T73S04 Session 38A Homework – Ratcheting
Last Update: 5/12/12

Mentor Guide Questions

1.47 Discuss the major factors influencing the outcome of a finite element analysis of a structure beyond strict shakedown.

1.48 Discuss what options may be available for a structure which is outside the R5 global shakedown limit.

Numerical Questions

1) Work through the derivation of the four bar model in the case which leads to ratcheting and check that you understand the derivation of Figure. 10 in the session 38A notes.

2) A structure approximates to a four-bar situation. The coefficient of expansion, yield strength and Young's modulus of the bar material are 17 x 10^{-6} /^{o}C, 120 MPa and 160 GPa respectively and can be assumed temperature independent. The primary load (F) is provided by a deadweight of 3.3 tonnes, borne jointly by the four bars which each have a cross-section of 1 cm^2.

(i) What is the maximum cyclic temperature change of the inner bars (ΔT) which avoids ratcheting?

(ii) If the cyclic temperature change is $\Delta T = 100^{o}C$ what is the ratchet strain per cycle?

(iii) If the tolerable ratchet strain is 5%, what is the maximum number of operating years before remedial action is required (assume 10 cycles per year)?

3) Bilinear Isotropic Hardening: Optional question for those who enjoy this sort of mathematical stuff…(see Figure over page)…

A material has a bilinear isotropic hardening behaviour. It is subject to a strain controlled cycle from an applied strain of zero to an applied strain of ε_a, then back to an applied strain of zero, etc. The cyclic behaviour on a stress-strain plot is shown below. On first application of the strain ε_a the stress is σ_1, which can be assumed to be in excess of the initial yield stress (as shown). After completing a load cycle back to zero strain and then back to strain ε_a again the stress increases to σ_2, as shown below. Similarly, after n complete load cycles (following the initial loading) the stress is σ_n. By finding the stress and strain at each point around the first cycle, or otherwise, show that,

$$\sigma_2 = \alpha\sigma_1 + \beta$$

where, $\alpha = (1 - 2\lambda)^2$ and $\beta = 2(1 - \lambda)E_p\varepsilon_a$ and $\lambda = \frac{E_p}{E}$, and E is Young's modulus and E_p is the plastic modulus (i.e., the slope of the plastic part of the bilinear stress-strain plot). Hence immediately conclude that,

$$\sigma_{n+1} = \alpha\sigma_n + \beta$$

By taking the limit $n \to \infty$ and summing the infinite series show that the stress after a large number of cycles becomes,

$$\sigma_\infty = \frac{E\varepsilon_a}{2}$$

Convince yourself that isotropic hardening automatically implies that the stress at the zero strain end of the cycle is an equal compressive stress. Hence conclude that the stress range is $E\varepsilon_a$. Hence conclude that the stable cycle is simply elastic cycling.