Tutorial Session 30: T73S04 – R5V2/3 Prerequisites

Last update 21/1/15

Relevant to Knowledge & Skills items 1.1, 1.6, 1.7, 1.8, 1.9, 1.10, 1.11, 1.12, 1.13

The purpose of R5V2/3: example of hysteresis loops and where the damage occurs; Failure modes addressed as prerequisites; Primary & secondary stress categories & limits; significance of creep; significance of load cycling; cyclically enhanced creep (refer to 27); strict shakedown; global shakedown; How these are demonstrated in R5; The shakedown factor, K_S; Ratcheting; Shakedown reference stress

Qu.: What is R5V2/3 for?

R5V2/3 is primarily about the assessment of the initiation of creep-fatigue cracks in a structure which is assumed to be initially defect-free.

Qu.: What does "initiation" mean?

I shall use the word "initiation" to mean the formation of a small crack where there was previously no crack. This is what R5V2/3 is all about.

However, there is a degree of terminological confusion. Some authors also use the word "initiation" to refer to the start of crack growth from an existing defect. The phrase "crack initiation" thus becomes ambiguous. I shall not use the word "initiation" in this sense.

There can be a period of time in which a crack exists, and is under load at creep temperatures, but has not yet started to grow. This is correctly called the "incubation period". Thus, the correct terminology, starting from a defect-free structure, is: initiation, then incubation, then crack growth. However, there may not be an incubation period (e.g., if the crack initiates and then grows by the same mechanism).

But the terminological confusion is compounded by the fact that, in fatigue, the initiation of a very shallow crack (20 microns) is referred to (in R5) as "incubation", and – worse still – the methodology for correcting lab endurance data involves consideration of microcrack growth – ending in a crack which is defined as the "initiated crack depth". So we have incubation and crack growth *before* a crack is considered to have initiated! Oh dear.

Qu.: How deep is a crack which has just "initiated"?

This is partly User specified. There is an explicit need to choose an "initiated" defect size in the fatigue damage part of the procedure, R5V2/3 Sections 8.1, 8.2, alluded to above. This size should usually be no greater than the crack size which corresponds to failure in the fatigue tests which provide the fatigue endurance data for the assessment. However, the User may choose a shallower crack as the "initiated" size. This may be sensible for thin structures.

The creep part of the procedure evaluates damage at the most onerous point. So any finite "initiated" crack size is a conservative interpretation as regards the creep component. Consequently the "initiated" crack depth can be identified with the choice made for the fatigue assessment.

If the initiation assessment is to be followed by a crack growth assessment, it may be best to use the smallest initiation defect for which the crack growth procedure can be assumed to be valid.

In all cases the “initiated” crack depth *should* be small compared with the section thickness so that the section compliance is negligibly affected. Otherwise the initiation assessment, being based upon the uncracked body stresses, may be invalid.

Qu.: What about reheat crack sizes, for example?

There is a snag in the above advice regarding the initiation crack size. Some mechanisms, e.g., reheat cracking, can lead to deep cracks forming almost instantaneously. (Strictly speaking the initial growth phase is just very rapid). For example, the superheater headers’ S4 nozzle welds at HYA/HRA. These are 64mm thick (316H) and produced reheat cracks which, when first discovered, had depths of perhaps 10mm-20mm. In principle, these cracks will have initiated at a single sub-surface grain and subsequently grew. But the growth phase was so rapid that it was not resolved between inspections. The reason is that the creep damage accumulated over the whole of the ~23mm deep tensile zone of the residual stress field. As soon as a crack initiated, it zipped through the remaining part of the residual tensile zone due to the already high levels of creep damage.

In cases like this it may be necessary to base an assessment on the most onerous point (sub-surface in this example), but then to concede a large “initiated” crack depth – covering the whole of the region in which damage is likely to be high. This is not because there is anything wrong in principal with considering a small “initiation” crack depth (e.g., 1mm). Rather it is because the latter course throws the burden on the subsequent crack growth assessment, which must now address very rapid crack growth through highly pre-damaged material. This approach may be possible, but is more work to carry out.

Qu.: What is the mechanism of creep-fatigue damage?

Any structural damage mechanism requires the irreversible absorption of energy. The energy is supplied by the applied loads. But the irreversible absorption of energy means there must be some plasticity or creep strain.

In the case of fatigue crack growth we have seen (Session 22) that the reverse yielding at the crack tip creates a zone of cyclic plasticity which is responsible for the energy absorption and hence the damage and the crack growth.

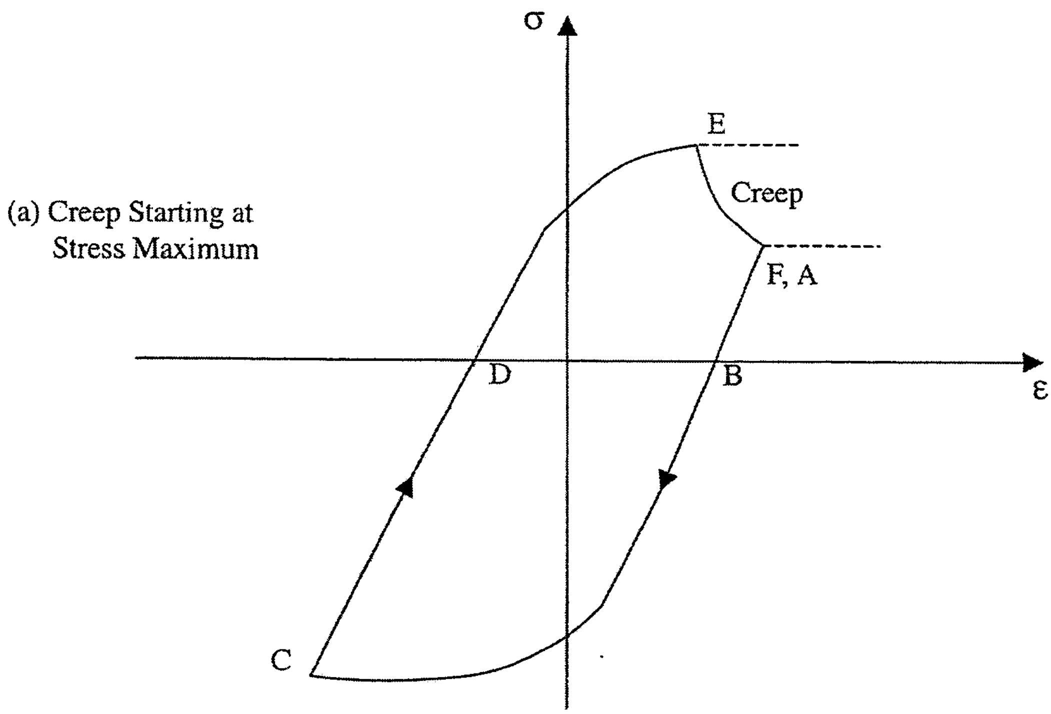
The classic creep-fatigue mechanism for the initiation of a crack from an initially defect-free structure involves a stress concentration which is sufficient to cause cyclic plasticity. On its own this would just cause fatigue. But if the structure operates at creep temperatures, creep damage will accumulate as well.

The key feature of creep-fatigue is the synergistic interaction of plasticity and creep. With no cyclic plasticity, any secondary stresses would relax under the action of creep. In the long term, therefore, the creep damage would be driven by the primary loads alone. Provided the structure is well designed, primary stress driven creep damage should be modest. However, cyclic plasticity will often cause the stress to be reset on each load cycle. Each dwell at high temperature may therefore be forced by the effects of the load cycle to start creeping from the same (reset) stress. In effect the secondary part of the stress is being renewed on every cycle, thus considerably enhancing the creep damage. This is the creep-fatigue damage mechanism. When active, it can be a very virulent damage mechanism.

The creep-fatigue damage mechanism consists of cyclic plasticity due to load cycling resetting the dwell stress on each cycle, and consequently a high rate of creep damage accumulation (in addition to the usual fatigue damage).

Qu.: Where do hysteresis loops come in?

Figure 1: Illustrative Hysteresis Loop



The construction of stress-strain hysteresis loops is central to an R5V2/3 assessment. We shall have a lot more to say about this in Sessions 32 and 33. For now note the following,

- The energy absorbed per cycle, which does the damage, is the area within the hysteresis loop*;
- The fatigue part of the damage is related to the overall strain range (roughly $\varepsilon_A - \varepsilon_C$, though there are some refinements to this in the procedure);
- The creep part of the damage is related to the creep strain, which we shall see is $\varepsilon_{creep} = \varepsilon_F - \varepsilon_E + (\sigma_E - \sigma_F)/\overline{E}$ per cycle.

Construction of the hysteresis cycle is key to an R5V2/3 assessment since it provides,

- The strain range from which the fatigue damage is calculated, and,
- The start-of-dwell stress, σ_E, from which the creep damage is calculated.

Strictly, of course, the area within a stress-strain hysteresis loop is the energy* ***density* *at the point of the structure assessed. You would need to integrate this over the volume of the structure to get the total energy.*

Qu.: What failure modes must be addressed as a prerequisite to an R5V2/3 assessment?

Assessments must be carried out to ensure avoidance of failure by,

- Plastic collapse;
- Creep rupture;
- Ratcheting;
- Cyclically enhanced creep (= creep ratcheting);

In addition to these failure modes, an R5V2/3 assessment is valid only if the following is also satisfied,

- Avoidance of excessive deformation

These issues are discussed individually in more detail below.

Qu.: How is avoidance of plastic collapse ensured?

The simplest assessment against plastic collapse is satisfaction of the primary stress limits, Equs.(6.1), (6.2) in R5V2/3. When the creep exponent, n, is >2, as it usually will be, these are similar to the requirements of many structural design codes, i.e.,

$$P_m \leq 0.67 S_y \tag{1}$$

$$P_L + P_B \leq S_y \tag{2}$$

where S_y is the lower bound 0.2% proof stress, and,

P_m = the primary membrane equivalent stress

P_L = the local primary membrane equivalent stress ($P_L \geq P_m$)

P_B = the primary bending equivalent stress

For the definitions see R5V2/3 Section 3.6.

Qu.: What subtleties are involved in the definitions of P_m, P_L, P_B ?

In my opinion both R5 and ASME are horribly confusing about the definitions – especially of the bending component, P_B . For example, in calculating P_m, P_L, P_B for a pipe, should linearisation be carried out across the whole pipe diameter, or just across the wall thickness? In other words, is P_B wall-bending or global bending? I make some observations on these issues in Appendix A. Below is some very brief guidance – with no guarantee!

Qu.: Global Bending or Wall-Bending?

My simple guidance on the global bending versus wall-bending issue is as follows,

- When carrying out an R5V2/3 assessment, P_B should generally be interpreted as wall-bending, so the linearisation is carried out across the wall thickness. This is consistent with the stress limit given by (2) above.
- When carrying out an ASME III NB assessment for a pressure vessel, P_B should generally be interpreted as wall-bending, consistent with R5. So again the

linearisation is carried out across the wall thickness and the stress limit is equivalent to (2)[1], i.e., $1.5S_m$.

- However, ASME III NB treats pipes in a different manner, explicitly defining P_B as global bending, i.e., carrying out the linearisation with respect to the whole pipe cross-section. My reading of ASME is that the corresponding stress limit is not then $1.5S_m$ but rather αS_m, where α is the relevant fully plastic moment factor (see ASME III NB-3221.3), e.g., $4/\pi = 1.273$ for bending-only.

Essentially, ASME III NB makes piping a special case because it also functions as a piping code (NB-3600). In contrast, in R5 you would usually assess a point on a pipe in the same way as any other pressure vessel. That's my interpretation – but I could be wrong!

Qu.: How is the linearisation done for multi-axial stressing?

I believe that the R5V2/3 Section 3.6 definition of P_B is **wrong** for multiaxial stressing. It says that you should evaluate P_B by,

- linearising the stress components;
- the Mises combination of the membrane (constant) part of each component is P_L;
- subtracting the membrane part from each component to leave 'bending components';
- the Mises combination of these bending components is P_B.
- then form the sum $P_L + P_B$

This is what Section 3.6 implies, but this is **wrong**.

Qu.: How can P_B be defined correctly?

It can't.

P_B does not exist as a separate quantity.

Qu.: Eh?

It makes no sense to add two quantities, P_L and P_B, both of which are Mises combinations of components - because the Mises function is non-linear.

In my opinion the only quantity which is defined is what is usually denoted "$P_L + P_B$". The notation suggests that this is the sum of two quantities, but really it is not. Although P_L is well defined, there is no sensible quantity P_B, nor is one necessary. The stress limit is defined with respect to "$P_L + P_B$" and only "$P_L + P_B$" need be defined.

[1] That is, when restricted by the low temperature (plastic) properties. R5 addresses the creep issues separately.

Qu.: My definition of $P_L + P_B$

I offer the following definition, which I hope is what people have actually used in practice (is it?)...

> *The linearised primary stress components are the stresses which, if varying linearly through the section, would give the same force and moment resultants as the elastically calculated distribution. The stress quantity "$P_L + P_B$" is then the maximum equivalent stress calculated from the multiaxial components of these linearised primary stresses in the section.*

In common with the existing R5 definition, this definition involves linearising the stress components *before* forming the Mises combination, rather than the reverse.

However, it differs from the existing R5 definition in that neither bending components nor a purely "bending Mises" is calculated. The procedure is simply,

- linearise the stress components;
- form the Mises combination of the linearised components on each of the two surfaces;
- whichever surface gives the larger result defines $P_L + P_B$

The quantity "$P_L + P_B$" should not be interpreted as the sum of two quantities. It may be tempting to define a quantity "$P_L + P_B$" – P_L but this quantity is meaningless in my opinion. It should not be regarded as a bending stress (and no such quantity, P_B, is separately required by the R5 procedure).

I believe my definition is consistent with R5V2/3 Appendix A2, last sentence of §A2.3.2.

Qu.: Where is the Mises stress formed from the linearised stresses greatest?

The maximum is always on one of the two surfaces and hence it suffices to calculate the Mises combination on the two surfaces only. The proof is left as an exercise for the reader.

Qu.: Is this just pedantry?

No.

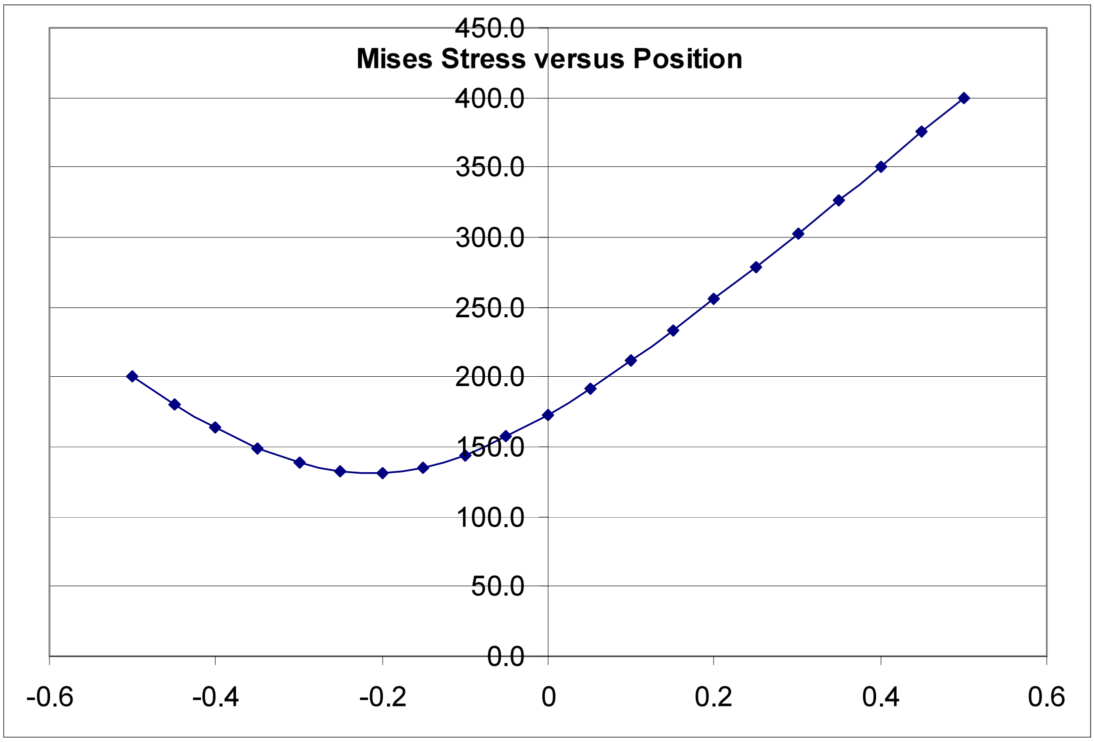
The difference is numerically large. Here's an example.

Suppose the stresses linearised through the wall were,

	x	y	z
membrane	300	200	100
bending	200	300	0

The existing R5V2/3 definition gives $P_L \pm P_B$ = 438 and -91, whereas the actual Mises stresses on the two surfaces are 400 and 200, and my definition gives "$P_L + P_B$" = 400. Hence my definition gives a result 10% smaller.

The R5 stress P_B is 265 in this case (P_L is 173) and it is hard to attach any particular meaning to this quantity considering how the Mises stress actually varies through the thickness (see graph below). Note that the maximum principle stress is always positive, so a "signed Mises" stress is identical.



Qu.: Is that the only way of ensuring plastic collapse is avoided?

No.

Other options are discussed in R5V2/3 Appendix A5, Section A5.2.1.

Option 3 is to use lower bound plastic limit load analysis – in other words, to use (or derive) a collapse solution. The loads to include in such a limit load analysis are the primary loads, including and thermal or displacement controlled loads which are judged to be sufficiently long range that they may act in a primary manner. This must cover the worst combination of loading and temperature to which the structure is subject (i.e., including short-term fault loading). The reference stress derived from the limit analysis must not exceed two-thirds of the lower bound 0.2% proof stress ($0.67S_y$).

Qu.: How is avoidance of excessive plastic deformation ensured?

R5V2/3 Equs.(6.3), (6.4) give limits on the combined primary + secondary stress ***range*** which are intended to ensure avoidance of excessive plastic deformation. They are, assuming n>2,

Ferritics: $$\Delta(P_L + P_B + Q) \le 2.0S_y \quad (3)$$

Austenitics: $$\Delta(P_L + P_B + Q) \le 2.7S_y \quad (4)$$

where Q is, in some sense, a linearised secondary equivalent stress. But we run into the same problem with Q as we did with P_B...

Qu.: How is Q defined in R5V2/3?

R5V2/3 Section 3.6 defines Q as a separate quantity. I believe this is wrong, or at best inappropriate. In any case it is not necessary. The quantity Q is not required as a separate quantity in the R5V2/3 procedure.

Qu.: How is $\Delta(P_L + P_B + Q)$ defined?

My definition, which I think is what R5 actually wants, is,

- For the total loading (primary and secondary loads) under operating condition 1 linearise all stress components;
- Repeat this for loading condition 2;
- Form the range of each linearised component (condition 1 - condition 2) on both surfaces;
- Form the Mises combination of these linearised component stress ranges;
- $\Delta(P_L + P_B + Q)$ is the largest value which results from any choice of operating conditions 1 and 2.

Qu.: Is that the only way of ensuring excessive plastic deformation is avoided?

No.

The alternative is full elastic-plastic cyclic FEA (as discussed in R5V2/3 Appendix A12). But, of course, you can only trust the result of such an FEA if you are sure that the constitutive law / flow rule are correct.

In practice the stress range limits can be a problem – and full elastic-plastic cyclic FEA is not exactly convenient. To my mind failure to satisfy the stress range limits need not be fatal to an assessment if the global shakedown criterion is met (see below). It seems to me that this also guards against excessive plastic deformation.

Qu.: Do the stress range limits protect against fatigue?

No.

Whilst they look like they might, being limits on stress ranges, fatigue is assessed separately in R5V2/3 as part of the crack initiation assessment.

Qu.: Do the stress range limits protect against ratcheting?

A common misconception is that the stress range limits, (3,4), constitute a shakedown assessment. They do not, and R5 does not say they do. Shakedown is assessed in a different manner, described below.

Qu.: Are the stress range limits crucial?

I'm committing blasphemy here, but in view of the fact that,

- Fatigue is assessed separately, and,
- Shakedown is assessed separately, and,
- Satisfaction of the global shakedown criteria protects against excessive deformation,

I suggest that failure to satisfy the stress range limits, (3,4), need not be considered fatal to an R5V2/3, subject to these other assessments being favourable.

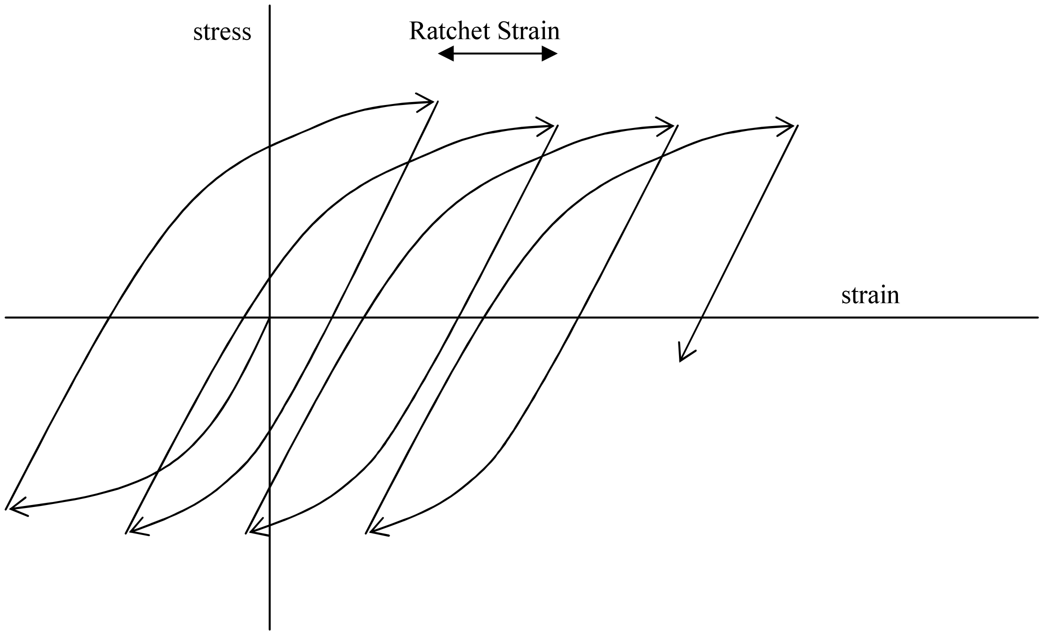
Qu.: How is avoidance of creep rupture ensured?

This has been covered in the sessions on T73S06. The key parts of R5V2/3 are Section 6.5 and Appendix A5, Section A5.2.2.

Qu.: What is ratcheting?

Ratcheting occurs if the stress-strain hysteresis loop does not close, but instead moves forward to greater plastic strain on each cycle. The increment of plastic strain on each cycle is the ratchet strain. It looks roughly like…

Figure 2: Illustration of Ratcheting



Qu.: Is ratcheting a failure mechanism?

No.

Strictly speaking, ratcheting is a deformation mechanism, not a failure mechanism. However, if it occurs it is likely to be serious and lead to failure due to ductility exhaustion, or correlated fatigue damage.

In practice failure due to ratcheting is rare on our plant. This is likely to be due to good design against primary loads. However, it may also be that ratcheting is not identified as such, but classified as fatigue.

Qu.: Is ratcheting a structural phenomenon or a material phenomenon?

Yes.

(See session 38A).

Qu.: What is shakedown?

Shakedown is the avoidance of ratcheting.

Qu.: What is strict shakedown?

Ignoring creep for the moment, in strict shakedown all parts of the structure cycle elastically after the first few loading cycles.

Qu.: What is global shakedown?

In global shakedown limited parts of the structure may cycle with an elastic-plastic hysteresis loop, but this hysteresis loop is closed and does not move progressively to larger strain.

Qu.: Is creep included when assessing ratcheting?

Yes and no.

Ratcheting is often considered to be a purely elastic-plastic phenomenon. This is the case in R5. So in R5 the ratcheting (or shakedown) assessment does not address creep effects. In case of doubt this could be called a plastic ratcheting (or shakedown) assessment.

However, the physical reality is that creep can contribute to the ratcheting mechanism. It can drive the hysteresis cycle in Figure 2 to progressively larger strains. But in R5 the assessment of this mechanism is called "cyclically enhanced creep". It is an unfortunate piece of terminology in my opinion. Cyclically enhanced creep is just ratcheting due to creep: there is no difference other than terminology, in my opinion. (A cautionary warning, though, Bob Ainsworth appears to have a different opinion - I must address that some time).

So the ratcheting/shakedown assessment required by R5 is elastic-plastic only because the creep part of the mechanism is addressed separately.

Qu.: How is avoidance of ratcheting ensured?

In R5 ratcheting is avoided by satisfying the global shakedown criterion.

Qu.: What is the "shakedown factor", K_S?

K_S is a material-dependent factor which must be found experimentally and varies with temperature. Examples are given in R5V2/3 Figures 5.1, 5.2 (reproduced below). It is the factor by which the cyclic stress amplitude can exceed the yield stress and still produce shakedown.

Qu.: What is the criterion for strict shakedown?

A structure is in strict shakedown if the steady cyclic state involves all points of the structure moving up and down the elastic line on the stress-strain diagram. Plasticity occurs only in the first few cycles, whilst the steady cyclic state is being established. Thereafter the structure behaves elastically.

Strict shakedown occurs if the steady cyclic Mises stress is *everywhere* within $K_S S_y$ *at all times* during the cycle. Note that this Mises stress is the total Mises stress, including the shakedown residual stress, not just the elastic Mises stress. It also includes peak (F) stresses, not just linearised stresses.

The criterion for strict shakedown is thus,

If the total Mises stress is everywhere less than $K_S S_y$ at all times during the load cycle, then the structure is in strict shakedown. (The total Mises stress includes the residual stresses established in the steady state cycle).

Figure 3a: K_S for 316 (R5V2/3 Issue 3 Rev.002)

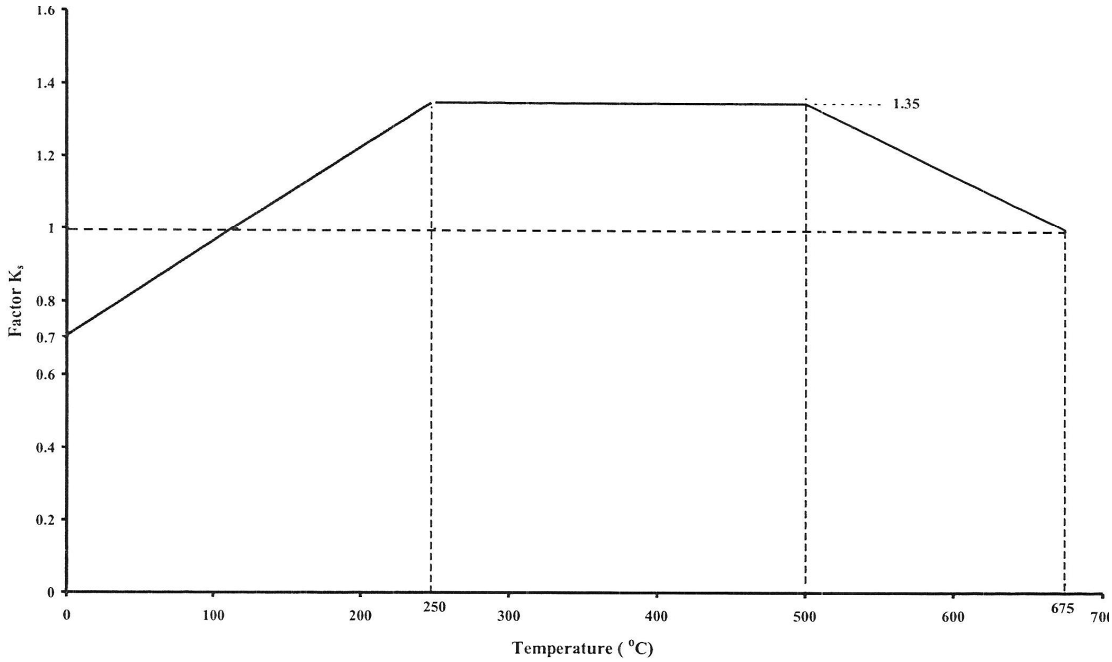
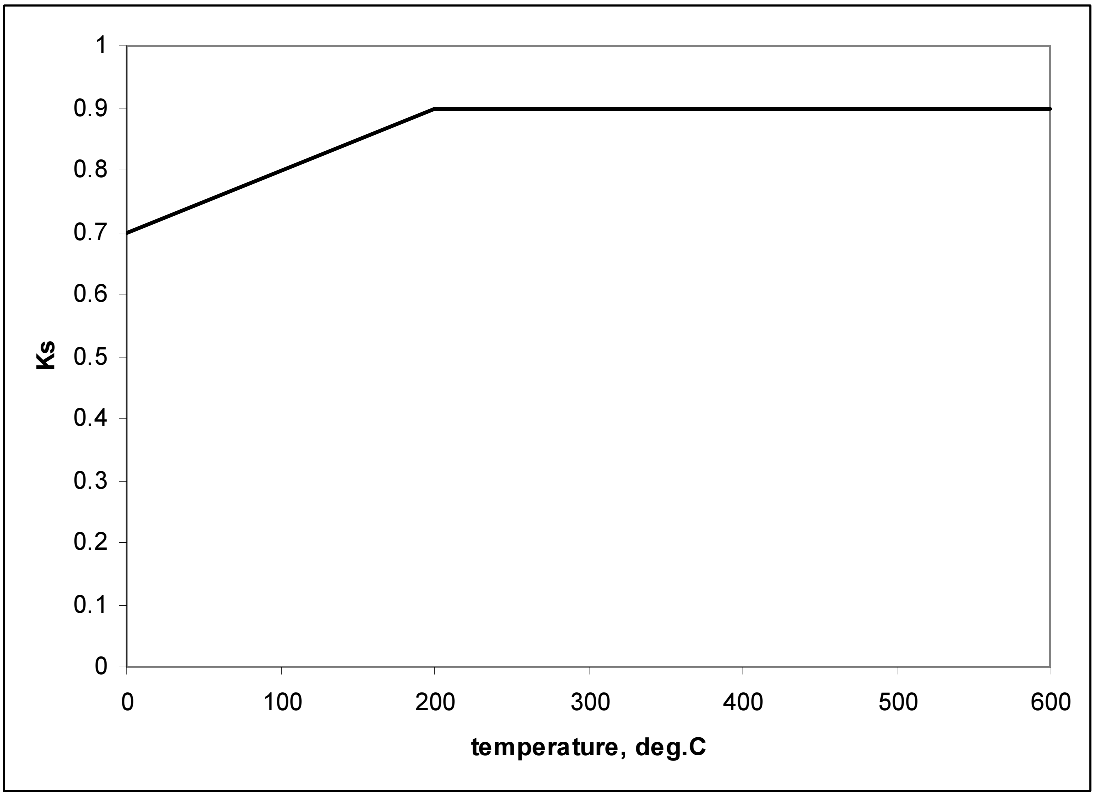


Figure 3b: K_S for CMV and maybe other ferritic steels (R5V2/3 Issue 3 Rev.002)



Qu.: What is the global shakedown criterion?

The global shakedown criterion in R5V2/3 is this,

If at least 80% of every section of the structure has a Mises stress which is less than $K_S S_y$ at all times during the load cycle, then the structure is said to be in "global shakedown". (The total Mises stress includes the peak stresses and the residual stresses established in the steady state cycle).

Qu.: How is strict or global shakedown established rigorously?

The Shakedown Theorem states that if any self-equilibrating stress field (i.e., such that $\sigma_{ij,j}^{residual} = 0$) can be found such that the Mises stress formed from components $\sigma_{ij} = \sigma_{ij}^{elastic} + \sigma_{ij}^{residual}$ does not exceed $K_S S_y$ at any location or at any time during the loading cycle, then the structure will strictly shakedown.

The self-equilibrating stress need not be the actual residual stress which occurs. That any such field exists is sufficient to establish strict shakedown.

If the above holds over at least 80% of all structural cross-sections, then this establishes global shakedown.

Global shakedown is taken to ensure the avoidance of ratcheting (and also, I think, avoidance of excessive plastic deformations).

Qu.: Is it easy to apply this criterion?

No.

It is not a simple matter in general to optimise the residual stress field assumed. If a simple assumption for the residual stress successfully produces a Mises stress less than $K_S S_y$ at all times, and over 80% or more of every section, then job done.

But if not, how can you be sure that some other assumption for the residual stress might not succeed in doing so? The optimal residual stress is difficult to establish.

Qu.: What is the "simple test for global shakedown" in R5V2/3 Sect.6.6.1?

The 'simple test' of Section 6.6.1 makes the simplifying assumption that the residual stress is zero. The global shakedown criterion is otherwise the same, i.e., the requirement that the Mises stress be less than $K_S S_y$ at all times over 80% of the section.

As written in Issue 3 Rev.002, the 'simple test' also requires that the linearised elastic Mises stress be less than KsSy across the whole section at all times, R5V2/3 Equ.(6.11). However I believe this is not really necessary and have raised User Query 165 on this matter.

Note that failure to meet the simple criterion based on the elastic Mises stress does not necessarily imply that shakedown will not occur. You would just need to do the job properly, i.e., find a residual stress field closer to optimal.

Qu.: What's magical about the 80% figure in the global shakedown criterion?

Nothing.

The important thing is that there is a big elastic core which prevents plastic distortion getting out of hand.

Could a figure of 70% or 60% or 50% be used instead? Quite possibly, but the smaller the elastic core the more doubt there is. However there is no obvious cliff edge.

Qu.: Is the R5 strict shakedown criterion $\Delta\bar{\sigma}_{elastic}^{\max} \leq (K_S S_y)_{top} + (K_S S_y)_{bottom}$?

No.

R5 does not say it is, and it would be wrong to do so. $\Delta\bar{\sigma}_{elastic}^{\max} \leq (K_S S_y)_{top} + (K_S S_y)_{bottom}$ is not a generally applicable criterion for shakedown – contrary to popular belief. The issue has become confused partly because design codes (e.g., ASME) do imply that a "twice yield" type limit on elastic stress range is indeed a shakedown assessment.

Qu.: But is $\Delta\bar{\sigma}_{elastic}^{\max} \leq (K_S S_y)_{top} + (K_S S_y)_{bottom}$ a rule-of-thumb guide to shakedown?

Based on the Bree analysis (of which more below) – yes, the criterion $\Delta\bar{\sigma}_{elastic}^{\max} \leq (K_S S_y)_{top} + (K_S S_y)_{bottom}$ can be a reasonable rule-of-thumb guide to shakedown ***provided that the primary stress is less than half yield***, $\sigma_{primary} < 0.5\sigma_y$.

Note that this rule-of-thumb is not in R5.

Qu.: Can the creep-fatigue mechanism occur in a structure within strict shakedown?

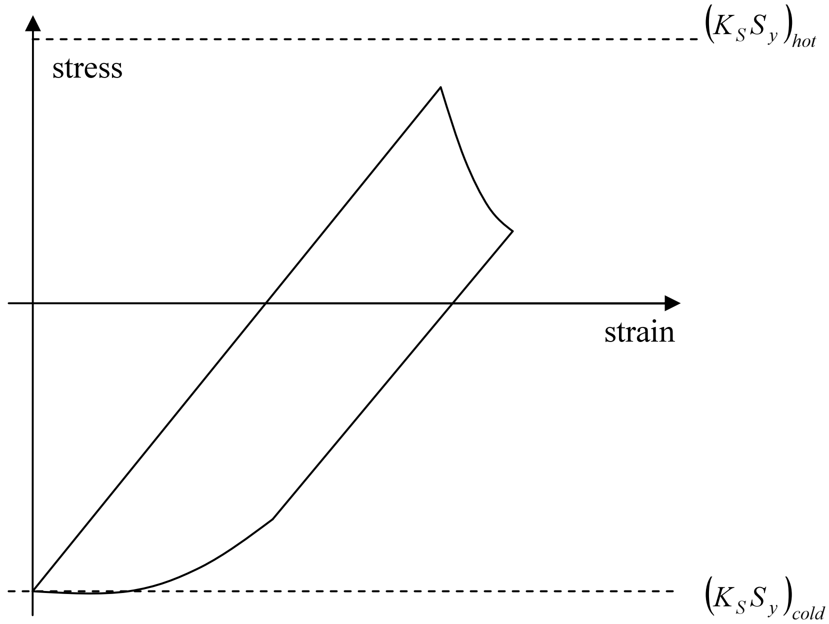
Oddly, yes.

The creep-fatigue mechanism, as opposed to creep and fatigue as separate mechanisms, is defined by the resetting of the creep dwell stress by cyclic plasticity and the resulting synergy between creep and plasticity. So a hysteresis loop is essential for the creep-fatigue mechanism to be active. Can we get a hysteresis loop in strict shakedown?

You might think not, since a structure within strict shakedown cycles elastically – so there is no hysteresis loop. But this is only true when we confine attention to elastic-plastic strains. When creep strains occur, a hysteresis loop may be generated even when the elastic-plastic (short-term) cycles are within strict shakedown, i.e., when the cycles would be purely elastic in the absence of creep. An example is shown in Figure 4.

Of course, it may still be necessary to perform an R5V2/3 assessment even when there is no hysteresis loop and hence no creep-fatigue as such. This is because of the other damage/failure modes which must be considered. And cracks could still be formed under the action of creep which is undisturbed by cyclic plasticity (e.g., reheat cracking).

Figure 4: Hysteresis with $\Delta\bar{\sigma} \le (K_S S_y)_{cold} + (K_S S_y)_{hot}$



Qu.: So is an R5V2/3 creep-fatigue initiation assessment all about hysteresis loops?

Largely, yes, because it is the hysteresis loop which resets the dwell stress and this is the essence of the creep-fatigue mechanism.

If the structure is outside strict shakedown but within global shakedown, and the dwell occurs at creep temperatures, then the creep-fatigue mechanism will certainly be active. So structures outside of strict shakedown but within global shakedown are the main province of non-trivial creep-fatigue assessments.

However, as we have seen in Figure 4, hysteresis loops can also arise due to the combined effect of creep and plastic strains, so that the creep-fatigue mechanism can occur even in strict shakedown when the cycles would be elastic in the absence of creep, and when we may have $\Delta\bar{\sigma}^{\max}_{elastic} \le (K_S S_y)_{top} + (K_S S_y)_{bottom}$.

If the structure is outside global shakedown then it fails the R5V2/3 assessment. It fails R5V2/3 – but you may still be able to make a case based on quantifying the ratcheting, but that would be beyond R5. See session 38A.

Qu.: Are R5V2/3 assessments required in strict shakedown?

Yes, certainly.

At least such assessments must assess creep and fatigue considered separately. This may still be an onerous assessment and lead to crack formation, because…

- Fatigue alone can initiate cracks at temperatures below the creep regime;
- Forward creep due to steady primary stresses can also lead to failure without any contribution from fatigue;
- Monotonically relaxing secondary stresses, undisturbed by load cycling, can also cause crack initiation, especially if sufficiently triaxial and in a material with poor triaxial creep ductility (reheat cracking).

But, in addition, in certain circumstances a hysteresis loop may be generated even in strict shakedown – and hence the creep-fatigue mechanism may be active with the dwell stress being re-primed at each cycle.

Qu.: Why is the stress range criterion not sufficient to guarantee strict shakedown?

One reason is that the stress range criterion, e.g. $\Delta(P_L + P_B + Q) \le 2.7S_y$ for austenitics, involves the linearised stresses only. So there could be a severe SCF causing peak stresses (F-stresses) which would cause the total elastic stress range $\Delta\overline{\sigma}_{elastic}^{\max}$ at the point to be substantially larger than $\Delta(P_L + P_B + Q)$. But we have seen that even $\Delta\overline{\sigma}_{elastic}^{\max} \le (K_S S_y)_{top} + (K_S S_y)_{bottom}$ does not ensure strict shakedown. Actually the stress range criterion does not even guarantee global shakedown, let alone strict shakedown, as we can see from…

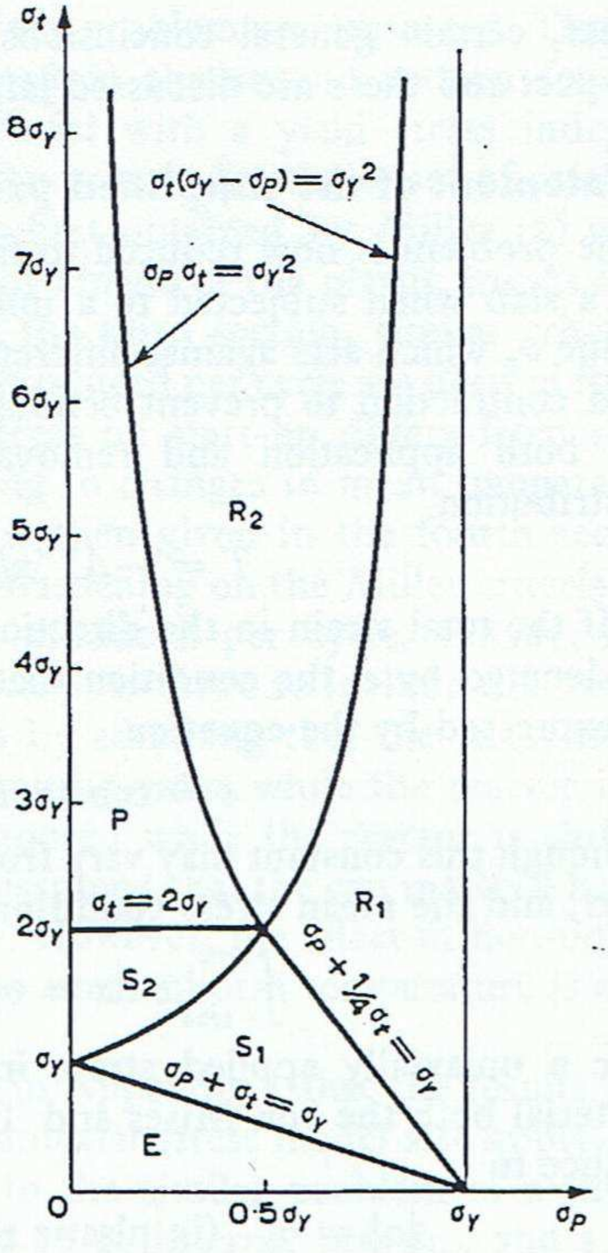
Qu.: What is the Bree Diagram?

The Bree diagram is a plot of the secondary elastic stress range against the primary stress. The original diagram, reproduced as Figure 5 below, was derived for uniaxial stressing with a constant primary membrane stress and a cycling secondary bending stress. Despite being derived for a rather specific geometry and loading, the Bree diagram is worth studying carefully since it illustrates the archetypal shakedown/ratcheting behaviour of a structure under combined primary and secondary loads.

The original Bree paper, Ref.1, is rather terse. I have re-derived the diagram in gory detail. However, now I have had more practice at doing these types of analyses I realise that it can be done far more painlessly using some simple graphical constructions, a method which is illustrated on a variant problem here.

Note that if the primary stress (σ_p) is sufficiently large in Figure 5 ratcheting occurs for very small secondary stress ranges (σ_t).

Figure 5: The Bree Shakedown Diagram



Stress régime	Can behaviour
R_1 and R_2	Ratchetting
S_1 and S_2	Shakedown after first half-cycle
P	Plastic cycling
E	Elastic

Bree Model - Implications

- Note that ratcheting occurs for a stress range (σ_t) which is less than $2\sigma_y$ if the constant primary membrane stress (σ_p) exceeds $0.5\sigma_y$. Indeed, the stress range required to cause ratcheting becomes vanishingly small as $\sigma_p \to \sigma_y$.
- This illustrates why the stress range criterion cannot guarantee shakedown.
- It also illustrates why the criterion $\Delta\bar{\sigma}_{elastic}^{max} \leq (K_S S_y)_{top} + (K_S S_y)_{bottom}$ is an unreliable guide to shakedown (though probably correct provided the primary stress is less than half yield – my "rule-of-thumb").

The Bree model produces a very different shakedown diagram if the primary membrane stress is assumed to cycle in-phase with the secondary bending stress.

Figure 6a: The Shakedown Diagram when Primary Stress Cycles In-Phase of In Exact Anti-Phase

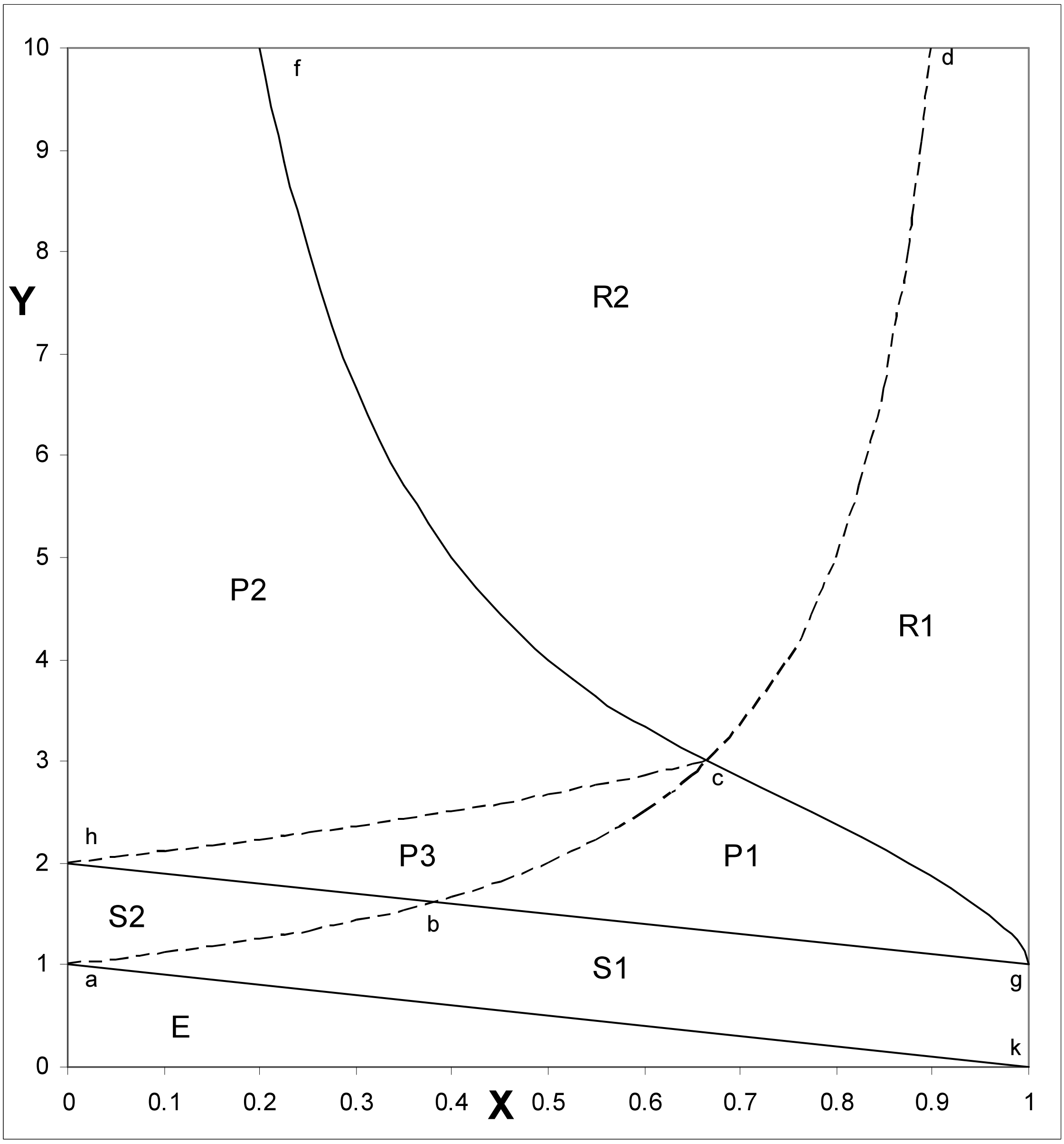
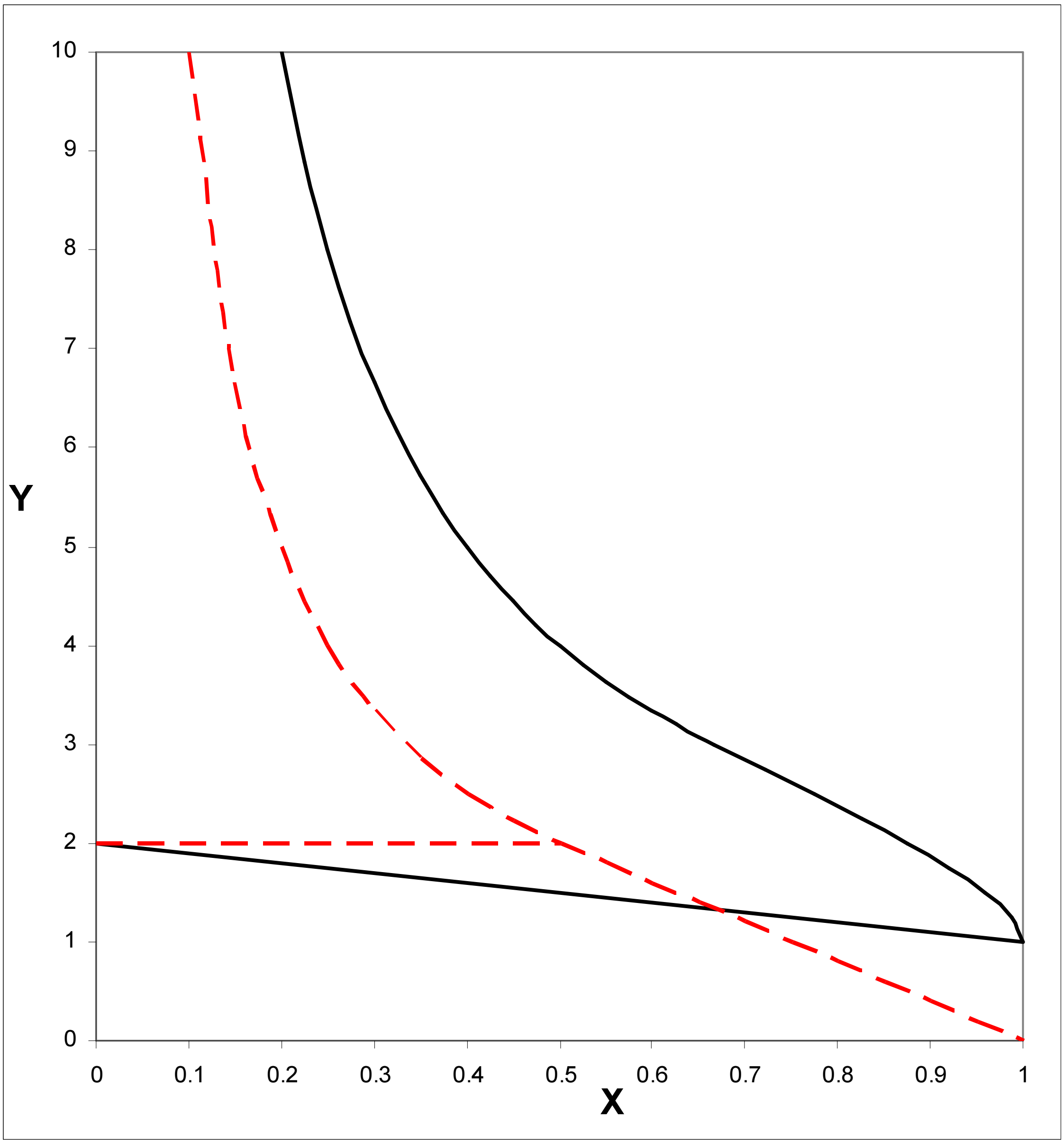


Figure 6b: Shakedown and ratcheting regions for in-phase or exact anti-phase cycling primary load compared with those of the original Bree problem (non-cycling primary load). Continuous black lines are for in-phase or exact anti-phase cycling primary load, Ref.[2]; Dashed red lines are the original Bree case with constant primary load, Ref.[1]. For both problems, the upper region is ratcheting, the lower region is elastic or strict shakedown to elastic cycling, and the middle region is stable plastic cycling (global shakedown).



WARNING: Work yet to be published shows that if the primary load cycles out-of-phase with the thermal load (as opposed to exact anti-phase) then the ratchet boundary, though above the Bree boundary, is much closer to it than for the in-phase or exact anti-phase cases.

Qu. What is the "shakedown reference stress"?

R5V2/3 Section 7.2.1 defines the shakedown reference stress, $\bar{\sigma}_{ref}^{S}$, and the corresponding temperature. My reading of R5 is that there are really two different "shakedown reference stresses" used for different purposes, as follows:-

Definition 1 relates to the use of the shakedown reference stress as a dwell stress for the assessment of creep damage and is,

- The steady cyclic stress field, $\sigma_S(x,t)$, is first estimated in some way. This is the stress field at any time t during the load cycle, after the stress-strain hysteresis loop has shaken down to its steady cyclic state. It can be obtained from the elastic stress field at the time by adding the shakedown residual stress field, $\sigma_S(x,t) = \sigma^{elastic}(x,t) + \sigma^{residual}(x)$.
- For this purpose it is conservative to use the full stress field including peak stresses (F-stresses). However, it is permissible to use the linearised stresses (without peak stresses).
- For each dwell period in the creep regime, the time to rupture at stress $\sigma_S(x,t)$ is found, using the temperature, T_S, relevant at that time. The point within the structure, and time during the loading cycle, which minimise the rupture time are identified. The Mises equivalent stress formed from $\sigma_S(x,t)$ at this point and time is the shakedown reference stress, $\bar{\sigma}_{ref}^{S}$, and the corresponding temperature is the shakedown reference temperature, T_{ref}^{S}.
- The shakedown residual stress should be optimised so as to minimise the resulting shakedown reference stress, $\bar{\sigma}_{ref}^{S}$, whilst at the same time respecting the global shakedown constraint.

The last requirement is difficult to fulfil in practice and often renders Definition 1 problematical to employ in real plant assessments.

Definition 2 relates to the use of the shakedown reference stress as a 'core stress' for the assessment of cyclically enhanced creep. It differs from Definition 1 in the following respects,

- The peak (F) stresses must be included, but,
- In finding the most onerous $\sigma_S(x,t)$ attention is confined to the 'elastic core' region, defined as the region which is within strict shakedown (i.e., the ">80%" region).

Qu.: Do we really need the shakedown reference stress to assess creep?

As regards the creep component of damage contributing to crack initiation - no.

You can perform R5V2/3 initiation assessments without using the shakedown reference stress to assess the creep damage. Personally, I never use it in real assessments. The alternative which I will advocate in these tutorials is to estimate the start-of-dwell stress by construction of the hysteresis loop.

BUT a core stress of some sort is required to assess cyclically enhanced creep, though there are alternatives to the above shakedown reference stress.

Qu.: What is cyclically enhanced creep?

Cyclically enhanced creep refers to the threat of gross section creep failure due to the accumulation of creep strains cycle-on-cycle. Cyclically enhanced creep differs from creep rupture in two ways,

- Creep rupture assumes constant loading, whereas cyclically enhanced creep accounts for the effects of cycling;
- Creep rupture relates only to primary loads, whereas cyclic creep can be enhanced by secondary loads.

Many people have difficulty understanding what cyclically enhanced creep actually is. It's very simple…

Cyclically enhanced creep is creep ratcheting

Cyclically enhanced creep would not be evident from an R5V2/3 hysteresis cycle construction. (Nor would ordinary plastic ratcheting). But, in principle, it should be evident from a non-closed cycle in a finite element analysis (just like plastic ratcheting) – always supposing you get the constitutive properties right (that's a big "suppose", by the way).

Plastic ratcheting occurs if non-closure of the hysteresis loop is due to plastic strains. Creep ratcheting occurs if non-closure of the hysteresis loop is due to creep strains.

Qu.: How is avoidance of excessive cyclically enhanced creep assessed?

R5V2/3 Section 7.5 provides the procedure for calculating a gross ligament creep usage factor (called W) which includes the effects of cyclically enhanced creep. Two alternative methods are provided. Either,

[1] Use the shakedown reference stress, $\bar{\sigma}_{ref}^{S}$, as defined above, and the corresponding shakedown reference temperature, to find a rupture time from the stress/time-to-rupture curve. This provides a conservative estimate of W as the time-fraction, using Robinson's rule if necessary.

Or,

[2] A less conservative method is provided in Section 7.5.3. The method works by defining an alternative "elastic core stress", σ_{core}, in terms of the primary and secondary stresses, and then using this core stress in the stress/time-to-rupture curve. However, this alternative method is of limited applicability.

Qu.: What is the 'core stress' based method for cyclically enhanced creep?

There was an error in this procedure which has been corrected in the 2014 revision (R5 Issue 3 Rev.002). The method is based on Ref.[2]. The core stress is defined in terms of dimensionless quantities: X is the primary membrane stress normalised by S_y, and Y is the secondary bending stress range normalised by S_y. The core stress is defined by R5V2/3 Equs.(7.19a-c),

For $Y \leq 1 - X$: $\sigma_{core} = XS_y$ (Region E)

For $1 - X < Y < \frac{1}{1-X}$: $\sigma_{core} = \left\{Y + 1 - 2\sqrt{Y(1-X)}\right\}S_y$ (Region S_1)

For $Y \geq \frac{1}{1-X}$: $\sigma_{core} = XYS_y$ (Regions S_2 and P)

The regions refer to the Bree diagram (see Figure 5 above), noting that the above expressions for the core stress are a by-product of the Bree analysis.

Inserting the core stress into an appropriate rupture expression at the shakedown reference temperature gives the cyclically enhanced creep damage term, W.

Qu.: When is this 'core stress' based method applicable?

It is applicable only in the following rather restrictive conditions,

- The primary stresses approximate to membrane in nature;
- The primary stresses are non-cycling;
- The secondary stresses approximate to a bending stress, and cycle between zero and their maximum value;
- The secondary stresses are transient, and do not occur during the creep dwell.

The last of these conditions was not made explicit prior to R5V2/3 Issue 3 Rev.002, but is now stated.

The method is therefore not applicable if thermal stresses persist during steady operation.

Qu.: Is this 'core stress' method consistent with design codes?

The same 'core stress' equations appear in (at least) two places in ASME:-

[1] The thermal ratcheting equations of ASME III NB-3222.5 for a linear temperature distribution provide the same limit on Y as Equs.(7.19b,c), as can be seen by substituting "core stress = Sy".

[2] In ASME III NH Appendix T, para T-1332(c), dealing with creep-fatigue assessment, the same definition of core stress can be found.

Note that $\bar{\sigma}^S_{ref}$ and σ_{core} are both supposed to be representative of the gross section, and hence indicative of rupture.

Qu.: Does the W damage term add to other damage terms?

No. It is a stand-alone assessment and is not added to other damages, e.g., the creep rupture damage, U, or the cyclic creep-fatigue damage, Dc. This is because W assesses creep ratcheting and hence is a different mechanism.

Qu.: Is this 'core stress' method conservative?

Yes, perhaps overly conservative. Note that $\bar{\sigma}_{ref}^{S}$ or σ_{core} are implicitly assumed not to relax during the dwells. This seems grossly pessimistic to me, since secondary stresses may be a large part of $\bar{\sigma}_{ref}^{S}$ or σ_{core}. As a result the assessment of cyclically enhanced creep in R5V2/3 often presents the assessor with a problem which may be an artificial problem.

Qu.: Is W always greater than U?

Since W includes the secondary stresses, and cyclic effects, it is reasonable to ask whether it will always exceed U, which does not include these effects.

But U includes the effects of any stress-raiser, via the χ factor and the rupture reference stress. The W term does not include this effect, because it is intended to represent the gross 'elastic core'. So if secondary stresses are small, and there is a large SCF effect, then U could be greater than W. Consequently both assessments are required in general.

Qu.: Are there other procedures for cyclically enhanced creep?

The extremely restricted applicability of R5V2/3 Section 7.5.3, and its generally very conservative outcome, motivates research into alternatives. Some observations and current work areas are noted below.....

Qu.: Secondary Stress Distribution - ASME

The Bree-based core stress is limited to a linear (bending) thermal stress distribution. ASME III NB-3222.5 also provides a limit on Y for a quadratic variation of thermal stress – thus extending the Bree analysis beyond bending dominance. Although the ASME advice relates to shakedown (i.e., plastic strains) by the same reasoning this could form the basis of a core stress for the assessment of cyclically enhanced creep for a quadratic distribution of secondary stress.

Some work is required to re-cast the ASME equations into the right form. I am not aware that anyone has done this, but it may be an easy and valuable extension of current R5 advice.

Qu.: Other Shakedown Analysis

There are a number of new shakedown analyses, either issued or shortly to be issued, which could be used to find alternative core stresses, e.g., for,

- Primary membrane stress varying in-phase or in anti-phase with the thermal loads;
- Primary membrane stress varying out-of--phase with the thermal loads;
- Constant but biaxial primary membrane stresses, for a plate or a cylinder (hence covering a pressurised cylinder).

The method would be as per the existing 'core stress' method, and hence be applicable only for transient thermal stresses, not steady state thermal stresses.

Qu.: What about steady state thermal stresses?

The core stress determined from the Bree analysis fails to be helpful if the thermal stresses persist during the creep dwell because the core stress is not constant across the section and always includes regions at the yield stress.

An alternative approach has been suggested here. This approach uses a modification of the Bree analysis together with the isochronous creep stress at 0.2% creep strain. The analysis provides an expression for the creep ratchet strain per cycle. Ductility exhaustion can then be used as the basis of the creep ratcheting assessment.

The existing work applies to the Bree loading but could be carried out for the alternative loading cases defined above.

Qu.: What criteria determine if creep is insignificant?

This was covered in Session 23. If creep is insignificant by these criteria then all the creep parts of the R5V2/3 procedure can be omitted. The assessments of creep rupture, cyclically enhanced creep and the creep part of creep-fatigue damage are all omitted.

Qu.: What criteria determine the insignificance of cyclic loading?

The criteria are given in R5V2/3 Section 6.6.2. Cyclic loading may be deemed insignificant if all three of the following criteria are met,

- The most severe cycle is within strict shakedown*;
- Elastically based fatigue damage is less than 0.05;
- Creep behaviour is unperturbed by cyclic loading. For a creep dwell at the top of the cycle this means $\Delta\bar{\sigma}_{el,\max} < \sigma_{ref}^{R} + (\mathrm{K_S} S_y)_{bottom}$

**R5V2/3 Issue 3 Rev.002 appears to commit the sin of conflating equ.(6.14) with the shakedown criterion. I suspect this needs correction.*

Qu.: What does the "elastically based fatigue" criterion mean?

The true fatigue damage, if cycling *were* significant, would require the elastic-plastic strain range. However, the above criterion requires only the elastic strain range. This would in general be wildly inappropriate, but is perfectly reasonable here in view of the requirement to be within strict shakedown. Any cycling is thus elastic. I would expect the limit of 0.05 on elastic fatigue damage to easily be met for the numbers of cycles we usually deal with (i.e., violation would require very large numbers of cycles for elastic strain ranges).

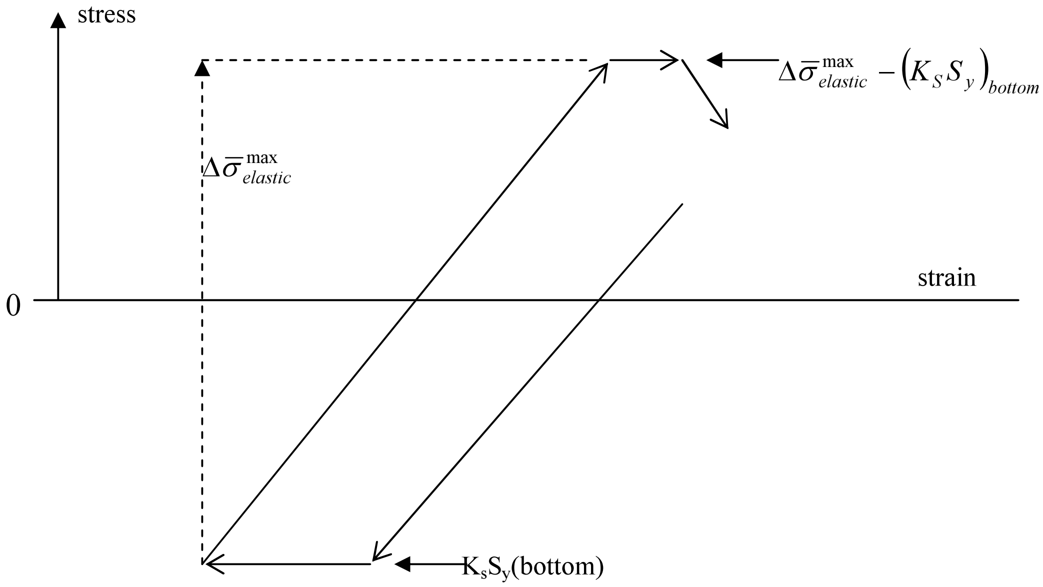
Qu.: When is creep unperturbed by cyclic loading?

The criterion to assess whether creep is perturbed by cyclic loading is given in R5V2/3 Section 6.6.2. Assuming the creep dwell occurs at the tensile end of the hysteresis loop this criterion is,

$$\Delta\bar{\sigma}_{elastic}^{\max} \le \sigma_{ref}^{R} + (K_S S_y)_{bottom} \quad (5)$$

Here σ_{ref}^{R} is the rupture reference stress, and hence depends only upon the primary loads. The reasoning behind this is that the dwell stress would normally be estimated to be $\Delta\bar{\sigma}_{elastic}^{\max} - (K_S S_y)_{bottom}$ (see diagram below). But this only applies if the result exceeds σ^R_{ref} [and if it is less than $(K_S S_y)_{top}$ as well, but we're not concerned with that limit here]. The reason is that the dwell stress cannot be less than the limit determined by the primary stresses. So if $\Delta\overline{\sigma}^{\max}_{elastic} - (K_S S_y)_{bottom} \leq \sigma^R_{ref}$ the dwell stress would be σ^R_{ref}, and hence unaffected by the cyclic loading.

Figure 7: Illustrating the Origin of Equ.(5)



References

[1] J.Bree, "Elastic-Plastic Behaviour of Thin Tubes Subject to Internal Pressure and Intermittent High-Heat Fluxes with Application to Fast Nuclear Reactor Fuel Elements", *Journal of Strain Analysis* (1967) **2**, 226-238.

[2] W.J.O'Donnell and J.Porowski, "Upper Bounds for Accumulated Strains due to Creep Ratcheting", J.Press.Vess.Tech. (Transactions of ASME), **96** (1974) 150-154.

Appendix A: Wall-Bending Versus Global Bending in R5 and ASME

1. General

The R5 and ASME stress categories and their limits appear at first glance to be very similar. However care is needed in moving between the two codes since, in truth, there are a myriad of differences. For example,

- R5 uses the symbols P_m, P_L, P_B, Q, F to mean the *Mises equivalent* stress in these categories. ASME uses these symbols to represent the six stress *components* in each of these categories, the stress limits being imposed upon the *Tresca* stresses derived therefrom.
- The primary membrane stress, P_m, is defined by ASME as including only mechanical loads / pressure, whereas R5 includes also thermal or displacement controlled loads in this category if they are deemed sufficiently long range to have a largely primary character ("stresses which cannot be completely relaxed by small local creep or plastic strains", R5V2/3 §3.6).
- The local primary membrane stress, P_L, is distinguished from P_m in ASME by the inclusion of discontinuity effects (and seismic loads). In R5 the distinction relates to the size of the region over which P_L acts, specifically whether it contains a collapse mechanism. The spirit of the two definitions is similar.

2. R5 Definition of "Structural Section" and Hence of P_B

The definitions provided in R5V2/3 §3.6 do not make clear whether P_B is intended to be a global stress or a wall-bending stress since reference is made simply to "the section" or "the structural section". R5V2/3 Appendix A2 §A2.3.2 addresses the matter explicitly in Equ.(A2.3) defining the coordinates used in the linearisation as "Cartesian coordinates…taken with respect to the centroid as origin and along the principal axes of inertia, Figure A2.1". From Figure A2.1 this appears to define the linearisation as being across the whole section of the body – hence producing the global bending component. However, I believe this is misleading and is not what R5V2/3 really intends to be used to define P_B.

I interpret P_B to be a wall-bending stress. More generally, I interpret all stress linearisation in R5V2/3 to mean linearisation across the wall thickness. I believe this is the accepted custom, but R5V2/3 perhaps needs to clarify that this is indeed what is intended.

3. ASME Definition of P_B

The ASME use of P_B is best explained by considering the specific examples given in ASME III NB Table NB-3217-1 (general pressure vessels) and NB-3217-2 (piping). For general vessels, Table NB-3217-1 makes it clear that "bending across the full section" is classed as P_m not P_B. The stress limit is therefore $S_m = 0.67S_y$ (assuming below the creep regime), ***not*** $1.5S_m = S_y$. Hence, in Figures NB-3221-1 and NB-3222-1, the ASME phrase "average across any solid section" appears to mean "averaging across any wall thickness".

There is an exception given as Note (1) to Fig. NB-3221-1, namely that the "bending component of primary stress for piping shall be the stress proportional to the distance from the centroid of the pipe cross section". Consistent with this footnote, Table NB-3217-2 identifies P_B with "Sustained mechanical loads, including weight", which are expected to give rise to predominantly global bending.. However it would be a mistake to think that the relevant stress limit is therefore $P_L + P_B = 1.5S_m$. Instead §NB-3221.3 applies and the relevant fully plastic section factor must be used. For pure bending and a thin pipe this would give a stress limit $P_B = 1.273S_m$ (i.e., the plastic limit load factor for a thin cylinder, $4/\pi$).

That R5 contains no such modified stress limits according to gross body section shape demonstrates, I think, that the R5 P_B must be interpreted as wall-bending. ASME is in accord with this in general but confusingly changes the definition to be global bending in the case of pipework.

This is my exceedingly brief attempt to make some sense of these stress categorisations, but I confess I lack confidence – partly due to the apparent contradictions in both codes. I'd be most grateful if anyone can explain to me the definition in ASME III NB-3213.7: "Bending Stress. Bending stress is the component of normal stress that varies across the thickness. The variation may or may not be linear." Eh?