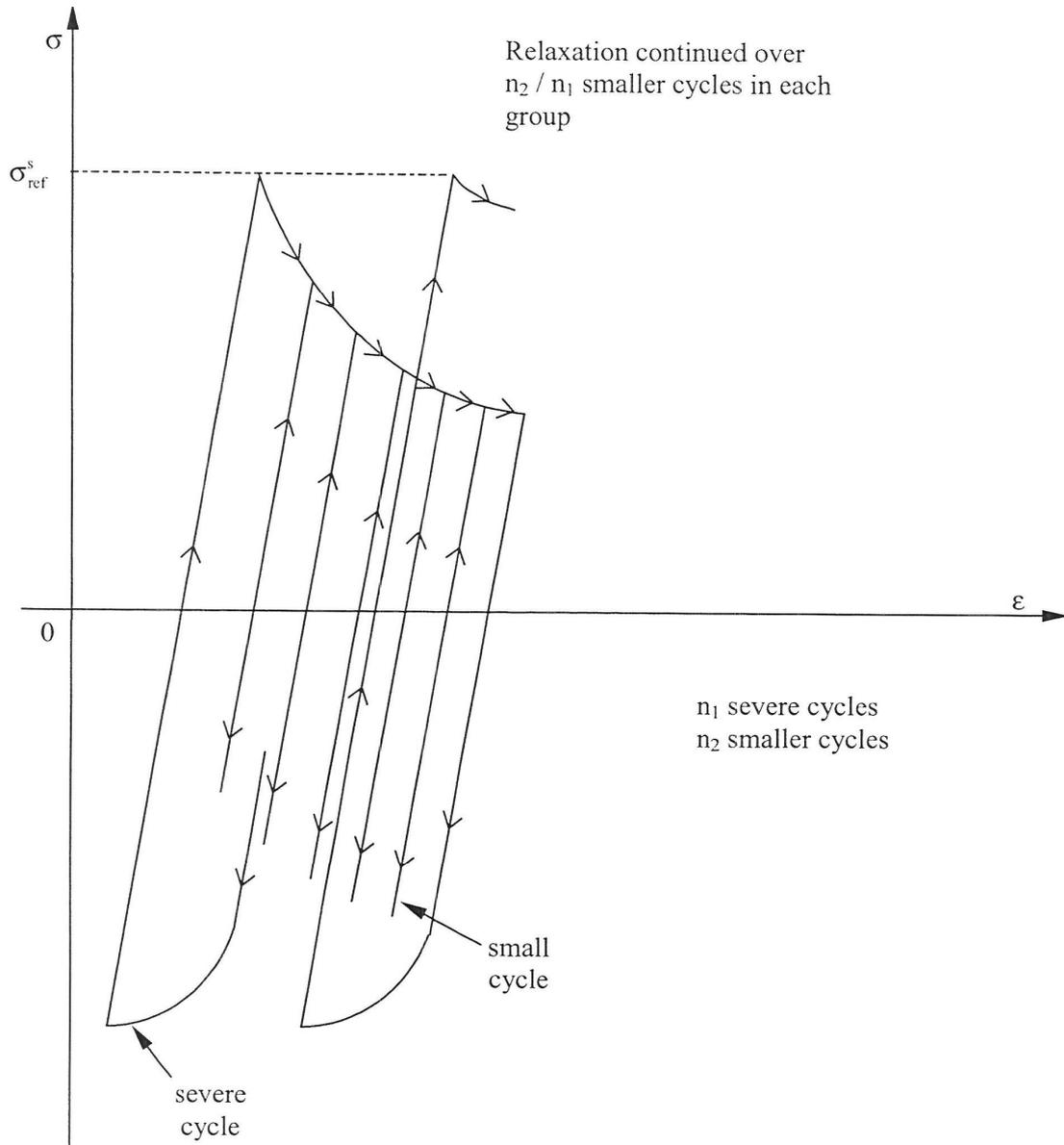
The continuity of creep hardening between cycles, or the absence thereof, is one of the most numerically dominant uncertainties in the outcome of an R5V2/3 assessment. For 316H parent *my view* is that primary creep reset is valid at all temperatures, though you can ameliorate the effect by application of a "zeta factor" or the equivalent, see Ref.[3]. BUT at March 2015 formal R5 advice remains as per Ref.[2], i.e., continuous hardening in 316H parent below 550°C

Existing advice in this area is being considered under the HiTBASS programme which is due to produce advice in late 2015, so advice may be subject to change soon.

Qu.: How are two different types of cycle assessed?

When the lesser load cycle is elastic (as illustrated by Figure 3 below) the methodology is simple and has been described in [Session 32](#).

Figure 3: Illustration of Small (Elastic) Cycles Superimposed on Large Cycles



Qu.: But how are interacting cycles assessed when both are elastic-plastic?

There are two cases which are simple:-

- [1] If there are a relatively small number of cycles of lesser severity, the pragmatic approach is simply to assess all cycles as being of the more severe kind. However, if the majority of the cycles are of lesser severity, this will be unduly conservative.
- [2] If large numbers of cycles of type 1 occur consecutively, followed by large numbers of cycles of type 2, then it will probably be adequate to assess both cycle types as if they alone occurred, and add the damage from each. In this case, because there are few consecutive cycles of differing type, and because we can assume that each cycle type achieves its steady cyclic state, there is little interaction between the cycle types.

But in general the procedure would be:-

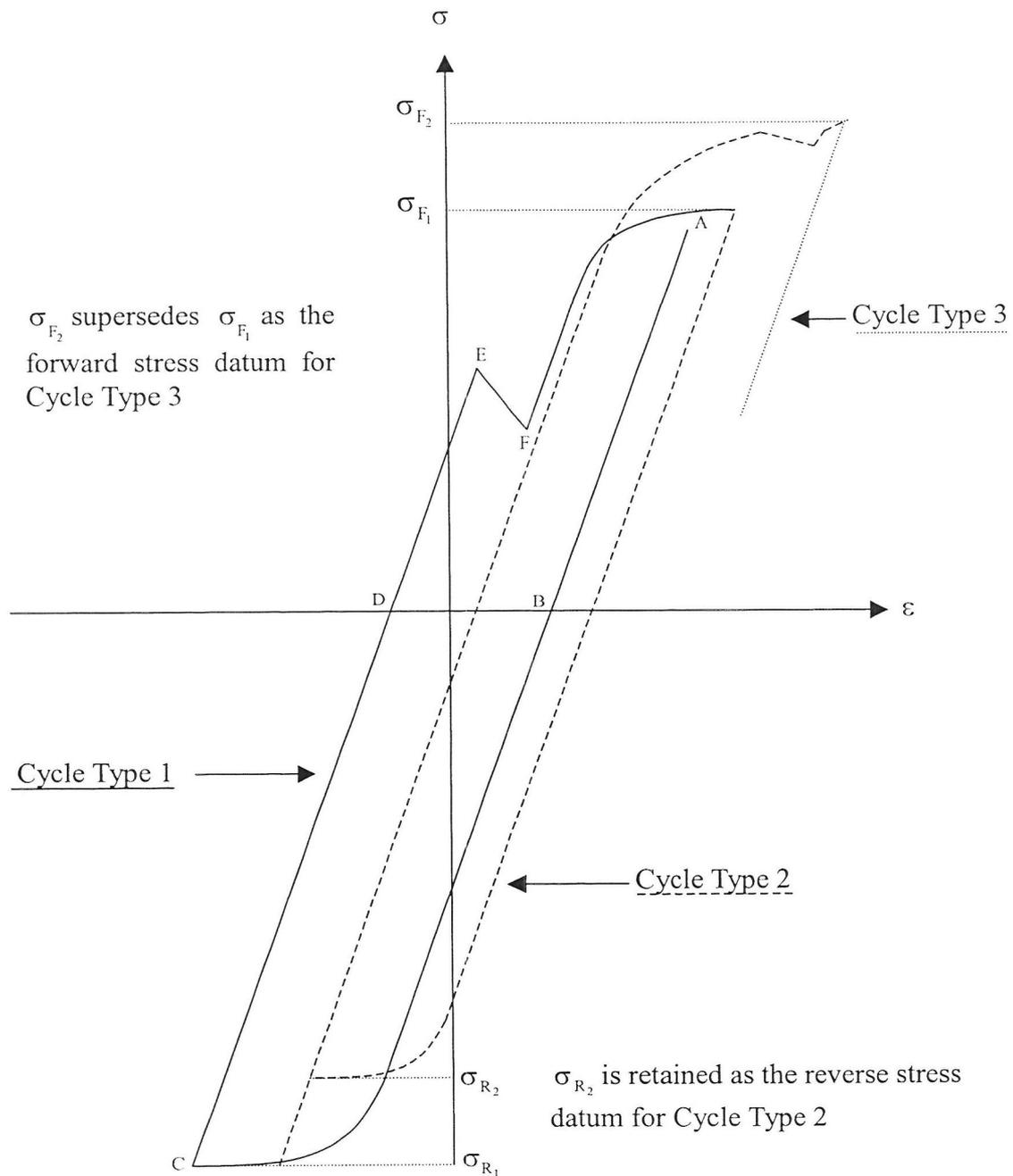
R5V2/3 Appendix A7, Section A7.6.2, provides some (rather brief) guidance on how to assess genuinely interacting hysteresis cycles. This is my understanding of the advice:-

Suppose that cycles of differing types occurred one after the other. Suppose, for example, that cycles of type 1 have occurred sufficiently often to achieve the steady cyclic state, and then a cycle of type 2 occurs, followed by a cycle of type 3, etc. These latter cycles will not achieve their steady cyclic states.

We assume that the cycle of type 2 starts at the peak of the preceding cycle type 1, and similarly that the cycle of type 3 starts at the peak of the preceding cycle of type 2. The procedure for calculating the hysteresis loop for the cycle type 2 is,

- Calculate the size and shape of the hysteresis loop for cycle type 2 exactly as described above (and hence using the same methodology as for cycle type 1);
- The only difference is the absolute positioning of the hysteresis loop for cycle type 2 along the stress axis;
- Instead of using Equ.(5) to define the absolute position of the hysteresis loop (via defining the reverse stress datum, σ_D) the position of the loop for cycle type 2 is assumed to be set by the forward stress datum (σ_{F1}) determined for cycle type 1. This is illustrated by Figure 4.

Figure 4: Interacting Cycles and Forward & Reverse Stress Datums



Similarly, cycle type 3 is evaluated in the same way and positioned along the stress axis so that its forward stress datum equals that from the previously established cycle type 2, i.e., σ_{F_2} , as shown above.

Qu.: Does the interaction make the damage larger or smaller?

Yes!

The modification of the position of the hysteresis loop for cycle type 2 has no effect on the strain range, and hence on the fatigue damage. However, the absolute magnitude of the dwell stress is affected, and hence the creep damage is affected.

Suppose cycle type 1 is less severe than cycle type 2 (in terms of the elastic stress range). This means that the forward stress datum for cycle type 1 (σ_{F_1}) will be less than that which

would occur for cycles of type 2 if they reached their steady cyclic state. Consequently the cycle of type 2 is placed lower down the stress axis by the above construction than the steady cyclic loop for cycle type 2. Hence, the interaction between the cycle types results in *less* creep damage from the cycle of type 2 (i.e., less than would be found by ignoring the interaction).

But the forward stress datum at the end of cycle type 2 (σ_{F2}) would be greater than that after cycle type 1 (σ_{F1}) – see Figure 3. Hence, if cycle type 2 were followed by another cycle of type 1 then the creep damage due to the non-steady-state cycle type 1 would be greater than that for the steady state cycles of type 1.

In other words, for this particular example, a given load cycle produces less damage if it is preceded by lesser cycles, but produces more damage if it is preceded by greater cycles.

However, this is because the dwell occurs under tensile stress and because we have assumed that the cycles ‘join’ at the top end. If the cycles were joined at the bottom end, this reasoning would be reversed: a given load cycle would produce more damage if it were preceded by lesser cycles, but would produce less damage if it were preceded by greater cycles.

Qu.: Is this cycle interaction methodology satisfactory?

No.

The methodology ignores the tendency of cycles to symmetrise. In practice, cycles tend to symmetrise very rapidly (far faster than the tens of cycles required to reach the steady cyclic state). The methodology is unrealistic in that the positioning of the first cycle has an undue residual influence on the positioning of all subsequent cycles. In reality the past cycles very rapidly become irrelevant. A more realistic cycle interaction methodology has been used in Ref.[4] – but without any justification.

References

- [1] R.A.W.Bradford, “HPB/HNB: Superheater Bifurcations’ Creep-Fatigue Crack Initiation Assessments to R5 under Full and Reduced Power Operation”, E/REP/BBAB/0008/AGR/08, August 2009.
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- [3] R.A.W.Bradford and P.J.Holt, “Materials Data Inputs to Probabilistic R5V2/3 Creep-Fatigue Crack Initiation Assessments of HYA/HAR Superheater Bifurcations and Adjacent Tubing in 316H Parent and HAZ Materials”, E/REP/BBAB/0022/AGR/12, November 2012.
- [4] R.A.W.Bradford, “Heysham I/Hartlepool Power Stations: BIFINIT-RB: A Probabilistic Creep-Fatigue Crack Initiation Program for the HYA/HAR Boilers’ Superheater Bifurcations; Theory, User Guide and Application to HAR Reactor 2”, E/REP/BBAB/0023/AGR/12, December 2012.