T73S03 (R5V4/5/7) – Session 42:
FCG, CCG and Continuum Damage

Last Update: 14/5/16

Fatigue crack growth; Paris Law versus small crack law; Definition of ΔK_{eff} and physical meaning; fcg within cyclic plastic zone; When are plasticity corrections to ΔK_{eff} required: ΔJ ; fcg threshold ΔK_{eff} and its physical origin (recap); Effect of dwells on apparent fcg rate: is it really creep?; C reference stress formula; C(t) estimation formula for primary loading alone; Effect of plasticity on the latter; Redistribution time; Derivation of $da/dt = A.C^{*q}$ from continuum damage mechanics; Sensitivity to constraint (factor of 50?); Derivation of relation of q to n, and A to ε_f ; Inconsistency of reheat crack initiation assumptions & ccg rates*

Fatigue Crack Growth (fcg)

Qu.: Is there more than one method for fcg in R5V4/5?

Yes.

Qu.: Is the method for combining fcg and ccg independent of the method for fcg?

No.

The two possibilities are,

- Method I: Use the Paris Law for fcg and use an $\dot{a} - C(t)$ law for ccg and combine the two simply by linear addition; or,
- Method II: Use the "small crack fcg law", in which case ccg is not calculated separately but rather creep enters as a non-linear correction to the fcg.

Qu.: When should Methods II be used rather than Method I?

The discriminating factor is whether the crack lies entirely within the cyclic plastic zone, r_p.

If the crack tip lies outside the cyclic plastic zone then Method I should be used and the total crack growth is simply $fcg + ccg$, where fcg is given by a Paris Law and ccg by integrating an $\dot{a} - C(t)$ law.

If the crack tip lies within the cyclic plastic zone then Method II should be used. This is described below.

Qu.: How is the size of cyclic plastic zone calculated?

r_p is found using the methods of R5V2/3 (see for example R5V2/3 §7.1.4). r_p is that part of the section which is outside strict shakedown (elastic cycling). Recall that the global shakedown criterion of R5V2/3 is that the sum of the cyclic plastic zone sizes, r_p, at the inner and outer surfaces should be less than 20% of the section. This r_p should not be confused with the cyclic plastic zone which occurs at the tip of the crack itself, r_p^{crack}.

Qu.: What is the Method II crack growth procedure?

This is addressed in R5V4/5 §10.7.3.2. The total (creep-fatigue) crack growth is found using,

$$\frac{da}{dN} = \frac{1}{\left(1 - D_c^{surface}\right)^2}\left(\frac{da}{dN}\right)_f \qquad (1)$$

where the fcg term $\left(\frac{da}{dN}\right)_f$ is found using the small-crack law (below). The creep damage which appears in (1) is that at the surface from which the crack is growing. It should be found using R5V2/3 methodology, i.e.,

$$D_c^{surface} = \int \frac{\dot{\varepsilon}}{\varepsilon_f} dt \qquad (2)$$

This is essentially an uncracked body surface creep damage, as per R5V2/3, except that R5V4/5 recommends that the elastic follow-up, Z, for the cracked body be used. However, the strain rate is determined from the uncracked body surface stress. Relaxation is taken into account in the integration, as per the usual R5V2/3 procedure. The creep ductility, ε_f, includes strain rate and stress state effects - and might deploy the stress modified ductility approach if appropriate.

However, note that when Method II is relevant there is necessarily a hysteresis cycle which will reset the start-of-dwell stress on every cycle. So the surface creep damage, (2), is to be understood in the R5V2/3 sense, summing the damage over every cycle. As with R5V2/3 there will be an issue of continuous creep hardening versus primary creep reset.

Qu.: What if $D_c^{surface} = 1$?

If $D_c^{surface} = 1$ then the crack growth rate in (1) is infinite. Since a crack is being assumed in the assessment, it is entirely possibly that this is due to such a prediction of creep crack initiation. In this case the R5V4/5 procedure is to assume that the crack extends over the size of the cyclic plastic zone. So you should set $a = r_p$ and continue the assessment using Method I. Effectively this assumes that the growth rate through the region of cyclic plasticity is very rapid. A more refined assessment could try to establish the variation of D_c through the cyclic plastic zone, and use the Method II approach starting from some intermediate crack depth $0 < a < r_p$.

Qu.: What is the 'small crack' (Method II) fatigue law?

The small crack fcg law in R5 and R66 is,

$$\frac{da}{dN} = B'a^Q \ \text{mm/cycle} \qquad (3)$$

where a is the crack depth in mm. R66 provides recommendations for the parameters B' and Q for a range of materials. An upper bound for all these materials is given by,

$$Q = 1 \ \text{and} \ B' = 26{,}100\Delta\varepsilon_t^{2.85} \qquad (4)$$

for a total strain range of $\Delta\varepsilon_t$. This bounds data obtained at high temperatures (550^{o}C and above).

Qu.: What is the Paris Law (Method I)?

The Paris Law for fcg is,

$$\frac{da}{dN} = C\Delta K^m \quad (5)$$

where m is typically between 3 and 4.

Error Trap: The numerical parameters in R66 for the small crack fcg law, (3), gives the result in mm/cycle, whereas for the Paris Law, (5), the result is in m/cycle

See the session 22 notes for the bulk of the required material on Paris Law fatigue crack growth http://rickbradford.co.uk/T73S02TutorialNotes22.pdf. This includes,

- Where the Paris Law comes from in terms of continuum damage;
- Definition of the effective stress intensity factor range, ΔK_{eff}, to account for partial crack closure effects when using the Paris Law;
- The fatigue crack growth threshold, ΔK_0, and its physical basis.

A brief recap follows…

Qu.: How is ΔK_{eff} defined?

When is a crack not a crack? When it is permanently closed. If the stresses were always compressive over the crack then the crack might as well not be there – and, in particular, fatigue crack growth would not be expected. Moreover commonly, the applied stress may range between compressive and tensile values. So the crack may be closed over part of the load cycle and open over the rest of the cycle. How does this affect fcg? The advice in R5 and R66 is as follows: define the minimum and maximum SIFs as $K_{\min}$ and $K_{\max}$, where the latter may be negative. Put,

$$R = K_{\min} / K_{\max} \quad (6a)$$

and then define,

$$q = 1 \quad \text{for } R \geq 0 \quad (6b)$$

$$q = \frac{1 - 0.5R}{1 - R} \quad \text{for } R < 0 \quad (6c)$$

And then use,

$$\Delta K_{eff} = q\Delta K \quad (6d)$$

in the Paris law to evaluate fcg.

Qu.: Is the assumption of partial-closure implicit in Eqs.(6a-d) always valid?

No.

There is an assumption implicit in this advice – namely that the crack was not opened before being loaded. If the crack has a substantial gape under zero load, then it may be that the *whole* SIF range, ΔK, contributes to fcg because the crack faces are never in contact.

Qu.: What is meant by the fcg threshold, ΔK_0?

Fatigue crack growth laws like the Paris Law $\frac{da}{dN} = C\Delta K^m$ do not apply for arbitrarily small stress intensity factor ranges. In fact, below a certain stress intensity factor range there is no fatigue crack growth at all. The ΔK which is required to produce non-zero growth is the fatigue crack growth threshold, ΔK_{th} or ΔK_0.

Qu.: Why is there a fatigue crack growth threshold?

A crack cannot advance by less than one atomic spacing. The fcg threshold ΔK_0 is effectively the SIF range which would produce a growth of one atomic spacing. Hence ΔK_0 is such that $C\Delta K_0^m \approx 2\times 10^{-10} m$, which for experimentally determined upper bound values of C gives a lower bound threshold of $\Delta K_0 \sim 2MPa\sqrt{m}$. This is in good agreement with test data of the fcg threshold. A lower bound of $\Delta K_0 \sim 2MPa\sqrt{m}$ is generally used in practice. In truth such low values tend to occur only for high mean stresses. For small mean stresses ΔK_0 may be a factor of 2 or 3 larger.

Qu.: What is the valid range of the Paris Law fits?

Obviously the Paris Law s not valid for $\Delta K < \Delta K_0$, when the growth is zero.

R66 gives maximum SIF range $\Delta K_{\max}$ (or sometimes a value for $K_{\max}$, but since R is generally very small this is almost the same thing). These are generally in the range 40 – 50 $MPa\sqrt{m}$.

R66 suggests that extrapolation beyond $\Delta K_{\max}$ may be acceptable provided that $K_{\max} < 0.75K_{Ic}$ for brittle materials and $\sigma_{ref} < 0.7\sigma_f$ for ductile materials.

For CMn steels explicitly modified Paris-type laws are given for $K_{\max} > 0.75K_{mat}$.

Qu.: What plasticity correction to ΔK_{eff} may be required?

If there is widespread yielding then ΔK cannot be expected to be adequate to parameterise fatigue crack growth, since it does not capture the plastic strain contribution. The relevant parameter is ΔJ, the range of the elastic-plastic fracture parameter, J, which takes the elevated reference strain range into account.

The fcg law is then re-expressed in terms of ΔJ, i.e.,

$$\frac{da}{dN} = C(E'\Delta J)^{m/2} \tag{7}$$

Qu.: How is the cyclic J-range found?

The cyclic J-integral, ΔJ, can be found by replacing stress, strain and displacement in the usual J-integral by their ranges. A more practical tool for assessments is to use the reference stress formula, $\Delta J = \frac{\Delta\varepsilon_{ref}}{\Delta\sigma_{ref}}\Delta K^2$. Here the reference strain is the elastic-plastic strain corresponding to the reference stress range $\Delta\sigma_{ref}$. For example, assuming a Ramberg-Osgood fit, $\Delta\varepsilon_{ref} = \frac{\Delta\sigma_{ref}}{E} + \left(\frac{\Delta\sigma_{ref}}{A}\right)^n$, where $n = \frac{1}{\beta}$.

However, it is also necessary to take account of crack closure effects. This can be done conservatively using,

$$\Delta J = q\left[\frac{q}{E} + \frac{(\Delta\sigma_{ref})^{n-1}}{A^n}\right]\Delta K^2 \tag{8}$$

This is equivalent to R5V4/5 Equ.(A3.9). See R5V4/5, Appendix A3 for further details.

Creep-Fatigue Interactions in Crack Growth

Qu.: What creep-fatigue interactions are there?

There are a rather bewildering variety of creep-fatigue interactions. We have already seen one in the Method II growth law, which combines fcg and ccg in a non-linear manner. The other issues relating to creep-fatigue interactions in R5V4/5 are,

- **§9.2**: test for insignificant cyclic loading, satisfaction of which means that ccg can be calculated as if it were steady creep (unperturbed by load cycling). Hence this criterion relates to whether fatigue influences ccg.
- **§9.3(i)**: the criterion for whether to use R66 Section 10 or Section 12 fcg laws. The former are based on continuous cycling, whereas the latter employed creep dwells between the load cycles. Hence this criterion relates to whether creep influences fcg.
- **§9.3(ii)**: Prior creep damage on the ligament can affect the subsequent fcg rate.

These three issues are discussed in turn below.

Qu.: What are the criteria for insignificant cyclic loading in R5V4/5 (§9.2)?

The criteria are in two parts, for the gross behaviour when assumed uncracked and for the crack tip region.

For the gross, uncracked structure, the criteria are given as in R5V2/3 Section 6.6.2. This aspect of cyclic loading may be deemed insignificant if all three of the following criteria are met,

- The greatest elastic Mises stress range, $\Delta\bar{\sigma}_{el,\max}$, is less than the sum of $K_S S_y$ at the two ends of the cycle;
- The elastically based fatigue damage is less than 0.05;
- Creep behaviour is unperturbed by cyclic loading.

The crack-dependent parts of the checks for insignificant cyclic effects are,

- The fatigue crack growth does not exceed 10% of the creep crack growth, and,
- The cyclic plastic zone at the crack tip is small compared with the characteristic dimensions (i.e., the crack size, the ligament size and the thickness).

The cyclic plastic zone size is given by,

Plane Stress: $$r_p^{crack} = \frac{1}{2\pi}\left(\frac{\Delta K}{2\sigma_y}\right)^2 \tag{9}$$

or 1/3 of this in plane strain, where σ_y is the cyclic yield strength (0.2%). All five of the above conditions must be met for cyclic loading to be insignificant.

Personally I would add the requirement to be within strict shakedown to this list. I suspect that the first criterion, above, namely $\Delta\bar{\sigma}_{el,\max} < (K_S S_y)_c + (K_S S_y)_{nc}$, is intended to ensure this – but actually this criterion is *not* sufficient to ensure strict shakedown.

Qu.: What if the above criteria are not met?

If the above criteria are met then creep is unperturbed by load cycling and ccg can be evaluated using R5V4/5 Appendix A2 (augmented by Ref.[1]). Prior to Issue 3 of R5 this is what used to be called "Volume 4". This is discussed in greater detail in session 43, http://rickbradford.co.uk/T73S03TutorialNotes43.pdf.

If creep is perturbed by load cycling then ccg must be evaluated using R5V4/5 Appendix A3 (augmented by Ref.[1]). Prior to Issue 3 of R5 this is what used to be called "Volume 5". This is discussed in greater detail in session 44A, http://rickbradford.co.uk/T73S03TutorialNotes44A.pdf.

Qu.: Should fcg data with dwells be used [§9.3(i)]?

This is a very important decision because fatigue crack growth laws based on tests which include dwells at creep temperatures can be far more onerous than those based on continuous cycling tests (e.g., by about an order of magnitude – compare R66 Sections 10 and 12). In practice the fcg laws based on continuous cycling are more commonly used because the R5V4/5 assessment itself takes account of the ccg part separately.

However, R5V4/5 §9.3(i) specifies an exception. This is when creep is perturbed by cyclic loading but the fcg is less than 10% of the ccg. In this case the use of fcg data based on tests including dwells "relevant to the service application" is recommended. Given that service dwells are likely to be of the order of 1000 hours or more this will be problematical! However, the use of R66 Section 12, as a minimum, appears motivated.

Qu.: Is the effect of dwells on fcg really due to creep?

In addition to creep, there is another degradation mechanism which becomes active if the specimen is held for a period at high temperature: oxidation. One school of thought is that we should really talk about creep-fatigue-oxidation crack growth. Certainly there is evidence of substantial amounts of crack face oxide at sufficiently high temperatures.

If this is true then the environment will affect the growth rate. CO_2 might be worse than air, for example. The way to test this would be to carry out careful comparison tests in an inert atmosphere (e.g., argon).

Qu.: How does prior creep damage affect fcg [§9.3(ii)]?

In principle the fcg rate can be accelerated through material previously damaged by creep [see R5V4/5 §9.3(ii)]. One school of thought is that this is relevant only for very heavy creep damage, i.e., if $D_c > 0.8$ (BS7910). Another is that fcg should be factored by $1/(1 - D_c)$ at all damage levels. In either case my opinion is that the D_c in question should be a best estimate.

C^* and $C(t)$ Estimation Formulae for Primary Loads Only

Qu.: How is C* estimated for primary loading alone?

We've already met the reference stress formula for C* in session 29. It is analogous to the reference stress estimate for J,

$$C^* = \frac{\dot{\varepsilon}_{ref}^{PR}}{\sigma_{ref}^{PR}} K_{PR}^2 \tag{10}$$

where "PR" stands for "primary loads".

Qu.: How is C(t) estimated for primary loading alone?

Again, we've already met the reference stress formula for C(t) in session 39, Equ.(22), in a more general form which incorporates both primary and secondary loads and plasticity. In the case of primary loads only, and no plasticity, this reduces to,

$$C(t) = f(\tau) \cdot C^*, \tag{11a}$$

where,

$$f(\tau) = \frac{(1+\tau)^{1+n}}{\left[(1+\tau)^{1+n} - 1\right]}, \tag{11b}$$

and,

$$\tau = \frac{\varepsilon_{ref}^c}{\varepsilon_{ref}^e} \tag{11c}$$

and n is the creep index, $\dot{\varepsilon}_c \propto \sigma^n$. All quantities are understood to be due to primary loads alone, and C^* is given by (10). In practice n is usually replaced in (11b) by,

$$n = \frac{q}{1-q} \tag{11d}$$

The denominator in (11c) is the elastic strain, $\varepsilon_{ref}^e = \sigma_{ref} / E$.

Qu.: Where does the relation between n and q come from?

The relation between n and q comes from the continuum damage derivation of the $da/dt = A.C^{*q}$ growth law, as we shall see below shortly.

Qu.: What is the redistribution time?

The redistribution time is when the creep strain reaches the elastic strain, i.e., $\varepsilon_{ref}^c(t_{red}) = \varepsilon_{ref}^e$. The parameter τ in (11a,b,c) is therefore effectively a non-linear dimensionless time measure which reaches 1 at the redistribution time.

Qu.: What are the short and long time behaviours of C(t) as given by (11)?

The function $f(\tau)$ takes account of redistribution. The limiting behaviours of the function $f(\tau)$ are,

$\tau \to 0$: $$f(\tau) \to \frac{1}{(1+n)\tau} \quad \text{(singular)} \tag{12a}$$

$\tau \to \infty$: $$f(\tau) \to 1 \tag{12b}$$

The latter conforms to the expectation that $C(t) \to C^*$ as $t \to \infty$.

The former models the "C(t) spike" at early times prior to redistribution.

Qu.: How does plasticity affect the C(t) estimate?

Again this follows from the more general reference stress formula for C(t) which was presented in session 39, Equ.(22), and which incorporates both primary and secondary loads as well as plasticity. In the case of primary loads only, but now including plasticity, it reduces to,

$$C(t) = f(\tau) \cdot C^*, \tag{13a}$$

$$f(\tau) = \frac{(1+\tau)^{1+n}}{\left[(1+\tau)^{1+n} - 1\right] + \dfrac{\varepsilon_{ref}^{p}}{\varepsilon_{ref}^{ep}}} \tag{13b}$$

$$\tau = \frac{\varepsilon_{ref}^{c}}{\varepsilon_{ref}^{ep}} \tag{13c}$$

In (13b,c) the superscripts $^{c,\ e,\ p}$ refer to creep, elastic and plastic strains respectively, with ^{ep} meaning the sum of the elastic and plastic strains.

Now that plasticity has been included we find that the behaviour as $t \to 0$ is no longer singular. Instead we find, as long as the plastic strain is non-zero,

As $t \to 0$: $$C(t) \to \frac{\varepsilon_{ref}^{ep}}{\varepsilon_{ref}^{p}} C^* \tag{14}$$

Hence, if the plastic strain is small compared with the elastic strain then at $t = 0$ $C(t) >> C^*$ and we still have a "*C(t)* spike" at early times, though not actually an infinite spike.

But if the plastic strain is large compared with the elastic strain then even at early times we have $C(t) \approx C^*$ and there is no *C(t)* effect.

This illustrates the importance of plasticity in creep crack growth.

Continuum Damage Mechanics Derivation of $\dot{a} = A.C^{*q}$

Qu.: What is Continuum Damage Mechanics (CDM)?

Continuum damage mechanics treats the material as a continuum and has some model for calculating damage in terms of stresses and strains. The particular CDM we shall assume here will use the simple ductility exhaustion damage model. A small material region (e.g., a grain or a finite element) is taken to fail when its creep strain reaches some creep ductility. This is just what is used to define creep damage in a crack initiation assessment to R5V2/3. The term CDM is used when the damage model in question is applied to all points (i.e., over the continuum). Pedants will note that this is not strictly true when CDM is implemented using finite element analysis, as is usual, because then the damage is calculated only at a discreet set of points (Gauss points or nodes).

Qu.: Can CDM be used to model crack growth?

Yes.

In fact the use of CDM in crack growth is technically stronger than its use to predict the fracture of a stationary crack.

In the latter case there is a snag if the usual sharp crack tip fields are employed since the stresses and strains are divergent at the crack tip. So, whatever finite ductility or finite strength were used, the crack would appear to fracture at virtually no load. This is, of course, the reason why parameters K and J are used in fracture mechanics instead of near-tip stresses and strains. In fracture an effective fracture strength, σ_{fs}, could be defined only if some characteristic fracture distance, d_f, were introduced. For example, $\sigma_{fs} = K_{Ic} / \sqrt{2\pi d_f}$.

Interestingly this problem does not occur in creep crack growth. Although the strain rate is divergent at the crack tip, the creep strain which accumulates in a given material element towards which the crack is advancing is finite. However, like the fracture case, this creep strain does include a characteristic length-scale dependence. In the case of ccg this length scale is the size of the creep zone, r_c (at least, it is if the crack tip fields are C* controlled). So, whereas the characteristic scale in fracture is a very small, near-tip distance, d_f, in ccg the characteristic scale, r_c, is large – the whole of the creeping zone[1].

Qu.: How is $\dot{a} = A.C^{*q}$ derived from CDM?

Imagine a material element which starts a distance r_c from the crack tip, and lies on the line along which the crack will grow. Since r_c is, by definition, the boundary of the creep zone from the initial crack tip position, the material element initially has zero creep strain (zero damage). The distance of this material element from the crack tip, r, will reduce as the crack grows towards it. So we write $r(t)$. The creep strain rate in this material element can therefore be written $\dot{\varepsilon}_c(r(t))$, where the notation

[1] In physicists' jargon, fracture would be said to have an ultraviolet catastrophe, whereas ccg has an infrared catastrophe.

emphasises that the strain rate will increase as the material element becomes nearer to the crack tip. The ductility exhaustion failure criterion is thus,

$$\int_0^{t_c} \dot{\bar{\varepsilon}}_c\left(r(t)\right)dt = \varepsilon_f \tag{15}$$

In (15), the creep ductility, ε_f, is that relevant to the state of stress, and the strain rate, prevailing within the creep zone. In truth this will vary with position in the creep zone and hence ε_f should really appear within the integral, i.e., the ductility exhaustion failure criterion is strictly,

$$\int_0^{t_c} \frac{\dot{\bar{\varepsilon}}_c\left(r(t)\right)}{\varepsilon_f\left(r(t)\right)}dt = 1 \tag{15b}$$

This refinement is ignored here for simplicity and we shall use (15). The upper limit of the integration in (15) is the time, t_c, for the crack to growth the distance r_c. In contrast, the instantaneous crack growth rate is,

$$\dot{a} = -\frac{dr}{dt} \tag{16}$$

So we can change the integration variable in (15) from t to r to give,

$$-\int_{r_c}^{0} \frac{\dot{\bar{\varepsilon}}_c(r)}{\dot{a}} \cdot dr = \varepsilon_f \tag{17}$$

The crack is assumed to have already incubated and the structure is assumed to be in secondary creep give by $\dot{\varepsilon}^c = B\sigma^n$. Moreover, steady creep conditions are assumed to have been achieved so that the crack tip fields are controlled by C^* [as opposed to $C(t)$]. Hence the explicit expression for the strain rate is,

$$\dot{\varepsilon}_{ij}^c(t) = B\left[\frac{C^*}{BI_n r}\right]^{\frac{n}{n+1}} \hat{\varepsilon}_{ij}(\theta, n) \tag{18}$$

Substituting (18) into (17) and carrying out the integral gives,

$$\dot{a}\varepsilon_f = \int_0^{r_c} \dot{\bar{\varepsilon}}_c dr = B\left[\frac{C^*}{BI_n}\right]^{\frac{n}{n+1}} \tilde{\tilde{\varepsilon}}(\theta, n)\int_0^{r_c} \frac{dr}{r^{\frac{n}{n+1}}} = \frac{(n+1)B^{\frac{1}{n+1}}}{I_n^{\frac{n}{n+1}}} C^{*\frac{n}{n+1}} \cdot \tilde{\tilde{\varepsilon}}(\theta, n) \cdot r_c^{\frac{1}{n+1}} \tag{19}$$

Note that the crack growth rate, $\dot{a}$, has implicitly been assumed constant and taken out of the integral. This approximation assumes sufficiently small amounts of growth that C*, and hence $\dot{a}$, do not vary significantly over the period of interest.

Equ.(19) is a growth law of the form $\dot{a} = A.C^{*q}$ and we can see that,

$$q = \frac{n}{n+1} \tag{20}$$

$$A = \frac{(n+1)B^{\frac{1}{n+1}}}{I_n^{\frac{n}{n+1}}} \cdot \frac{\tilde{\tilde{\varepsilon}}(\theta, n)}{\varepsilon_f} \cdot r_c^{\frac{1}{n+1}} \tag{21}$$

Qu.: How well does (20) represent crack growth data?

A typical austenitic primary creep index is $n \sim 4$, for which (20) gives $q \sim 0.8$, which agrees pretty well with experimental data (i.e., the slope of the $\dot{a} = A.C^{*q}$ trend line on a log-log plot). Common q values fitted to ccg data are up to 0.9, which correspond using (20) to $n = 9$, which is also a typical creep index in secondary creep.

Qu.: How important is the r_c dependence in (21)?

For a typical primary creep index of $n \sim 4$ ($q = 0.8$), the dependence upon r_c is weak, i.e., $r_c^{0.2}$. Thus, a 10 fold increase in r_c causes only a 58% increase in growth rate. For an equally likely $n = 9$ ($q = 0.9$), more typical of austenitic secondary creep, the ccg dependence is even weaker, $r_c^{0.1}$, and a 10 fold increase in r_c would result in only a 26% increase in crack growth rate.

Qu.: How sensible is the ductility dependence in (21)?

Equ.(21) implies that crack growth rates would be 10 times faster for a material with a ductility of 1% compared with a material with a ductility of 10%. This seems reasonable.

Qu.: How well does (21) represent crack growth data?

Comparison of q with prediction of Equ.(20)

Consider 316ss at 525-550°C. The growth rate data fit of R66 Rev.007 (i.e., prior to 2010) Table 11.2 gave q = 0.811, whereas longer term test data, including HAZ tests reported in Ref.[3], are consistent with R66 Rev.008 (or Rev.009) which give q = 0.891.

The RCC-MR deformation law (R66 Table 5.3) gives n_1 in primary creep to be ~4.15 and n, in secondary creep, to be 8.2 to 9.06. The former might be expected to relate better to the short term ccg tests and gives, according to Equ.(20), $q = \frac{n}{n+1} = 0.806$, in good agreement with the shorter term tests. Similarly, the secondary creep index gives $q = \frac{n}{n+1} =$ 0.89 to 0.90, in good agreement with the longer term ccg test results.

Comparison of *A* with prediction of Equ.(21)

To compare the experimental and predicted magnitudes of the coefficient A we assume the longer term tests and secondary creep, and shall use 525°C. Hence the RCC-MR parameters are $n = 9.06$ and $B = 4.15 \times 10^{-29}$ (units: MPa, m and hours).

To evaluate Equ.(21) we shall need the HRR field solution to provide I_n and $\tilde{\tilde{\varepsilon}}(\theta, n)$ and we shall also need the creep ductility appropriate to the state of stress near the crack tip. The value of I_n is 3.03 to 4.60 (plane stress and plane strain respectively, Ref.[4]). Considering $\theta = 0$ the HRR field data are,

Table 1: HRR Dimensionless Parameters $\tilde{\sigma}_{ij}, \tilde{\varepsilon}_{ij}$ on $\theta = 0$ for $n = 9$ (from Ref.[4])

Component	Strain		Stress	
	Plane Stress	Plane Strain	Plane Stress	Plane Strain
x	0.0590	-0.0163	0.6348	1.7184
y	0.7938	0.0163	1.1463	2.4609
z	-0.8528	0	0	2.0897
Mises	0.9525	0.01882	0.9946	0.6430
Hydrostatic	0	0	0.5937	2.0897
Spindler*	-	-	0.46	~10^{-5} !!

**using p = 2.38, q = 1.04 (as for reheat cracking assessments of 316H)*

Clearly the use of the extremely highly constrained conditions near a perfectly plane strain, sharp crack tip has resulted in the Spindler fraction being extrapolated beyond its range of applicability. However, we ***do*** know from the phenomenon of reheat cracking that the correct Spindler fraction under conditions of high constraint can be of the order of 0.1. All we can really say is that, in this context, the correct factor for a plane strain crack tip might be even smaller than 0.1, perhaps a lot smaller. Webster & Ainsworth suggest ~0.02.

Plane Stress Prediction

The ccg data is derived from 2-inch CTS, with a ligament of order ~25mm. Hence we shall assume $r_c \sim 10$ mm (a rough guess only, but the result is not terribly sensitive to this).

Finally, for 316 at ~525°C, the lower bound and best estimate uniaxial creep ductilities are respectively ~2.6% and ~10.7%.

Putting these data into (21) gives,

$$A = \frac{(n+1)B^{\frac{1}{n+1}}}{I_n^{\frac{n}{n+1}}} \cdot \frac{\tilde{\tilde{\varepsilon}}(\theta,n)}{\varepsilon_f} \cdot r_c^{\frac{1}{n+1}} \approx \frac{10.06 \times \left(4.15 \times 10^{-29}\right)^{1/10.06} \times 0.9525 \times (0.01)^{1/10.06}}{0.46 \times 3.03^{9.06/10.06} \times (0.026 to 0.107)} = 0.073 \text{ to } 0.264$$

This compares with the BE and UB values of A from Ref.[2] of 0.088 and 0.221 respectively (consistent with the q value of 0.891). The prediction is remarkably close, spanning the bulk of the measured range.

This concurs with the findings of Webster & Ainsworth (graphs given in session 41, http://rickbradford.co.uk/T73S03TutorialNotes41.pdf) where experimental crack growth rates were generally close to the plane stress prediction.

Plane Strain Prediction?

Because of the uncertainty regarding the Spindler fraction, a plane strain prediction is problematical. For sake of argument let us adopt Webster & Ainsworth's suggestion that the multiaxiality factor on ductility is ~0.02 (i.e., 1/50), Ref.[5]. This is in stark contrast to the Spindler fraction of ~0.46 in plane stress (Table 1). The ductility near a plane strain crack tip is thus ~0.04 times that near a plane stress crack tip.

Now note that the Mises equivalent strain near the plane strain crack tip, and on $\theta = 0$, is a factor of 50 times ***smaller*** than that near the plane stress crack tip, i.e., the ratio of the $\tilde{\tilde{\varepsilon}}(\theta,n)$ is 0.01882 / 0.9525 = 0.020 (p.strain over p.stress).

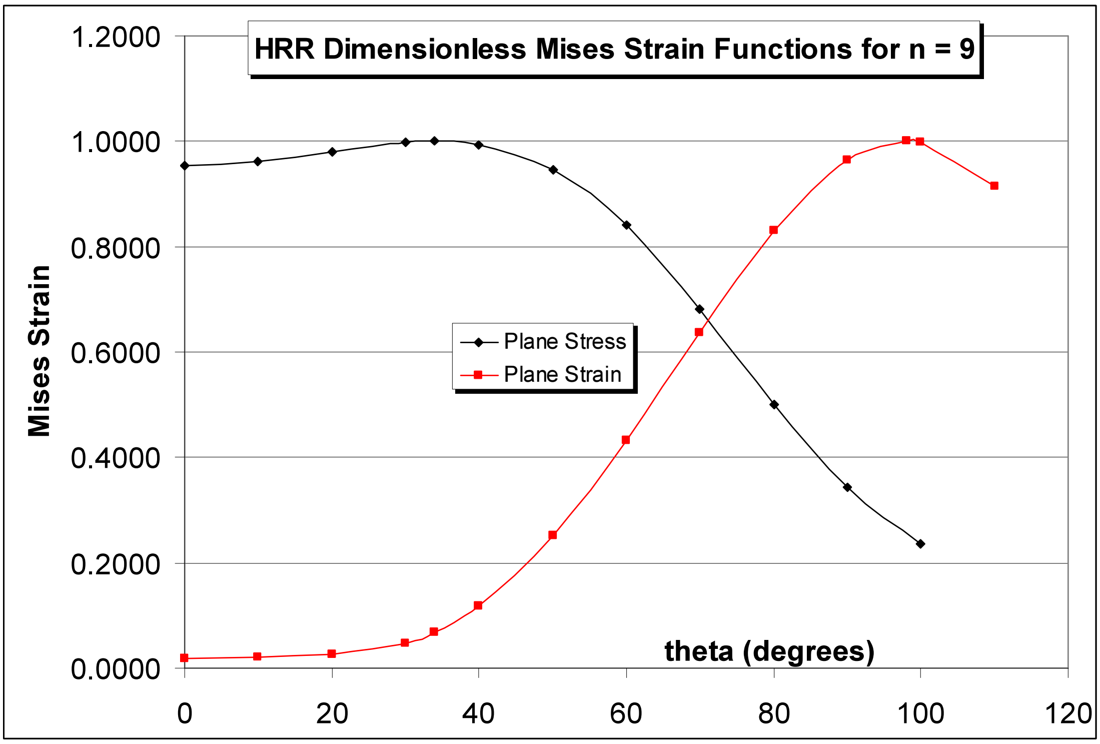
This appears to suggest that the plane strain crack growth rate should be a factor 0.02/0.04 = 0.5 times the plane stress crack growth rate. How does this anomalous prediction that the plane strain crack growth rate be smaller than in plane stress come about? And why do Webster & Ainsworth, Ref.[5], make a very different claim?

Part of the reason becomes clear when we look at how the dimensionless HRR Mises strain function varies with θ, as shown in Figure 1 below. The plane strain value near $\theta = 0$ is very small, 0.0188, but it increases to unity at an angle of 98^o. In contrast, the plane stress function is close to unity at $\theta = 0$, and indeed for all angles less than ~45°.

Webster & Ainsworth claim that the plane strain growth rate exceeds the plane stress growth rate by up to a factor of 50, i.e., simply by the (reciprocal of) the multiaxial ductility factor. The basis of this claim is that they consider Equ.(21) to apply at that angle which maximises the function $\tilde{\bar{\varepsilon}}(\theta, n)$ shown in Figure 1. The maximum value of $\tilde{\bar{\varepsilon}}(\theta, n)$ is unity for both plane stress and plane strain – the difference being the angle at which this occurs. Webster & Ainsworth attach no significance to this angle.

But is this reasonable?

Figure 1



Qu.: What is the relevance of the angle?

Experimental data is intended to relate to self-similar crack propagation, i.e., on $\theta = 0$. In the case of side-grooved specimens, such self-similar growth is effectively forced. Consequently the use of $\theta = 0$ in Equ.(21) seems to me to be reasonable. For plane stress this gives excellent results. For plane strain the result of using $\theta = 0$ is not necessarily in contradiction with experiment, since the prediction depends upon the unknown multiaxial ductility factor. On the other hand, the (implicit) use of $\theta = 98^o$ by Webster & Ainsworth seems strange. Whilst a specimen without side-grooves might well tend to grow the crack at some angle, growth approaching, or exceeding, 90^o seems unlikely. And in any case, actual laboratory data is constrained to refer to $\theta = 0$.

The issue might be confused by crack tip blunting. This will permit greater strains to arise in the blunting zone ahead of the crack, on $\theta = 0$, where the triaxiality is most severe. However, for many of the steels of interest to us – especially for long term tests – the cracks remain sharp as they grow.

I tentatively suggest that these considerations imply that the effect of plane strain conditions on creep crack growth are not so marked as Webster & Ainsworth suggest, i.e., not a factor of 50. Indeed it is feasible that, in some circumstances, there may be little difference between the plane stress and plane strain growth rates. The data of Webster & Ainsworth, Ref.[5], does suggest this, in that most data appears to stick stubbornly to near the plane stress prediction – even for specimens which one might have expected to be quite highly constrained.

In summary, whilst the plane stress prediction using continuum damage mechanics based on ductility exhaustion is in remarkably good agreement with experimental ccg data, there are difficulties with the equivalent plane strain prediction.

Qu.: Can FEA be used with CDM to model ccg?

In principle, yes - by removing failed Gauss points or reducing material strength at these points. But the above observations sound a note of caution. Mises creep strains ahead of the crack tip might be heavily constrained due to triaxiality, whilst the creep ductility is dramatically reduced in simple formulations for the same reason. Damage near the crack tip thus becomes indeterminate (in the limit, zero divide by zero). Do not have naive belief in the numbers coming out of such an attempt at modelling ccg.

Qu.: What ccg law can be used if you have no experimental ccg law?

If no specific ccg data are available, but creep ductility *is* available, R66 §11.3.5 suggests, following Nikbin, that you should use q = 0.85 with an upper bound of $A = \frac{0.003}{\varepsilon_f}$, where ε_f is the absolute creep ductility, or a conservative 'mean' of $A = \frac{0.0003}{\varepsilon_f}$.

References

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[2] C D Hamm (ed.), AGR Materials Data Handbook, R66 Revision 008, BEGL (2010), or now at Rev.009 as of July 2011.

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