T73S03 Tutorial Session 44A – CFCG with Significant Cycling

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Creep-fatigue crack growth for combined primary plus secondary loading and significant load cycling: Ainsworth, Dean & Budden (again); Definition of dimensionless crack velocity; Assessment of creep crack growth if C correlation is invalid; Provision for enhanced crack growth rates prior to steady cyclic conditions; Simplified assessment of enhanced creep crack growth during redistribution when $t > t_{red}$; Combinations of materials data bounds: creep strain rate and creep crack growth law parameters, for (a)ferritic materials, (b)austenitic materials*

Qu.: Recap - Criteria for insignificant cyclic loading in R5V4/5 (§9.2)

The criteria are in two parts, for the gross behaviour when assumed uncracked and for the crack tip region. For the gross, uncracked structure, the criteria are given as in R5V2/3 Section 6.6.2. This aspect of cyclic loading may be deemed insignificant if all three of the following criteria are met,

- The greatest elastic Mises stress range, $\Delta\bar{\sigma}_{el,\max}$, is less than the sum of $K_S S_y$ at the two ends of the cycle. (I suspect that this criterion is intended to ensure strict shakedown, though it doesn't really);
- The elastically based fatigue damage is less than 0.05;
- Creep behaviour is unperturbed by cyclic loading.

The crack-dependent parts of the checks for insignificant cyclic effects are,

- The fatigue crack growth does not exceed 10% of the creep crack growth, and,
- The cyclic plastic zone <u>at the crack tip</u> is small compared with the characteristic dimensions (i.e., the crack size, the ligament size and the thickness).

The crack-tip cyclic plastic zone size is given by,

Plane Stress: $$r_p^{ct} = \frac{1}{2\pi}\left(\frac{\Delta K}{2\sigma_y}\right)$$

or 1/3 of this in plane strain, where σ_y is the cyclic yield strength (0.2%). All five of the above conditions must be met for cyclic loading to be insignificant.

Qu.: Recap – What are the key steps when cycling is **in**significant?

Grossly over-simplifying, the key steps of the ccg calculation are,

(i) Obtain the elastic loads under normal operation;

(ii) Calculate relaxation of the secondary loads due to plasticity;

(iii) Allow for relaxation of the secondary loads due to cracking;

(iv) Calculate the creep relaxation continuously over life.

(v) Estimation of $C(t)$ as discussed in Session 43.

Qu.: What changes if cycling is significant?

The key changes relate to the fact that if cycling perturbs creep then the loads that are used in the ccg assessment must be those derived from a hysteresis loop construction. These differ from the elastic loads because they are modified by cyclic plasticity. Moreover, the start-of-dwell loads will be reset on every cycle, so the case of significant cycling is potentially far more onerous than insignificant cycling.

Qu.: Where is the procedure for cfcg with significant cycling?

The main sources are R5V4/5 Appendix A3 and Ainsworth, Dean & Budden, Ref.[1].

Qu.: How are the start-of-dwell loads found?

Note firstly that the first sentence of R5V4/5 Appendix A3, §A3.4.1 which states, "*For cyclic loading the shakedown analysis of V2/3 leads to an uncracked body reference stress at the time in the cycle corresponding to the creep dwell*" – is wrong! It is wrong because R5V2/3 only gives you the start of dwell stress at a point – usually at a point of peak stress – it does not give you a reference stress (which relates to the whole section).

However, the sentiment is right. It is necessary to carry out some form of 'shakedown analysis' (for which read 'hysteresis loop construction') in order to find the load resultants which prevail at the start of the creep dwell. Ref.[1] provides a more accurate statement of this objective.

The loads in question are to be understood as linearised over the section of interest. Hence we want to find the membrane and bending load resultants, *N* and *M*, in each of (say) the axial and hoop directions. Once we have these we can proceed as for the case of insignificant cycling since the reference stress can be defined by R5V4/5 App.A3, Equ.(A3.16) in terms of the current load resultants. That is by, $\sigma_{ref} = f(N_t, M_t, N_h, M_h, a)$. Alternatively, the references stress might, where applicable, be defined by the SIF-based formula, R5V4/5 App.A3, Equ.(A3.11). See Session 43 for a discussion of the difference.

Qu.: So – how ***are*** the start-of-dwell loads found?

The principle is that, after shakedown to some stable elastic-plastic cyclic behaviour, the stress $\sigma_{ij}(\bar{r},t)$ at any point $\bar{r}$ and time t can be found in terms of the elastic stress at that position and time, $\sigma_{ij}^{el}(\bar{r},t)$, and the shakedown residual stress field, $\rho_{ij}(\bar{r})$, which does not vary over time and which must be self-equilibrating, thus,

$$\sigma_{ij}(\bar{r},t) = \sigma_{ij}^{el}(\bar{r},t) + \rho_{ij}(\bar{r}) \qquad (1)$$

Hence, providing we know the elastic stresses, $\sigma_{ij}^{el}(\bar{r},t)$, and can calculate the shakedown residual stresses, $\rho_{ij}(\bar{r})$, we can find the net (real) stresses $\sigma_{ij}(\bar{r},t)$, from which we can find the load resultants across any section of interest.

Qu.: But how are the shakedown residual stresses found?

In general the determination of the shakedown residual stresses is the tricky bit. A wheeze that is often employed is to assume that they are proportional to the secondary stresses at some time t_1 (i.e., at some point around the cycle, probably when the secondary stresses are most onerous),

$$\rho_{ij}(\bar{r}) = -\alpha\sigma_{ij}^{S}(\bar{r}, t_1) \tag{2}$$

The net stresses are thus,

$$\sigma_{ij}(\bar{r}, t) = \sigma_{ij}^{P}(\bar{r}, t) + \sigma_{ij}^{S}(\bar{r}, t) - \alpha\sigma_{ij}^{S}(\bar{r}, t_1) \tag{3}$$

In (2,3), σ_{ij}^{P} and σ_{ij}^{S} are the ***elastic*** primary and secondary stresses respectively. The assumption (2) is possible because the secondary stresses are also self-equilibrating.

The time t_1 is chosen to minimise the excursions beyond KsSy, at either end of the cycle.

Qu.: Can the R5V2/3 Hysteresis Loop Construction Help?

The procedures are not explicit about this, but I suspect that the R5V2/3 construction of the dwell stress (at the most onerous point) could be of assistance in calculating the reference stress at the start of dwell. The reason is that the Mises stress formed from (1) evaluated at this assessment point should equal this R5V2/3 dwell stress. If we assume the approximation (2) then Equ.(3) should reproduce the start of dwell stress. We can therefore find the parameter α from,

$$Mises\left\{\tilde{\sigma}_{ij}^{P}(\bar{r}_0, t) + \tilde{\sigma}_{ij}^{S}(\bar{r}_0, t) - \alpha\tilde{\sigma}_{ij}^{S}(\bar{r}_0, t_1)\right\} = \sigma_0 \tag{4}$$

where σ_0 is the R5V2/3 dwell stress at the assessment point, $\bar{r}_0$. Having found α it is then possible to find the load resultants over any section because all the stress components and their variation are known from (3).

Warning: This is just my idea – no guarantees!

Qu.: Any other methods for finding the reference stress at dwell?

The obvious alternatively is FEA, which could provide the steady cyclic stress state.

Qu.: How are the load resultants found?

Whichever method is used to estimate the stresses at the start of the dwell, the final step is to integrate to find the load resultants using formulae like,

$$N_a = \int_{-t/2}^{+t/2} \sigma_a dx \text{, and} \qquad M_a = \int_{-t/2}^{+t/2} \sigma_a x dx \tag{5}$$

Qu.: "Relaxation" of load resultants due to the crack

Whichever of the above methods is used to estimate the start of dwell load resultants it is likely to be for the uncracked body. But the secondary load resultants also relax due to the cracking itself. This is taken into account by via crack size dependent γ-factors, such as,

$$N_t = N_t^{PR} + \gamma_{tm}(a) N_t^{S} \tag{6a}$$

$$M_h = M_h^{PR} + \gamma_{hb}(a) M_h^{S} \tag{6b}$$

These are exactly as in the procedure for insignificant cyclic loading. Their estimation may be problematical, but the use of methods like that of Ref.[2] may be one option.

Qu.: How is the reference stress calculated?

The reference stress at start of dwell is then given simply by the reference stress solution written in terms of the load resultants, i.e., by R5V4/5 App.A3, Equ.(A3.16),

$$\sigma_{ref} = f\left(N_t, M_t, N_h, M_h, a\right) \tag{7}$$

The load resultants in (7) are the combined primary-plus-secondary loads, including the effects of relaxation by the crack, (6a,b). This is generally more reliable than the scaling method based on the SIFs, i.e., R5V4/5 App.A3, Equ.(A3.11) - but the latter can also be used if applicable - which means if the out-of-plane secondary stresses are not too large.

Qu.: Does the CCG calculation now proceed as for insignificant cycling?

No, there are two more differences. The first is how relaxation is addressed. Integration of the relaxation equation,

$$\frac{d\sigma_{ref}}{dt} = -\frac{E}{Z}\left(\dot{\varepsilon}_{c,ref} - \dot{\varepsilon}_{c,ref}^{PR}\right) + \frac{da}{dt}\cdot\frac{\partial \sigma_{ref}}{\partial a} \tag{8}$$

is over the <u>dwell period only</u>. At the start of the next dwell, the reference stress starts again from its start-of-dwell value.

BUT the start-of-dwell reference stress is not the same on every cycle because of the crack size dependence of the loads, (6), and the fact that the crack is growing which influences the reference stress directly, (7).

Qu.: What creep hardening law is used with (8)?

Ref.[1] and R5 advise the use of strain hardening.

Qu.: Is primary creep reset on each cycle?

Advice currently from HiTBASS is that resetting of primary creep should be assumed if the cyclic plastic strain exceeds 0.01%. This is based on 316 stainless steel, but I assume can be extended to other austenitic steels. It now applies at all temperatures, unlike earlier advice.

Qu.: What is the last difference in the ccg procedure?

The final difference in the ccg procedure is potentially of considerable benefit. Recall that plasticity is beneficial in terms of reducing the "$C(t)$ spike". But since we are assessing a case with plastic cycling it follows that there must be substantial plasticity. The advice of R5 Vol.4/5 App.A3 is therefore to ignore the "$C(t)$ spike" altogether. The $C(t)$ estimation formula is thus given by R5 Vol.4/5 App.A3 Equ.(A3.45),

$$\frac{C(t)}{C^*} = \left(\frac{\sigma_{ref}\dot{\varepsilon}_{c,ref}}{\sigma_{ref}^{PR}\dot{\varepsilon}_{c,ref}^{PR}}\right) \tag{9}$$

This is the same as putting $C(t) = \sigma_{ref}\dot{\varepsilon}_{c,ref}R'$, where $R' = \left(K^{PR}/\sigma_{ref}^{PR}\right)^2$ is the usual length scale. Physically, this is the assumption that cyclic plasticity results in full redistribution of the crack tip fields immediately.

Qu.: In summary, what are the differences in the ccg procedure?

In summary the main differences in the CCG calculation compared with insignificant cyclic loading are,

Insignificant Cycling	Significant Cycling
Obtain the elastic loads under normal operation	Estimate the start-of-dwell stress via shakedown/hysteresis analysis
Calculate relaxation of the secondary loads due to plasticity	Plasticity is implicit within the above shakedown/hysteresis analysis
Allow for relaxation of the secondary loads due to cracking	Allow for relaxation of the secondary loads due to cracking in the same way
Calculate the creep relaxation continuously over life	Calculate the creep relaxation over the current dwell, then re-prime the start-of-dwell stress with allowance for crack growth
$C(t)$ estimated using R5V4/5 App.A3 Equ.(A3.33)	$C(t)$ estimated using R5V4/5 App.A3 Equ.(A3.45), which omits the "$C(t)$ spike" and hence may be of considerable benefit

Qu.: What about Fatigue Crack Growth?

Fatigue crack growth is calculated in the same way as described in session 42, http://rickbradford.co.uk/T73S03TutorialNotes42.pdf.

Recall that this may involve the use of ΔJ rather than ΔK - indeed because cyclic is significant this is more likely. Also recall the procedure for crack closure effects - including its modification for cyclic plasticity effects.

Qu.: What is the crack velocity validity criterion?

The dimensionless crack velocity is required by Ref.[1] to remain sufficiently small, specifically,

$$\lambda = \frac{\dot{a}(\sigma_{ref})^2}{EC(t)} < 1 \qquad (10)$$

Note that (10) is the most generally applicable form the of the crack velocity criterion, as made clear in Ref.[1], Equ.(30). It involves the combined load reference stress, σ_{ref}.

Qu.: What if the crack velocity criterion is not met?

Of course we already know that C^* is not always the correct parameter to use to calculate ccg. More generally $C(t)$ should be used, for a start. But neither parameter is valid if $\lambda > 1$, nor if the creep index is small (n < 3). For large dimensionless crack velocities ($\lambda >> 1$) R5 advises a correlation of creep crack growth rate with K (see R5V4/5 Appendix A2, Equs.(A2.28-29). However I don't know where $\dot{a} - K$ relations might be found (I can't see any in R66).

Qu.: Is it right to assume the steady cyclic state from the start?

Not really, no.

R5V4/5 includes advice on how to calculate ccg during the transient period whilst the steady cyclic state is being established. In our plant this transient period may have happened decades ago and will be irrelevant to, say, the crack growth over the next operating period. However if you have to estimate growth from start of life – or if you are assessing a newly installed component – then the following procedure should be used…

(1) Qu.: What is the dwell stress on first loading?

The reference dwell stress on first loading is denoted $\sigma_{ref}^{cyc=1}$. It can be found using the elastic stresses for the operating condition followed by application of the Neuber construction to account for initial (not cyclic) plastic relaxation (see R5V4/5 A3.4.1).

(2) Qu.: How long does it take to establish the steady cyclic state?

R5V4/5 §10.6 gives guidance on calculating t_{cyc}, the time to establish steady cycling. It is estimated by a formula which is a crude approximation to the time required to relax the first cycle dwell stress, $\sigma_{ref}^{cyc=1}$, down to the steady cycle dwell stress, σ_{ref}. Specifically, R5V4/5 Equ.(10.10) defines t_{cyc} by,

$$\varepsilon_c\left(\frac{\sigma_{ref}^{cyc=1}+\sigma_{ref}}{2},t_{cyc}\right)=\frac{Z\left(\sigma_{ref}^{cyc=1}-\sigma_{ref}\right)}{E} \qquad (11)$$

(3) Qu.: How is early cycle ccg calculated?

This is addressed in R5V4/5 §10.7.1.3. For times before the steady cyclic state is reached, i.e., for $t < t_{cyc}$, an average C-parameter is employed. R5V4/5 §10.7.1.3 expresses this in terms of C* as follows,

$$\overline{C}^* = \frac{\sigma_{ref}^{cyc=1}+\sigma_{ref}}{2}\dot{\varepsilon}_c R' \qquad (12)$$

where the creep strain rate is evaluated at the average reference stress, $\left(\frac{\sigma_{ref}^{cyc=1}+\sigma_{ref}}{2}\right)$.

Note that whilst the notation suggest that this is a "C^*" value, by virtue of (9) it is actually a cyclic value of $C(t)$. This average crack parameter is plugged into the usual growth formula which is integrated over the dwell period to get the creep crack growth per dwell.

Qu.: What is the scatter in creep data?

Huge.

Upper bound strain rates may be a factor of ~5 larger than mean rates, and lower bounds a factor of ~5 slower (even after HiTBASS has reduced this scatter somewhat). The 95% confidence interval might cover a spread of about a factor of 25.

The upper bound creep crack growth law typically differs from the best estimate by a factor of between 3 and 5, and the spread from lower bound to upper bound due to the ccg law alone can therefore be around a factor of 10 to 25 or so.

Consequently if all combinations of LB, BE and UB strain rate and crack growth law were considered the range of results would be very large indeed - a factor of several hundred between the LB/LB and UB/UB combinations.

Qu.: Is creep data correlated?

Yes.

We know that the parameter A in the ccg rate law is inversely proportional to creep ductility.

There is a correlation of creep ductility with creep strain rates (simply because the faster the material creeps, the more likely it is to accumulate a large strain before failure).

Consequently there will be an inverse correlation between the ccg law parameter, A, and strain rate.

There are almost certainly other correlations in creep data that are not so relevant here – such as an inverse correlation between creep rupture strength and strain rate (which is required to avoid unrealistic ductilities).

Consequently, it is not necessary to report the extreme range of assessment outcomes which would result from all combinations of LB, BE and UB options since many are unphysical.

Qu.: What combinations of bounds are recommended?

Here we confine attention to deformation rate / ccg law, in that order…

Ferritic Materials

R5V4/5 §11.1 recommends a base case using BE / BE and sensitivity studies addressing LB / UB and UB / LB combinations only.

Austenitic Materials

The R5 advice referred to above is not material specific. Hence the combinations BE / BE, LB / UB and UB / LB should be assessed and reported. However, in the case of 316ss, Ref.[3] has advised that the combination BE(deformation) / UB(ccg) also be assessed and reported. In practice this is generally the bounding case.

Whilst Ref.[3] is specific to 316ss, I suggest that the BE/UB combination should be considered for all austenitic materials. (The main reason why the requirement has been recognised for 316ss is simply because this material has been subject to long term ccg testing).

The arbitrariness of the recommended combinations is a shortcoming of deterministic assessments. Probabilistic methods are strongly motivated in creep.

References

[1] R A Ainsworth, D W Dean and P J Budden, "Creep and Creep-Fatigue Crack Growth for Combined Loading: Extension of the Advice in R5 Volume 4/5 Appendix A3", E/REP/BDBB/0059/GEN/04 (now at Rev.004 ?)

[2] R.Bradford, "A Structural Approach to the Calculation of J", Engineering Fracture Mechanics, Vol.29, No.6 (1988) 683-695. Preprint available here http://rickbradford.co.uk/MyPublications.html.

[3] D.W.Dean and D.N.Gladwin, "Creep Crack Incubation and Growth Behaviour of Type 316H Steels", E/REP/BDBB/0040/GEN/03, April 2004.