T73S03 (R5V4/5/7) – Session 43:
CCG under Primary-plus-Secondary Loads

Last Update: 28/6/16

C(t) estimation formulae for primary+secondary loads; R5V7 type approach: $(K_{TOT} / K_P)^2$ factor; Effect on redistribution time; Limitations of R5V7 method (ignores relaxation and plasticity); R5V4/5 C(t) estimation; Concept of total, or pseudo, reference stress: is it well defined?; Elements of the procedure when cyclic loading insignificant; Significance of out-of-plane secondary stresses; Competing effects of relaxation and crack growth on pseudo-reference stress; Prescription for service initiated cracks: reset the 'time' datum: what 'initial' quantities in C(t)/C mean; The "C(t) spike" and the importance of plasticity in ameliorating it.*

This session relates to steady loading, with creep unperturbed by load cycles, covering the methods of R5V7 and R5V4/5

R5V7

Qu.: What is the scope of R5V7 again?

R5V7 applies only to similar weldments in low alloy ferritic materials under steady loading (no cycles). It addresses both creep rupture and creep crack growth.

Qu.: Why are we starting with R5V7?

Because it's simpler than R5V4/5.

Qu.: How is ccg calculated in R5V7?

The usual growth law $\dot{a} = AC(t)^q$ is used together with a form of $C(t)$ estimation which is unique to Volume 7.

Qu.: How does the R5V7 estimation formula for C(t) differ from R5V4/5?

This is compactly given in R5V7 Appendix A6.

The reason for the R5V7 method being simpler than the more complete R5V4/5 method is that,

- Its treatment of secondary stresses is much simpler;
- It does not include relaxation;
- It does not include plasticity;

On the other hand there is one feature in R5V7 that is not in the (otherwise more general) R5V4/5 methodology. This is,

- Allowance for CMV weldment zone off-loading (the k factor)

Consequently, R5V7 may the method of choice for CMV weldments. Residual stresses are generally of less concern for CMV weldments due to PWHT.

Qu.: What is the R5V7 estimation formula for C(t)?

Essentially the same formula we have seen before is used,

$$C(t) = f(\tau) \cdot C^*, \tag{1a}$$

where,

$$f(\tau) = \frac{(1+\tau)^{1+n}}{\left[(1+\tau)^{1+n} - 1\right]}, \tag{1b}$$

However both terms C^* and τ are calculated in a slightly different way for a weldment with secondary stresses present.

Qu.: How is C* calculated in R5V7?

$$C^* = k \frac{\dot{\varepsilon}_c^{PR}}{\sigma_{ref}^{PR}} K_{PR}^2 \tag{2}$$

Equ.(2) involves,

σ_{ref}^{PR} the primary reference stress for the homogeneous body (i.e., as if the weld were not there).

$\dot{\varepsilon}_c^{PR}$ the creep strain rate corresponding to σ_{ref}^{PR}.

K_{PR} the SIF in steady creep operation due to the primary loads alone.

k the weldment zone off-loading factor for the zone being assessed. See http://rickbradford.co.uk/T73S06TutorialNotes26.pdf for definition.

Note in particular that the off-loading factor *k* is ***not*** used to factor the stress when determining the strain rate used in (2). The weldment zone used to calculate the strain rate is as follows,

- For hoop dominance the parent creep deformation should always be used (whatever weldment zone is being assessed), but the k factor relevant to the zone in question should be used in Equs.(2) and (3);
- For axial dominance the creep deformation appropriate to the weldment zone being assessed should be used (but *k* is, of course, 1 for all zones).

Note that the secondary stresses do not contribute at all to C*.

Qu.: How is the dimensionless time, τ, calculated in R5V7?

The dimensionless time is the only place where the secondary stresses enter the assessment in R5V7 App.A6. Previously we have defined τ as the ratio of the creep strain to the elastic strain. This is modified to,

$$\tau = \left(\frac{K_{PR}}{K_{TOT}}\right)^2 \frac{E\varepsilon_c^{PR}}{k\sigma_{ref}^{PR}} \tag{3}$$

where $K_{TOT} = K_{PR} + K_S$ is the sum of the LEFM primary and secondary SIFs.

Qu.: How does (3) make the growth rate faster?

We would expect that inclusion of (positive) secondary stresses would increase the growth rate. In (3) the inclusion of K_S makes τ smaller at any given time. This means that $f(\tau)$, (1b), is larger, and hence that $C(t)$ is larger. The presence of the secondary stresses is modelled by spending longer within the "C(t) spike".

Qu.: How does weldment zone affect growth rate?

<u>Effect of *k*</u>

There is a double whammy effect on C(t) when $k > 1$. Firstly C* increases in proportion to *k*, through (2), but also τ is smaller due to (3), which increases $f(\tau)$.

<u>Effect of strain rate</u>

BUT zones with $k > 1$ have slower creep rates (this being the reason why they have $k > 1$). So this tends to reduce C* through (2).

<u>Effect of propensity for growth (ductility)</u>

The different zones will generally have different ccg laws, $\dot{a} = AC(t)^q$. In particular, the coefficient A will be larger for zones with smaller creep ductility - which will tend to be the zones with slower creep rates and hence with $k > 1$.

<u>Effect of mixed HAZ</u>

R5V7 gives means of assessing mean growth rates through mixed HAZ in analogy with the mixed HAZ treatment of rupture. One of these is to evaluate coarse and refined HAZ growths separately and then to use the interpolation formula in terms of the mixing parameter α (see http://rickbradford.co.uk/T73S06TutorialNotes26.pdf), thus,

$$\dot{a}_M = \frac{1+\alpha}{\alpha + (\dot{a}_C / \dot{a}_R)} \dot{a}_C \qquad (4)$$

What is the overall effect? – See the homework for this session.

Qu.: Where does the R5V7 C(t) estimation formula come from?

These days it is more usual to employ the R5V4/5 formula for C(t). However, for interest, formulae of R5V7 type are derived here, http://rickbradford.co.uk/C_t_EstimationFormulae.pdf, which includes the effects of primary plasticity as well. This explains where the far-from-obvious factor of $\left(\frac{K_{PR}}{K_{TOT}}\right)^2$ in (3) comes from. However, I would not advise that you spend much time on these derivations.

R5V4/5

Qu.: What other checks are there in a cfcg assessment?

In addition to the precursor assessments (sessions 40A and 40B) and the crack growth calculation itself, it is always necessary to ensure that,

(i) The crack remains stable via an R6 assessment;

(ii) The ligament does not rupture, by integration of the time-fraction rupture damage corresponding to the (time dependent) reference stress;

(iii) The dimensionless crack velocity remains sufficiently small, i.e.,

$$\lambda = \frac{\dot{a}(\sigma_{ref})^2}{EC(t)} < \frac{1}{2} \qquad (5a)$$

Note that (5a) is the most generally applicable form the of the crack velocity criterion, as made clear in Ref.[1], Equ.(3). It involves the combined load reference stress, σ_{ref} (of which more below). R5V4/5 often quotes the particular case of (5a) for primary loads only, namely,

$$\lambda = \frac{\dot{a}(\sigma_{ref}^{PR})^2}{EC^*} = \frac{\dot{a}(\sigma_{ref}^{PR})^3}{kEK_{PR}^2\dot{\varepsilon}_c^{PR}} < \frac{1}{2} \qquad (5b)$$

Qu.: What is the structure of the R5V4/5 methodology?

The creep crack growth methodology under combined primary-plus-secondary loading was extensively revised in Revision 1 to R5 Issue 3 in 2012. The revised methodology was based on a report by Ainsworth, Dean and Budden, Ref.[1], but R5V4/5 itself can now be used directly without reference to this underlying report.

The expanded methodology for the calculation of creep-fatigue crack growth lies principally in three Appendices in R5V4/5,

- Appendix A2 deals with steady loading, with creep unperturbed by load cycles, under primary loads alone.
- Appendix A3 deals both with the effect of load cycles and also the issue of combined primary and secondary stresses (and hence relaxation).
- Appendix A4 deals with weldments.

The R5 structure is slightly unfortunate in that Appendix A3 deals both with cyclic loading and combined loading, which are quite distinct issues. In these notes I have structured the presentation differently, as follows,

- Last session (#42): Steady primary loads only
- This session (#43): Steady combined primary+secondary loads
- Next session (#44A): Cyclic loads
- Session #45: Weldments

This session in particular rests heavily on the Ref.[1]-based update of Appendix A3 as regards combined loading.

Qu.: What does R5V4/5 App.A2 bring to the party?

The C(t) estimation formula of App.A2 is similar to that of R5V7 (but without the secondary stress) – but it also includes a plasticity correction term. This is similar to the plasticity correction we have already seen in Equ.(22) of the session 39 notes http://rickbradford.co.uk/T73S03TutorialNotes39.pdf, but it may differ in detail. This will be subsumed in the discussion below.

Qu.: What is meant by a 'combined load reference stress'?

R5V4/5 and Ref.[1] part company with R6, R5V2/3 and R5V7 in using the term "reference stress" in a different way.

In R6, R5V2/3 and R5V7 the term "reference stress" relates to primary loads only and is well defined through the concept of the classical plastic limit state via $\sigma_{ref}^{PR} = (P / P_y)\sigma_y$.

In contrast, in R5V4/5 and Ref.[1] if the term "reference stress" is used without qualification it refers to a different concept – namely a type of reference stress which includes the effects of both primary and secondary loads. Such a reference stress cannot be defined via the plastic limit load. What, then, is the definition?

No general definition is given in R5V4/5 or Ref.[1], which is most unsatisfactory in my view. Instead specific formulae are given which are considered to be appropriate for specific geometries and loadings. In the past there have been heated discussions around whether a specific calculation of this type of reference stress is correct or not. But it is impossible to answer such a question in the absence of a definition of the quantity being estimated. The only definition which I can offer is this,

The combined load reference stress is that stress which, when used within the C(t) estimation formulae of R5V4/5 App.A3, produces an accurate estimate of C(t), e.g., as compared with finite element calculations.

Unfortunately this definition is not helpful as regards determining a formula for σ_{ref} in any given circumstance, short of carrying out an FEA.

Note that henceforth, in the context of R5V4/5, the notation σ_{ref} will refer to the combined load reference stress, in contrast to the usual (primary load) reference stress which will be written σ_{ref}^{PR}. Also the phrase "reference stress" shall be understood to mean the combined load reference stress unless qualified to the contrary.

Qu.: But surely R5V4/5 and Ref.[1] ***do*** give a simple formula for σ_{ref}?

R5V4/5 App.A3, Equ.(A3.11) and Ref.[1], Equ.(1) give the following very simple formula for the reference stress,

$$\sigma_{ref} = \frac{K_{TOT}}{K_{PR}} \sigma_{ref}^{PR} \qquad (6)$$

where $K_{TOT} = K_{PR} + K_S$. This would satisfy my concerns expressed above completely if it were not for the fact that this simple definition is only applicable when,

[1] The longitudinal secondary stresses are small, and,

[2] The transverse primary and secondary stress distributions are of similar form.

In [1,2], by transverse stress is meant the Mode I opening stress. By longitudinal stress is meant a stress parallel to the crack plane.

These limitations on the applicability of (6) were first made clear in Ref.[1] and constitute perhaps the most important development beyond the first issue of R5V4/5 Issue 3 App.A3.

Unfortunately both these conditions are most often *not* met. Consider welding residual stresses and a circumferential crack in a cylinder.

The hoop stress (longitudinal) is generally larger than the transverse (axial) stress, so [1] is not met. This is the most typical of situations and is what principally motivated the development of Ref.[1].

Consider also thermal stresses due to a through-wall temperature gradient. In this case the out-of-plane and in-plane (transverse) stresses are usually comparable, again violating [1]. Moreover, [2] is not usually true either. Pressure stresses, for example, generally have a different distribution from welding residual or thermal stresses.

In short, whilst (6) is neat it is generally not applicable. If it is, fine – use it.

Qu.: What does Ref.[1] put in place of (6)?

The advice on calculating σ_{ref} given in Ref.[1] depends upon,

(i) being able to express the true (primary) reference stress as a function of the primary load resultants, and,

(ii) being able to express the secondary stresses in the form of load resultants.

The "load resultants" are the net force and net moment across the section of interest, denoted N and M respectively. For a section of thickness t these correspond to membrane and bending stresses of $\sigma_m = N/t$ and $\sigma_b = 6M/t^2$. Such load resultants can be defined for the primary and secondary stresses separately, also for the two directions: in-plane (denoted t = transverse) and longitudinal (denoted h). Hence, in an obvious notation, the total in-plane force resultant is,

$$N_t = N_t^{PR} + N_t^S \tag{7}$$

And the total out-of-plane moment resultant is,

$$M_h = M_h^{PR} + M_h^S \tag{8}$$

and so on. Note that the secondary load resultants will be time-dependent due to relaxation of the secondary stresses. In (7,8) the load resultants are those at the time of interest. How these are found is discussed shortly.

It is assumed that some true (primary) reference stress solution has been identified and can be written as,

$$\sigma_{ref}^{PR} = f\left(N_t^{PR}, M_t^{PR}, N_h^{PR}, M_h^{PR}, a\right) \tag{9}$$

where a is the crack depth. (In general the crack may have both a length and a depth which would both appear in equ.9). The combined load reference stress is then defined by this same function with the argument replaced by the total load resultants, thus,

$$\sigma_{ref} = f\left(N_t, M_t, N_h, M_h, a\right) \tag{10}$$

Qu.: "Relaxation" due to the crack

We will deal with relaxation of the secondary stresses due to creep shortly. But note that, even under purely elastic conditions, the secondary load resultants will reduce below their value for the uncracked structure due to crack opening. This is made explicit by the notation adopted in Ref.[1] by taking a crack-depth dependent factor out of the secondary load resultants, re-writing the likes of (7) and (8) as,

$$N_t = N_t^{PR} + \gamma_{tm}(a) N_t^S \tag{11}$$

$$M_h = M_h^{PR} + \gamma_{hb}(a) M_h^S \tag{12}$$

and so on. Note that $N_t^S, N_h^S, M_t^S, M_h^S$ still retain a time dependence due to creep relaxation.

Qu.: How is creep relaxation calculated for a stationary crack?

If the crack were not growing, the relaxation due to creep alone would follow the usual relaxation equation,

$$\frac{\partial \sigma_{ref}}{\partial t} = -\frac{E}{Z}\left(\dot{\varepsilon}_{c,ref} - \dot{\varepsilon}_{c,ref}^{PR}\right) \tag{13}$$

where $\dot{\varepsilon}_{c,ref}$ is the creep rate at the current reference stress and creep strain $(\sigma_{ref}, \varepsilon_c)$, and $\dot{\varepsilon}_{c,ref}^{PR}$ is the creep rate at the primary reference stress and current creep strain $(\sigma_{ref}^{PR}, \varepsilon_c)$. It is particularly important to note when coding the integration of (13) that the primary strain rate $\dot{\varepsilon}_{c,ref}^{PR}$ is evaluated at the *same* current creep strain as $\dot{\varepsilon}_{c,ref}$. This means that this term cannot be integrated independently of the first term in the combined load. As elsewhere in R5, strain hardening is assumed.

Qu.: How is creep relaxation calculated for a growing crack?

By the chain rule, the total time derivative of the reference stress is,

$$\frac{d\sigma_{ref}}{dt} = \frac{\partial \sigma_{ref}}{\partial t} + \frac{da}{dt} \cdot \frac{\partial \sigma_{ref}}{\partial a} \tag{14}$$

The first term on the RHS of (14) is due to creep relaxation at a fixed crack size, whereas the second term is the relaxation due solely to the change of crack size with no creep. Substituting from (13) gives the relaxation equation which is actually recommended in Ref.[1] as Equ.(12),

$$\frac{d\sigma_{ref}}{dt} = -\frac{E}{Z}\left(\dot{\varepsilon}_{c,ref} - \dot{\varepsilon}_{c,ref}^{PR}\right) + \frac{da}{dt} \cdot \frac{\partial \sigma_{ref}}{\partial a} \tag{15}$$

The term $\frac{\partial \sigma_{ref}}{\partial a}$ is evaluated by differentiating (10), remembering that all five of its arguments are a-dependent by virtue of expressions like (11, 12).

In practice some form of computer code or spreadsheet is required to carry out these ccg calculations. Integration by time-stepping allows the currently calculated crack growth rate (and strain rates) to be used in (15) to evaluate the stress relaxation for the next time step. The differentiation of (10) wrt crack depth, a, would also be done numerically.

Qu.: How is σ_{ref} found immediately after a service crack initiates?

Given a starting reference stress, σ_{ref}^{sol}, the reference stress at any time follows by integration of (15) ***unless*** there is a service crack initiation. Then there is a discontinuous change in σ_{ref}, as illustrated in Figure 1. The value of the reference stress is then the so-called "initial" stress σ_{ref}^{0}, of which more below.

Qu.: Does the reference stress always increase when the crack is introduced?

Not necessarily. It will increase if primary loads are dominant. However if secondary loads are dominant then the relaxation of the secondary load resultants due to the introduction of the crack could possibly cause a reduction in the reference stress (though I suspect this is unusual – it would normally increase).

Qu.: Does the reference stress always reduce during creep dwells?

Not necessarily, though this would be typical.

Although I have referred to (15) as a relaxation equation, only the first term on the RHS is a relaxation (i.e., a negative change of stress). The second term will generally be positive, because the reference stress generally increases as the crack gets deeper at a given stress level, $\frac{\partial \sigma_{ref}}{\partial a} > 1$. Which of these two contrary effects wins will vary from case to case.

If the primary loads are dominant, the reference stress will increase with crack depth and with time.

If the secondary loads are dominant the matter is less clear. Both creep and the 'relaxation' due to crack opening, as measured by the $\gamma_{ij}(a)$ factors, will tend to reduce σ_{ref} over time as the crack grows. On the other hand, the reference stress formula, (10), will have an explicit crack size dependence which will tend to cause an increase in σ_{ref}. Most often a secondary load dominated case will have a reducing σ_{ref}, but this is not guaranteed.

For cases in which the primary and secondary loads are comparable it is difficult to guess whether σ_{ref} will increase or decrease.

Like shares, the reference stress can go up or down

Qu.: What is Ref.[1] Equ.(9) or R5V4/5 Equ.(A3.25) for?

Ref.[1] Equ.(9) or R5V4/5 Equ.(A3.25) are really horrible forms of relaxation equation written in terms of derivatives of the SIFs. Actually they are simply special cases of (15) when Ref.[1] Equ.(1) is valid [i.e., our (6)]. These complicated equations are only valid in that case - i.e., rarely.

Qu.: Do you need to find the relaxed secondary SIF?

Yes, in general.

Equ.(15) allows you to calculate the relaxing (or increasing) reference stress. But is it necessary also to calculate the relaxing secondary SIF?

It is not necessary to know the secondary SIF in order to calculate ***creep*** crack growth because C(t) depends only upon the primary SIFs and the various reference stresses and strains.

It may not be necessary to know the secondary SIF in order to calculate the fatigue crack growth either IF the secondary stresses do not cycle (such as, for example, welding residual stresses) - although even non-cycling secondary stresses may affect fcg via the closure factor, q.

If the secondary stresses cycle (e.g., thermal stresses) is it then necessary to find the relaxing secondary SIFs in order to calculate fatigue crack growth? The answer is again "no", assuming that the cyclic thermal *loads* are constant, i.e., the cycle is between two unvarying temperature distributions. The reason is that the stress range will not change due to the relaxation – which merely sets up a reverse stress in the cold condition. Strictly, crack closure effects may complicate things, but a reasonable approach is to base fatigue crack growth on the initial (elastic) secondary stress range. The secondary SIF range will increase, though, as the crack size increases.

So you *almost* do not need the relaxing secondary SIF in order to calculate crack growth. Unfortunately there is a complication – it is (sometimes) needed to calculate the so-called "initial" quantities σ_{ref}^0 and ε_{ref}^0, of which more below.

In any case, you also need to confirm crack stability. For this you will need the relaxing secondary SIF in the R6 assessment.

Qu.: How is the relaxed secondary SIF found if Equ.(6) defines the reference stress?

In the case that the reference stress is defined via Equ.(6) as $\sigma_{ref} = \frac{K_{TOT}}{K_{PR}} \sigma_{ref}^{PR}$ then the secondary SIF at any time follows immediately from knowledge of the reference stress, i.e.,

$$K_S = \left(\frac{\sigma_{ref}}{\sigma_{ref}^{PR}} - 1 \right) K_{PR} \tag{6b}$$

Qu.: How is the relaxed secondary SIF found if Equ.(10) defines the reference stress?

R5 is not clear about how K_S is to be found in the more general (and more usual) case that Equ.(10) is used to define the reference stress. However, if a scheme for finding the individual relaxed load resultants is adopted (such as that described below) then the secondary SIF follows immediately (i.e., if you have the loads and the SIF solution, then you know K_S).

Note that you cannot use (6b) to find the relaxed K_S if Equ.(10) has been used to find the reference stress. This would be inconsistent and would lead to an incorrect, discontinuous change in K_S.

Qu.: How could the relaxed secondary SIF be found consistently with (10)?

R5 does not seem to say. This para gives my suggestion.

The point is that knowing the relaxed reference stress does not tell you the values of the relaxed secondary load resultants, $N_t^S, N_h^S, M_t^S, M_h^S$. This is because knowing $\sigma_{ref} = f(N_t, M_t, N_h, M_h, a)$ does not permit you to find all four arguments. If only one of $N_t^S, N_h^S, M_t^S, M_h^S$ were significant then that would be OK, the equation could be solved. But if two or more are significant, what should we do?

If you have to do this I would advise that you check what others have done previously. However, I offer the following suggestion: assume that all four secondary load resultants $N_t^S, N_h^S, M_t^S, M_h^S$ remain in fixed proportion. This means that all of them will relax by the same factor, call it $\xi(t)$. The total load resultants can thus be written,

$$N_t = N_t^{PR} + \xi(t)\gamma_{tm}(a)N_{t0}^S \quad (16a)$$

$$M_t = M_t^{PR} + \xi(t)\gamma_{tb}(a)M_{t0}^S \quad (16b)$$

$$N_h = N_h^{PR} + \xi(t)\gamma_{hm}(a)N_{h0}^S \quad (16c)$$

$$M_h = M_h^{PR} + \xi(t)\gamma_{hb}(a)M_{h0}^S \quad (16d)$$

where N_{t0}^S, etc., refer to the initial values. Since there is now only one parameter, ξ, to find, knowing the value of $\sigma_{ref} = f(N_t, M_t, N_h, M_h, a)$ permits inversion to find ξ at each time step. Some iterative procedure will be required (or the EXCEL Solve facility).

Qu.: What is the Ref.[1] estimation formula for C(t)?

The formula recommended for general use is,

$$\frac{C(t)}{C^*} = \left(\frac{\sigma_{ref}\dot{\varepsilon}_{c,ref}}{\sigma_{ref}^{PR}\dot{\varepsilon}_{c,ref}^{PR}}\right)\left[\frac{\left(\varepsilon_{ref}/\varepsilon_{ref}^0\right)^{n+1}}{\left(\varepsilon_{ref}/\varepsilon_{ref}^0\right)^{n+1} - \left(\sigma_{ref}^0/E\varepsilon_{ref}^0\right)}\right] \quad (17)$$

Where C* is given by the usual formula involving only primary stress, primary strain rate and primary SIF, i.e., $C^* = \frac{\dot{\varepsilon}_{c,ref}^{PR}}{\sigma_{ref}^{PR}} K_{PR}^2$ for a homogeneous body.

Great care is needed to interpret the different stresses and strains in (17) correctly. Those in the first term in round brackets we have already defined above. Those in the second term in square brackets are defined as follows,

$\varepsilon_{ref} = \varepsilon_{ref}^e + \varepsilon_{ref}^p + \varepsilon_{c,ref}$	the sum of the elastic, plastic and creep strains at the stress σ_{ref}.
σ_{ref}^0	the "initial" value of the combined load reference stress, i.e., before creep relaxation. Note that σ_{ref}^0 is an elastic-plastic stress *after* stress reduction by plasticity.

$\varepsilon_{ref}^{0} = \varepsilon_{ref}^{e0} + \varepsilon_{ref}^{p0}$ — the "initial" reference strain, hence only the elastic and plastic strains corresponding to stress σ_{ref}^{0} on the stress-strain curve.

Note that plasticity effects are included in (17). The way in which the "initial" quantities σ_{ref}^{0} and ε_{ref}^{0} are found is defined below.

Qu.: What do the two terms in C(t) mean physically?

The first term, $\left(\frac{\sigma_{ref}\dot{\varepsilon}_{c,ref}}{\sigma_{ref}^{PR}\dot{\varepsilon}_{c,ref}^{PR}}\right)$, accounts for the secondary stresses and their relaxation.

The second term, $\left[\frac{\left(\varepsilon_{ref}/\varepsilon_{ref}^{0}\right)^{n+1}}{\left(\varepsilon_{ref}/\varepsilon_{ref}^{0}\right)^{n+1} - \left(\sigma_{ref}^{0}/E\varepsilon_{ref}^{0}\right)}\right]$, is the "C(t) spike" and hence models redistribution of both the primary and secondary stresses.

Qu.: To what does (17) reduce if there is no plastic strain?

The first term in (17) is unchanged (it involves only creep strains). In the second term we have,

$$\frac{\sigma_{ref}^{0}}{E\varepsilon_{ref}^{0}} \to \frac{\sigma_{ref}^{0}}{E\varepsilon_{ref}^{e}} = 1 \qquad \text{and} \qquad \frac{\varepsilon_{ref}}{\varepsilon_{ref}^{0}} \to \frac{\varepsilon_{ref}^{e} + \varepsilon_{c,ref}}{\varepsilon_{ref}^{e}} = 1 + \frac{\varepsilon_{c,ref}}{\varepsilon_{ref}^{e}} = 1 + \tau$$

where τ is the dimensionless time parameter defined as the ratio of creep to elastic reference strains. Hence the second term in (17) thus becomes simply $f(\tau)$ where the "C(t) spike" function f is given by (1b). Hence we end up with,

No Plasticity:
$$\frac{C(t)}{C^{*}} = \left(\frac{\sigma_{ref}\dot{\varepsilon}_{c,ref}}{\sigma_{ref}^{PR}\dot{\varepsilon}_{c,ref}^{PR}}\right) f(\tau) \qquad (18)$$

Note that this is a different estimation formula from (1a). The difference is that the first term in (18) explicitly models relaxation, which is ignored in R5V7's Equ.(1a).

Qu.: To what does (17) reduce if there is a great deal of plastic strain?

If the plastic strain is far larger than the elastic strain then,

$$\frac{\sigma_{ref}^{0}}{E\varepsilon_{ref}^{0}} \to \frac{\varepsilon_{ref}^{e}}{\varepsilon_{ref}^{e} + \varepsilon_{ref}^{p}} \to 0$$

So the entire second term in (17) becomes unity,

$$\left[\frac{\left(\varepsilon_{ref}/\varepsilon_{ref}^{0}\right)^{n+1}}{\left(\varepsilon_{ref}/\varepsilon_{ref}^{0}\right)^{n+1} - \left(\sigma_{ref}^{0}/E\varepsilon_{ref}^{0}\right)}\right] \to 1$$

Hence,

Lots of Plasticity:
$$\frac{C(t)}{C^{*}} \approx \left(\frac{\sigma_{ref}\dot{\varepsilon}_{c,ref}}{\sigma_{ref}^{PR}\dot{\varepsilon}_{c,ref}^{PR}}\right) \qquad (19)$$

Equ.(19) strictly gives $C(t) \approx \left(\frac{\sigma_{ref} \dot{\varepsilon}_{c,ref}}{\left(\sigma_{ref}^{PR}\right)^2} \right) K_{PR}^2$, but if the primary and secondary SIFs are in the same ratio as the primary and secondary reference stresses, then we get,

Lots of Plasticity: $$C(t) = \frac{\dot{\varepsilon}_{c,ref}}{\sigma_{ref}} K_{TOT}^2 \qquad (20)$$

In this case there is no "C(t) spike" because plasticity has washed it out and the C(t) formula is just the same as the C* formula but with primary stress, primary strain and primary SIF replaced by total stress, total strain and total SIF.

Remember, though, that the secondary stresses in (20) will be relaxing, so if we wait long enough for them to relax completely we will get $C(t) \to C^*$. In practice, for slow rates of creep, this condition may never be attained during plant lifetime.

Qu.: To what does (17) reduce if there is no secondary stress?

In this case the first term in (17) is unity and we are left only with a "C(t) spike" given by the second term,

$$\frac{C(t)}{C^*} = \frac{\left(1+\tau_{ep}\right)^{n+1}}{\left(1+\tau_{ep}\right)^{n+1} - \left(1 + \frac{\varepsilon_{ref}^p}{\varepsilon_{ref}^e}\right)^{-1}} \qquad \text{where,} \quad \tau_{ep} = \frac{\varepsilon_{c,ref}}{\varepsilon_{ref}^e + \varepsilon_{ref}^p} \qquad (21)$$

Note that once again if $\varepsilon_{ref}^p >> \varepsilon_{ref}^e$ then $C(t) \approx C^*$ at all times.

Qu.: Isn't there also a more accurate C(t) estimation formula?

Yes – R5V4/5, App.A3, Equ.(A3.35), which is,

$$\frac{C(t)}{C^*} = \left(\frac{\sigma_{ref} \dot{\varepsilon}_{c,ref}}{\sigma_{ref}^{PR} \dot{\varepsilon}_{c,ref}^{PR}} \right) \left[\frac{\left(\varepsilon_{ref} / \varepsilon_{ref}^0\right)^{n+1}}{\Phi\left\{ \left(\varepsilon_{ref} / \varepsilon_{ref}^0\right)^{n+1} - 1 \right\} + \left(1 - \sigma_{ref}^0 / E\varepsilon_{ref}^0\right)} \right] \qquad (22)$$

where the time-dependent parameter Φ is given by R5V4/5, App.A3, Equ.(A3.34). It lies between 1 and $Z/(Z-1)$. Equ.(22) reduces to (17) when $\Phi = 1$. For larger values of Φ Equ.(22) will give a reduced estimate of C(t). However Ref.[1] does not recommend (22) for general use.

Qu.: Why is "initial" as in "initial" σ_{ref}^{0} and ε_{ref}^{0} in inverted commas?

The word "initial" is in inverted commas because these quantities are not evaluated using the start-of-life crack size nor, in some cases, at the start-of-life stresses either. Before proceeding further note that we shall have in mind the possibility that the cracks initiate in service at some time t_d…

The methodology for service-initiated cracks is treated here. The methodology for original sin cracks is a special case of this with initiation time, t_d, set to zero.

Assuming that σ_{ref}^{0} increases with crack depth, then the "initial" quantities σ_{ref}^{0} and ε_{ref}^{0} are defined by using the stresses when the crack initiates (time t_d) together with either the initiation crack size, a_0, ***or*** the current crack size, a, whichever gives the larger C(t).

Qu.: How are the "initial" σ_{ref}^{0} and ε_{ref}^{0} calculated if Equ.(6) defines the reference stress?

It is important to remember that σ_{ref}^{0} and ε_{ref}^{0} are elastic-plastic quantities. If the inputs to your assessment are the elastic stresses, then we do not know either of these quantities. In particular it would be a mistake in general to derive an elastic value for σ_{ref}^{0} and then to find ε_{ref}^{0} by entering the stress-strain curve at this stress. This would over-estimate both quantities – possibly hugely over-estimating the plastic strain. This would be non-conservative, i.e., it leads to an under-estimate of C(t).

R5V4/5 App.A3, Equ.(A3.15) gives a prescription for finding σ_{ref}^{0} and ε_{ref}^{0}, i.e.,

$$\sigma_{ref}^{0}\varepsilon_{ref}^{0} = \frac{\left(\sigma_{ref}^{PR,0}\right)^2}{E}\cdot\frac{\left(1+VK_S^0/K_{PR}^0\right)^2}{F^2\left(L_r^0\right)} \tag{23}$$

where $K_r = F(L_r)$ is the R6 Failure Assessment Diagram (FAD) and V is the multiplicative factor used in R6 to account for secondary stress induced plasticity.

I have put superscripts 0 on the quantities on the RHS of (23), although this is not done in R5, to emphasise that all quantities are calculated at the so-called "initial" stresses and crack size defined in the box above. Equ.(23) can be solved if you also know the stress-strain curve. For a Ramberg-Osgood curve you will need to solve,

$$\sigma_{ref}^{0}\left(\frac{\sigma_{ref}^{0}}{E}+\left(\frac{\sigma_{ref}^{0}}{A}\right)^{1/\beta}\right) = \frac{\left(\sigma_{ref}^{PR,0}\right)^2}{E}\cdot\frac{\left(1+VK_S^0/K_{PR}^0\right)^2}{F^2\left(L_r^0\right)} \tag{24}$$

for σ_{ref}^{0} and thence find ε_{ref}^{0} also.

There may appear to be a chicken-and-egg problem involved in using Equ.(6b) to find the secondary SIF in terms of the reference stress and also using Equ.(24) to find the (initial) reference stress in terms of the SIFs. I think the position is this…

(i) Track creep relaxation prior to crack initiation – no SIFs involved – hence find the loads required to evaluate the "initial" quantities;

(ii) Hence evaluate the 'elastic' SIFs based on the above loads, K_S^0, K_{PR}^0;

(iii) Hence evaluate the "initial" reference stress and strain via Equ.(24);

(iv) Proceed to the next time step, evaluating relaxation using Equ.(15);

(v) Then use Equ.(6b) to find the relaxed SIF;

(vi) Repeat from (iv) for as many time steps as required.

So there is no conflict because (24) is used just once, prior to creep, whilst thereafter (6b) is used.

Qu.: How are the "initial" σ_{ref}^0 and ε_{ref}^0 calculated if Equ.(10) defines the reference stress?

In this case Equs.(23,24) would not be used.

In this case the "initial" reference stress follows from Equ.(10) so long as the load resultants are all known. The tricky thing here is that the secondary load resultants will relax instantaneously when the crack is introduced. This is taken into account, in principle, by the $\gamma_{ij}(a)$ factors. But you need to be able to calculate these, which may be a problem.

What about the plastic correction equivalent to Eq.(24)? This should also be implicit in the $\gamma_{ij}(a)$ factors, which should account for both elastic and plastic relaxation of the secondary load resultants.

R5 and Ref.[1] appear to avoid discussing this issue.

Ref.[2] discusses a general method of calculating the relaxation of the secondary load resultants due to introduction of a crack. Ref.[3] is an example application of the methodology.

Qu.: How are secondary stresses defined if an elastic-plastic FEA is available?

An elastic-plastic finite element analysis might be available for, say, welding residual stresses – though the same reasoning as laid down here could be used for any secondary load. Ideally the elastic-plastic FEA should have been carried out for,

[1] The total service loading (all primary and secondary loads), and,

[2] The primary loading alone.

The secondary stresses, and the associated load resultants, are then defined as the difference of these quantities between these two runs, [1] – [2]. This is very important since it can result in a substantially reduced residual stress compared to an evaluation of the residual stress prior to service loading.

In particular, the "initial" reference stress σ_{ref}^0 for an original sin defect can be found by inserting the load resultants obtained from [1] – [2] into (10). There is no need for the plasticity correction, such as accomplished by Equs.(23,24), because this is already implicit within the FEA.

The "initial" reference strain, ε_{ref}^0, is then found simply by entering the stress-strain curve at stress σ_{ref}^0.

However, it is commonly the case that an elastic-plastic FEA is available only for the uncracked body (e.g., from a welding simulation). Consequently the User is left with the problem of determining the $\gamma_{ij}(a)$ factors to use in Equ.(10). Again it is possible that Ref.[2] may be helpful.

Qu.: How are the elastic and plastic reference strains found for use in (17)?

The elastic reference strain is, of course, just $\varepsilon_{ref}^e = \frac{\sigma_{ref}}{E}$.

The advice of R5 is simply to take the plastic reference strain as equal to its "initiation" value at all times. I say this because Ref.[1], Equ.(39) can be re-written as $\varepsilon_{ref}^{e+p} = \varepsilon_{ref}^e + \varepsilon_{ref}^{p0}$ from which it follows that,

$$\varepsilon_{ref}^p = \varepsilon_{ref}^{p0} \tag{25}$$

Qu.: How are the creep strains found for use in (17)?

The creep strain is defined in the obvious manner as,

$$\varepsilon_{c,ref} = \int_{t_d}^{t} \dot{\varepsilon}_c\left(\sigma_{ref}(t'), \varepsilon_c(t')\right)dt' \tag{26}$$

where t is the assessment time and t_d is the time when the crack initiates (i.e., when the crack is deemed to appear, forming a new crack tip). Equ.(26) subsumes both the case of original sin, when $t_d = 0$, and service initiated defects, when $t_d > 0$.

In (26) the notation $\dot{\varepsilon}_c(\sigma, \varepsilon_c)$ denotes a creep deformation law in strain hardening form.

It is crucial to observed that, whilst (26) is the correct creep strain to use in (17) to estimate C(t), for service initiated defects, with $t_d > 0$, it is ***not*** the total creep strain which has accumulated over life. This is because creep strain has accumulated also prior to crack initiation. However, the total creep strain must be used in (26) to track the strain hardening. This is why the argument of the strain rate function has been written as ε_c in contrast to $\varepsilon_{c,ref}$. The total creep strain over life is,

$$\varepsilon_c = \int_0^{t_d} \dot{\varepsilon}_c\left(\sigma_{ref}(t'), \varepsilon_c(t')\right) dt' + \varepsilon_{c,ref} \tag{27}$$

Qu.: What is the import of using the time datum t_d in defining $\varepsilon_{c,ref}$?

The peak of the C(t) spike occurs at $\varepsilon_{c,ref} = 0$. Hence defining $\varepsilon_{c,ref}$ from the time datum t_d places the C(t) spike at the initiation time. If the total creep strain, ε_c, were used in the C(t) estimation formula, (17), instead, the C(t) spike would be placed at start of life, t = 0. But since crack growth is calculated only after the crack has initiated (obviously, because the crack isn't there before) the C(t) spike is probably over by then. So it would be non-conservative to use the total creep strain, ε_c. And it is correct to use $\varepsilon_{c,ref}$ because the C(t) spike should arise when a new crack tip forms – because only then is there suddenly an LEFM field which needs redistributing.

Qu.: What about incubation?

If the crack forms by a creep-fatigue mechanism then incubation is not relevant. However, if the crack forms by some other mechanism so that there is an incubation period, t_{inc}, then the procedure is simply this,

- Calculate exactly the same C(t) at any time as if there were no incubation;
- But only start the crack growing at time $t_d + t_{inc}$.

This works for both original sin ($t_d = 0$) and service initiated cracks ($t_d > 0$).

Qu.: What is the import of incubation?

Most obviously, if there is an incubation period there is less time for the crack to grow. However, if the incubation period is hundreds or a few thousands of hours, this may seem not to make a great difference in a life of ~250,000 hours.

Nevertheless, incubation can be disproportionately beneficial because <u>it gets you some way down the C(t) spike</u>. Incubation can effectively veto the period when the crack would have been growing fastest. It can therefore be very beneficial, more so than its duration in hours might suggest.

Qu.: What are the contrasting effects of initiation and incubation on C(t)?

- Initiation creates the "C(t) spike" because it creates a brand new crack tip
- Incubation ameliorates the "C(t) spike" because it takes you some way down it

Qu.: For service initiated defects is creep relaxation allowed prior to initiation?

Yes.

This is a matter of consistency with calculating creep strain accumulation from start of life, i.e., (27). Of course the second term on the RHS of (15) is zero and the first term is evaluated for the uncracked body: $\frac{d\sigma_{ref}}{dt} = -\frac{E}{Z_u}\left(\dot{\varepsilon}_{c,ref} - \dot{\varepsilon}_{c,ref}^{PR}\right)\Big|_{a=0}$, noting that the uncracked follow-up factor, Z_u, is generally different from the cracked body follow-up factor, Z. The recommended relationship used to be $Z = Z_u + 1$ but I suspect this is out of favour – seek advice.

Qu.: What is the start-of-life reference stress from which integration of (15) starts?

The point is that you cannot in general use the so-called "initial" reference stress σ_{ref}^0 as the starting reference stress when integrating,

$$\frac{d\sigma_{ref}}{dt} = -\frac{E}{Z}\left(\dot{\varepsilon}_{c,ref} - \dot{\varepsilon}_{c,ref}^{PR}\right) + \frac{da}{dt} \cdot \frac{\partial \sigma_{ref}}{\partial a} \tag{15}$$

The "initial" reference stress is not necessarily the same thing as the start-of-life reference stress, but (15) must always be integrated from start of life – because creep always starts at the start of life!

So I presume the right thing to do is to evaluate the start-of-life reference stress σ_{ref}^{sol} in exactly the same way that σ_{ref}^0 is calculated but using the start-of-life stresses and crack size. For a service initiated defect this will mean crack-free. For original sin σ_{ref}^{sol} might be the same thing as σ_{ref}^0 if the start-of-life crack size is limiting.

References

[1] R A Ainsworth, D W Dean and P J Budden, "Creep and Creep-Fatigue Crack Growth for Combined Loading: Extension of the Advice in R5 Volume 4/5 Appendix A3", E/REP/BDBB/0059/GEN/04, Rev.003, May 2010.

[2] R.Bradford, "A Structural Approach to the Calculation of J", Engineering Fracture Mechanics, Vol.29, No.6 (1988) 683-695. Preprint available here http://rickbradford.co.uk/MyPublications.html.

[3] R.A.W.Bradford, "Sizewell B Dry Store: Analytic Fracture Assessments of the Baseplate to Shell Weld of the MPC for the Axisymmetric Case", E/EAN/BBAB/0024/SXB/11, April 2011.

Figure 1

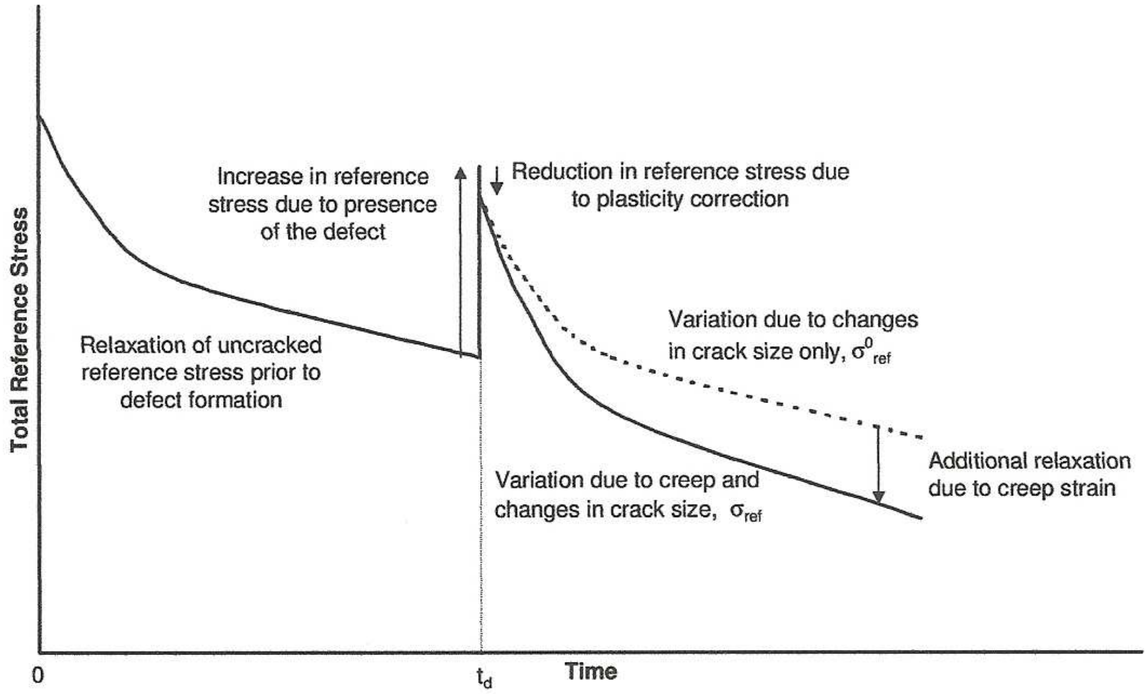

Figure 3 Schematic Variation of Reference Stress for a Service-Initiated Defect Formed at $t = t_d$