Session 39 - T73S03 (R5V4/5/7 Creep-Fatigue Crack Growth)

Last Update 27/3/16

Definition of C(t) and C: units; Time dependent J(t); Relation between C(t) & J and between C* & J; Relation between C(t) and J for short times; Qualitative variation of C(t) against t: primary & secondary loads; Crack tip fields for stationary crack in creeping material (no plasticity); Hence reasonableness of C(t) control of creep crack growth; Qualitative observations for moving crack tip, Hu-Riedel (HR) fields; Distinction between redistribution & relaxation (recap)*

Qu.: What is R5V4/5 for?

R5 V4/5 addresses how to calculate the growth rate of a crack due to creep, fatigue or creep-fatigue. The methodology is applicable to both parent material and weldments. Methodologies are given for each of the situations,

- Steady loads (no load cycles);
- Load cycles which do not perturb the creep behaviour;
- Load cycles which do perturb the creep behaviour (crack growth with hysteresis loops).

In addition, R5V4/5 presents three methodologies for assessing whether crack growth incubates (i.e., whether any growth takes place at all). We will take a close look at just one of these incubation methodologies (the σ_d method) in a later session.

R5V4/5 addresses the effects of both primary and secondary loads in the above methodologies. In the case of creep crack growth under combined loading, methods to calculate the relevant controlling parameter, C(t), are given which include,

- the effects of both relaxation and redistribution, and,
- the effects of plasticity.

Initially the creep crack growth methodology for combined loading, including relaxation effects, was within report E/REP/BDBB/0059/GEN/04 Rev.003 (May 2010) by Ainsworth, Dean & Budden. This was incorporated into R5 in May 2012.

Qu.: Why is R5V7 included in T73S03?

R5V7 is included in the scope of T73S03 because it also gives a methodology for the calculation of creep crack growth. It is restricted in scope, however, as follows…

- R5V7 is applicable only to low alloy ferritic weldments;
- R5V7 applies only to steady loading, and so does not address fatigue or creep-fatigue;
- The estimation formula for the C(t) parameter in R5V7, from which creep crack growth is calculated, takes account of redistribution but not relaxation nor plasticity;
- Whilst an incubation methodology is included in R5V7, this is a method for which the required material data is unlikely to be available. The more useful σ_d method appears only in R5V4/5.

Consequently if it is desirable to include any of the above features in an assessment of a low alloy ferritic weldment, R5V4/5 should be used rather than R5V7.

Qu.: What are relaxation and redistribution?

Relaxation refers to secondary loads only. It is the phenomenon whereby the accumulation of creep strains causes the secondary loads, and hence the secondary stresses, to reduce (relax). Relaxation cannot happen to the primary loads – by definition since primary loads are held constant. Sometimes the term 'relaxation' may be used in the context of plastic strains rather than creep strains. The phenomenon is the same.

Redistribution refers to the *stress distribution*, not to the loads. Redistribution can occur for either, or both, of the primary and secondary stress distributions. The load resultants do not change during redistribution. In other words, the net force and the net moment acting over a section ($\int \sigma \cdot dx$ and $\int \sigma x \cdot dx$) do not change. In redistribution the highest stresses over a section will reduce whilst the lowest stresses will *increase* in order to keep the load resultants fixed. Obviously, redistribution will not occur if the stresses are uniform across the section. Creep will cause redistribution. However plasticity will also cause redistribution of the elastically calculated stress distribution.

For secondary loads, relaxation and redistribution will both be happening at the same time.

Qu.: How is the problem of creep crack growth approached?

Way back in session 22 we saw that the mechanism of fatigue crack growth involved energy being dumped into the crack tip region on each cycle. This is due to the cyclic plastic zone set up by the load cycling. More generally we can say that *any* mechanism of crack advance must involve depositing irrecoverable energy into the near-tip region. This energy produces the damage that causes the crack growth. But irrecoverable energy (entropy increase) means plastic or creep strains. Fatigue crack growth is the plastic part. Creep crack growth is, obviously, the creep part.

How do we get a handle on the creep strains being developed near the crack tip? In other words, what are the crack tip fields in creep (stress and creep strain rate)?

Qu.: Recap – What are the elastic-plastic crack tip fields?

When there is plasticity but no creep, and assuming power law hardening, $\frac{\varepsilon}{\varepsilon_0} = \left(\frac{\sigma}{\sigma_0}\right)^n$, the crack tip fields are (often) the HRR fields (see session 17). These are characterised by J, specifically,

$$\sigma_{ij}(r,\theta) = \sigma_0 \left[\frac{J}{\sigma_0 \varepsilon_0 I_n r}\right]^{1/n+1} \tilde{\sigma}_{ij}(\theta,n) \quad (1a)$$

$$\varepsilon_{ij}(r,\theta) = \varepsilon_0 \left[\frac{J}{\sigma_0 \varepsilon_0 I_n r}\right]^{n/n+1} \tilde{\varepsilon}_{ij}(\theta,n) \quad (1b)$$

$$u_i(r,\theta) = \varepsilon_0 r \left[\frac{J}{\sigma_0 \varepsilon_0 I_n r}\right]^{n/n+1} \tilde{u}_i(\theta,n) \quad (1c)$$

The universal angular functions $\tilde{\sigma}_{ij}(\theta,n), \tilde{\varepsilon}_{ij}(\theta,n)$ and $\tilde{u}_i(\theta,n)$ have to be found by numerical means. In the case $n = 1$ they reduce to the LEFM angular functions. The angular dependence varies with hardening index, n. However, since they are subject to the same boundary conditions, they have qualitatively similar forms. Tabulated values may be found in Fong Shih's 1983 paper.

The HRR fields apply in small scale yielding – but they also apply for general yielding in many instances, e.g., for 'valid' fracture toughness specimens.

Note that the HRR fields depend upon the radial distance, r, only through the dimensionless ratio r/δ_0, where $\delta_0 = J/\sigma_0$. Moreover, their only dependence on the applied load is also via the ratio r/δ_0, since this is the only place in which J occurs. Hence, as load is increased, the graph of any of the field components against normalised distance, r/δ_0, remains unchanged.

Qu.: What creep strain rate do we assume?

Like the plastic case, the crack tip fields are developed for power law creep. Specifically,

$$\dot{\varepsilon}^c = B(t)\sigma^n \qquad \text{(uniaxial)} \tag{2a}$$

$$\dot{\varepsilon}^c_{ij} = \frac{3}{2}B(t)\bar{\sigma}^{n-1}S_{ij} \qquad \text{(multiaxial)} \tag{2b}$$

These expressions describe secondary creep if $B(t)$ is time independent, and otherwise can describe primary creep with an arbitrary time dependence [except that the stress and time dependences are assumed separable in Equs.(2)].

Qu.: How can the HRR fields, Equs.(1), be extended to creep?

The trick is to define a creep fracture parameter which can take the place of J in Equs.(1) so that these equations then give the correct crack tip fields in creep when strain and displacement are replaced by strain rate and displacement rate. This is easier than it might seem. Recall that in PYFM J is defined as,

$$J = \int_\Gamma \left\{ W_{ep}dy - \sigma_{ij}\frac{du_i}{dx}n_j ds \right\} \tag{3}$$

where,

$$W_{ep} = \int \sigma_{ij} d\varepsilon^{ep}_{ij} \tag{4}$$

and ε^{ep} is the elastic-plastic strain. Here Γ is some contour which encloses the crack tip and starts and finishes on opposite faces of the crack (see sessions 15 & 16). We know, therefore, that substitution of Equs.(1) into (3,4) will result in J popping out of the integration.

Consequently we define the creep fracture parameter, $C(t)$, as,

$$C(t) = LIM_{\Gamma \to 0}\left[\int_\Gamma \left\{ \tilde{W}_c dy - \sigma_{ij}\frac{d\dot{u}_i}{dx}n_j ds \right\} \right] \tag{5}$$

where $\dot{u}_i$ represents displacement rates, and $\dot{\varepsilon}$ represents strain rates, and $\tilde{W}_c$ is defined analogously to Equ.(4) but in terms of strain rates, i.e.,

$$\tilde{W}_c = \int \sigma_{ij} d\dot{\varepsilon}_{ij}^c \tag{6}$$

Because strain *rates* are involved in (5,6), the elastic and plastic strains do not feature. Consequently if we put, in analogy to Equs.(1),

$$\sigma_{ij}(t) = \left[\frac{C(t)}{B(t) I_n r}\right]^{\frac{1}{n+1}} \hat{\sigma}_{ij}(\theta, n) \tag{7}$$

$$\dot{\varepsilon}_{ij}^c(t) = B(t)\left[\frac{C(t)}{B(t) I_n r}\right]^{\frac{n}{n+1}} \hat{\varepsilon}_{ij}(\theta, n) \tag{8}$$

[These are deduced from (1) by noting that $B(t)$ replaces $\varepsilon_0 / \sigma_0^n$].

That these expressions are correct can be checked by substituting them into (5). Only terms which are products of stress and strain rate (or displacement rate gradient) occur, and it is readily seen that *B(t)* cancels in such terms. The integral becomes identical to that obtained by substituting the PYFM fields, (1), into (3) for *J*, except that *C(t)* replaces *J*. Consequently the integral must evaluate simply to *C(t)*, as required.

Hence, for a given creep behaviour, defined by *B(t)* and *n*, the parameter *C(t)* defined by (5) uniquely specifies the crack tip fields via (7,8).

However, just as there are limitations upon when the plastic crack-tip fields are the HRR fields, so the creeping HRR fields given by (7,8) will apply only in certain circumstances, clarified further below.

Qu.: Why is *C(t)* defined only using a small contour in (5)?

Unlike *J*, the contour integral definition of *C(t)* applies only for sufficiently small contours.

This is because we have no right to expect the integral in (5) to be contour independent. Recall that the contour independence of *J* only follows by analogy with non-linear elasticity, and hence is restricted (strictly) to proportional loading. In practice it is reasonable to expect *J* obtained from (3) to be contour independent for monotonically increasing loads. But we can expect gross deviations from contour independence if there are stress reversals (due to, say, unloading or relaxation). Since relaxation will be common in creep, contour independence of (5) is not expected.

Note that this is not the same thing as contour independence of the domain integrals in ABAQUS. These are mathematically transformed versions of (5), which are exactly equivalent and involve both an integral like (5) performed on a large contour plus an area integral. (True contour independence would imply that this area integral was zero, but this is not usually the case in creep).

Qu.: How big is the region within which Equs.(7,8) are valid?

Equs.(7,8) describe the stress and strain rate fields only sufficiently near the crack tip and obviously only within the region where creep is occurring.

At sufficiently early times the creep zone around the crack tip will be very small. There may then be a region surrounding the creep zone which is still sufficiently close to the crack tip to be controlled by the crack tip fields – but these will be the PYFM HRR fields since the creep strains are zero. This is turn may be surrounded by a region in which the strains are purely elastic but still dominated by the crack tip and hence in which the stresses are the LEFM fields.

Hence, for small scale yielding and for sufficiently early creep times, there is a hierarchy of size scales,

$$r_{HRR}^{c} < r_c < r_{HRR}^{p} < r_p < r_{LEFM} < L \tag{9}$$

where,

r_{HRR}^{c} is the size of the region in which the fields are given by Equs.(7,8), the creep HRR fields specified by $C(t)$;

r_c is the size of the creep zone (non-zero creep strains);

r_{HRR}^{p} is the size of the region surrounding r_c in which the fields approximate to the PYFM HRR fields specified by J;

r_p is the size of the plastic zone (non-zero plastic strains);

r_{LEFM} is the size of the region in which fields may approximate to LEFM form;

L is the ligament size.

If plasticity is sufficiently extensive, then there will be no LEFM region.

At longer times, creep will tend to obliterate both the PYFM and LEFM regions.

Qu.: When are Equs.(7,8) for the creeping crack tip fields valid?

The conditions under which Equs.(7,8) are valid are,

- The creep rate (deformation) law is of the form of Equ.(2), i.e., power law creep;
- Within the region where the creep strains near the crack tip are large compared with the elastic strains (r_{HRR}^{c}).This region will be very small at early times;
- For "small scale creep", when the creep zone around the crack tip is surrounded by a PYFM or LEFM zone, OR, for large scale creep in certain circumstances (analogous to "valid" specimens – essentially when sufficiently constrained);
- The total strain is small (<<1). Equs.(7,8) are not valid too close to a blunting crack tip;
- The crack tip is stationary or sufficiently slow moving.

The last condition is significant since we shall be using the C(t) parameter to evaluate creep crack growth – so the crack tip is not stationary. In the case that the crack tip is moving there is another class of crack tip fields which apply (under certain simplifying conditions): the so-called HR or Hu-Riedel fields. For a sufficiently slowly moving crack tip, the Hu-Riedel fields will be wholly contained within a zone in which the fields are of the form (7,8). Under these conditions we have,

$$r_{HR}^{c} < r_{HRR}^{c} < r_c \tag{10}$$

where r_{HR}^{c} is the size of the region in which the fields are of HR form. However, if the crack tip moves too quickly then r_{HR}^{c} grows and eliminates any HRR region. In this case the process zone fields are no longer controlled by C(t) and creep crack growth laws derived from standard tests may not be applicable. R5 includes crack velocity limits to guard against this possibility.

Qu.: What is the "process zone"?

The process zone ($r < r_{proc}$) is the region in which the metallurgical mechanisms are occurring which lead to creep crack growth.

Qu.: When does *C(t)* control creep crack growth?

C(t) controls creep crack growth if the process zone lies within the region dominated by the creep HRR fields, given by Equs.(7,8), i.e., if $r_{proc} < r_{HRR}^{c}$. It this case it does not matter what the mechanisms underlying the creep crack growth might be since, as long as they depend only upon the stresses and strain rates, they must be controlled by C(t). This is entirely analogous to "validity" in PYFM theory.

For sufficiently early times, or perhaps even for long times if the temperature is low, the size of the region controlled by C(t), r_{HRR}^{c}, may be very small, so that $r_{HRR}^{c} < r_{proc}$. In this case C(t) will not control crack growth. Forgetting this simple fact can lead to some confusion. An interesting and important example of this will be treated in a later session (namely crack growth in austenitic steels at temperatures around or below LOIC).

Qu.: Why don't creep strains contribute to fracture?

I risk confusing you by raising this point, but I can't resist.

The usual approach to assessment is,

- For fracture assessments we use R6, which is based on J - and J depends only upon the elastic and plastic strains, not on the creep strains (even at high temperatures when creep is occurring);
- For the assessment of creep crack growth we use R5V4/5, which is based on C(t) - and C(t) depends upon the creep strain (rate).

But in PYFM we learnt that the occurrence of plastic straining causes (for primary loads at least) an increase in *J*, and hence an increased propensity to fracture for a given toughness. This can be seen from (3,4) or from the reference stress approximation for *J*, i.e., $J \approx \frac{\varepsilon_{ref}}{\sigma_{ref}} K^2$. For given primary loads, plasticity increases the reference strain and hence increases *J*.

But in that case you might expect creep strains to cause an increase in *J*, which would become a time dependent parameter. Hence, if you wait until the creep strains become large enough, do you get fracture as a result of this time-dependent *J(t)* reaching the fracture toughness?

Actually this is almost correct – except for the use of the word "fracture". The relevant term is actually "stable tearing". But it *is* true that there is a time dependent *J(t)* parameter, and you *do* get stable crack growth when this exceeds a certain 'toughness' value. The J(t) parameter is defined by the obvious extension of (3,4), namely,

$$J(t) = \int_{\Gamma} \left\{ W_{epc} dy - \sigma_{ij} \frac{du_i}{dx} n_j ds \right\} \tag{11}$$

where,

$$W_{epc} = \int \sigma_{ij} d\varepsilon_{ij} \tag{12}$$

$$\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^p + \varepsilon_{ij}^c \tag{13}$$

In other words, *J(t)* just uses the total (elastic, plastic *and* creep) strains, and displacements, rather than only the elastic-plastic values.

Qu.: So creep strains do contribute to fracture then?

No.

They contribute to stable tearing.

Qu.: By why is this stable tearing not included in R5 assessments?

It is.

But you usually call it "creep crack growth".

And it is usually assessed using C(t), not J(t).

Creep crack growth is actually a form of stable tearing (although the metallurgical mechanisms may be very different in creep and plasticity – and hence have very different energy requirements and hence very different effective 'toughnesses').

Qu.: So are C(t) and J(t) alternative but equivalent creep parameters?

Yes.

And consequently they are related. In fact it is the relationship between C(t) and J(t) which lies at the heart of the derivation of the estimation formulae for C(t). These derivations are due originally to workers such as Ainsworth, Budden and Joch. My own version which includes a small generalisation to primary creep is,

$$C(t) = \frac{B(t) J(t)^{n+1}}{(n+1)\int_0^t B(t') J(t')^n dt' + D J(0)^n} \tag{14}$$

where D is the constant in the tensile stress-strain curve, $\varepsilon_{ij}^p = \frac{3}{2} D \bar{\sigma}^{m-1} S_{ij}$. You do not ever need to use such relations in applications. The point of the R5 procedure is to protect you from such things. The derivation of (14) together with the derivation of the most general C(t) estimation formula of R5V7 type can be found on my web site at http://rickbradford.co.uk/C_t_EstimationFormulae.pdf.

Qu.: Why are stable tearing and creep crack growth usually considered to be completely different things?

Despite the mathematical relationship between C(t) and J(t), and the fact that creep crack growth is, macroscopically, a form a stable tearing, the metallurgical mechanisms of fracture (short term tearing) and creep crack growth (long term tearing) will generally be very different. The longer timescales involved in creep give slow, but low energy, mechanisms time to become significant. The effective 'toughness' in creep can be far smaller than the fracture toughness. So in practice the two things are best considered as distinct.

The low energy requirement of creep crack growth mechanisms is the reason why, for example, residual stresses may cause substantial growth of a crack over a long timescale, despite being insufficient to cause fracture.

Qu.: What is C*?

You may be surprised at the concentration on C(t). You may have expected the main creep fracture parameter to be C*. The physical distinction between the two will emerge later. For now we note that the definition of C* is simply as the long-time limit of C(t),

$$C^* = LIM_{t\to\infty}\{C(t)\} \tag{15a}$$

This limit exists only if the long-time limit of the creep law is secondary (i.e., constant strain rates, $B(t) \to B(\infty)$ = constant). The stresses then also become constant in the long-time limit. In this case, C* is time independent because Equ.(5) depends only upon strain rates, displacement rates and stresses which all become constant in the long-time limit. If B(t) does not become constant, then C* is not defined. Hence C* is not defined if primary creep continues indefinitely or if the long term behaviour is a form of tertiary creep.

Qu.: How is C* related to J(t)?

You cannot relate either C(t) or C* to the ordinary PYFM J, since this depends only upon the elastic-plastic strains and hence cannot be related to quantities which depend upon the creep strain rate. However just as (14) relates C(t) to J(t), so C* can be related to J(t).

This relationship follows by taking the time derivative of (11). Those parts of the derivative of the integrand of (11) which involve the strain rate or displacement rate just reproduce the integrand of (5), the definition of C(t). The remaining terms involve the time derivative of the stresses. During primary creep, or transient creep (redistribution or relaxation), the stress do change, so their time derivatives are not zero. Consequently C(t) is <u>not</u> equal to dJ/dt. However, in the long time, steady secondary creep limit the stresses become constant and these terms vanish leaving only the term equivalent to (5) – and in this limit $C(t) \to C^*$. Hence,

$$C^* = LIM_{t\to\infty}\{C(t)\} = LIM_{t\to\infty}\left\{\frac{dJ}{dt}\right\} \tag{15b}$$

Readers may like to confirm that (14) is consistent with (15b).

Qu.: Is this how R5 uses the symbol C*?

Not quite.

It almost is, but R5 defines C* through the reference stress formula as,

$$C^* = \frac{\dot{\varepsilon}_{ref}^{PR}}{\sigma_{ref}^{PR}} K^2 \tag{16}$$

In (16) the superscript ^{PR} denotes primary quantities, so the reference stress is just the ordinary primary-load only reference stress and $\dot{\varepsilon}_{ref}^{PR}$ is the corresponding strain rate.

Hence if secondary creep is used to evaluate $\dot{\varepsilon}_{ref}^{PR}$ then (16) is the conventionally defined C* (within the accuracy of the reference stress approximation).

But R5 also uses (16) in primary creep. In that respect the R5 definition of C* differs from the conventional definition (which would define C* only in secondary creep). However, it could be argued that there is no universal agreement in the literature.

Qu.: What are the units of C* and C(t)?

The dimensionality of C* and C(t) is the same as dJ/dt and hence their units are MPa.m/hour if the units used to calculate it are: stresses in MPa, K in MPa√m, strain rate in absolute/hour.

Take care – some sources use seconds as the time unit. And always convert strains to absolute if your deformation law gives you strain in %.

Qu.: What is the typical order of magnitude of C* or C(t) for plant?

Obviously there is a wide range of values, but a feel for the sort of order of magnitude is provided as follows,

- Take the stress to be 100 MPa;
- Take a creep strain of 1% to be accumulated in 30 years (225,000 hours at 85% availability) and use the corresponding average strain rate;
- Take K to be 20 MPa√m.

These give C* from the reference stress formula, (16), to be ~1.8 x 10^{-7} MPa.m/hour. Actually this is rather a large C* for plant and would correspond to a rather nasty crack growth rate. Values closer to ~10^{-8} MPa.m/hour may be more typical of plant. These estimates will be relevant when we look at the typical C* levels used to derive creep crack growth data in session 41 (generally above 10^{-6} MPa.m/hour).

Qu.: What is C(t) in the short-time limit?

We will state without proof how C(t) may be estimated for very early times. The derivations may be found in http://rickbradford.co.uk/C_t_EstimationFormulae.pdf. In the following formulae the primary creep function is assumed to be $B(t) \propto t^{-m}$, i.e., $\dot{\varepsilon}_c \propto t^{-m}$, where $0 < m < 1$.

In these formulae, J is the usual time-independent, elastic-plastic J parameter, not including creep strains.

No Plasticity

For secondary creep as t→0: $$C(t) \to \frac{K_{TOT}^2}{(n+1)\phi E} \cdot \frac{1}{t} = \frac{J}{(n+1)\phi} \cdot \frac{1}{t} \qquad (17)$$

For primary creep as t→0: $$C(t) \to \frac{(1-m)K_{TOT}^2}{(n+1)\phi E} \cdot \frac{1}{t} = \frac{(1-m)J}{(n+1)\phi} \cdot \frac{1}{t} \qquad (18)$$

In (17,18), K_{TOT} is the total LEFM SIF, i.e., the sum of the primary and secondary SIFs. ϕ is a parameter which tends to unity if the secondary stresses are small, and tends to $\frac{Z}{Z-1}$ if the secondary stresses are dominant, where Z is the elastic follow-up. Specifically,

$$\phi = \frac{Z}{\left(\frac{\sigma_{ref}^{PR}}{\sigma_{ref}}\right)^n + (Z-1)} \tag{18b}$$

Hence, C(t) diverges at t = 0. The strength of the divergence in C(t) as t→0 is ∝1/t for both secondary and primary creep in the case of no plasticity. C(t) is proportional to J, which is just the LEFM J in this case.

Small Scale Plasticity

As $t \to 0$:

$$C(t) \to \frac{\varepsilon_{ref}^{ep}\dot{\varepsilon}_{ref}^{c}(t)}{\varepsilon_{ref}^{p}\sigma_{ref}} K_{TOT}^2 \approx \frac{\dot{\varepsilon}_{ref}^{c}(t)}{\varepsilon_{ref}^{p}} J \tag{19}$$

The superscripts ^{e,p,c} refer to elastic, plastic and creep strains respectively. They are evaluated at the total (pseudo-)reference stress, σ_{ref}, which is based on taking account of both the primary and the secondary loads (more of this concept later). The reference strain rate, $\dot{\varepsilon}_{ref}^{c}$, is also calculated at this total (pseudo-)reference stress.

Hence, for secondary creep the value of C(t) is finite at t = 0 and independent of t to first order. However, for primary creep C(t) is singular at t = 0, being proportional to the primary strain rate, and hence proportional to t^{-m}. The singularity is therefore less strong than for the case with no plasticity.

This is the first instance of a general theme: plasticity reduces the magnitude of C(t).

The limit, (19), is valid when the creep strain is small compared with the elastic-plus-plastic strain, $\varepsilon_{ref}^{c} << \varepsilon_{ref}^{ep}$.

Widespread Plasticity

The algebraic expression is identical to (19),

As $t \to 0$:

$$C(t) \to \frac{\varepsilon_{ref}^{ep}\dot{\varepsilon}_{ref}^{c}(t)}{\varepsilon_{ref}^{p}\sigma_{ref}} K_{TOT}^2 \tag{20}$$

However, if the plastic strain is far larger than the elastic strain then this becomes,

As $t \to 0$:

$$C(t) \to \frac{\dot{\varepsilon}_{ref}^{c}}{\sigma_{ref}} K_{TOT}^2 \tag{21}$$

Curiously, in the limit of very large plastic strains, C(t) is related to the LEFM K but *not* to the PYFM J, which is the opposite of what one might have expected.

Equ.(21) looks similar to Equ.(16) for C*, but the latter depends upon the primary stresses alone, whereas the former includes the (relaxing) secondary stresses in three terms in (21).

The limits, (20,21), are valid when the creep strain is small compared with the elastic-plus-plastic strain, $\varepsilon_{ref}^{c} << \varepsilon_{ref}^{ep}$.

Hence, the singularity in C(t) of strength t^{-m} persists for primary creep, though if secondary creep prevailed even at the earliest times (unlikely) then C(t) would be finite at t = 0.

Qu.: C(t) is generally singular at t = 0. Does this lead to infinite crack growth?

No.

We will see in a later session that although the singular C(t) implies an infinite crack growth rate at t = 0, the integrated crack extension over a finite time period is finite and hence (potentially) sensible.

However, the singular 'spike' at t = 0 in C(t) does tend to lead to a rapid jump in crack size initially – the "C(t) spike effect". Is this very rapid initial growth physically real, i.e., does it actually happen? It is not certain, perhaps not.

But we will see that this spike in C(t) at early times is strongly ameliorated by plasticity. In many cases this avoids too unrealistic a result in assessments.

Also, growth due to creep does not start immediately but requires an incubation period during which creep damage accumulates around the crack tip. By the end of the incubation period the magnitude of C(t) will have reduced, perhaps quite dramatically, from its very early values.

Qu.: How does C(t) vary over the whole range of times?

An expression which gives a reasonable guide to the time variation of C(t) over the whole range of times is,

$$C(t) = \frac{\dot{\varepsilon}^c_{ref}}{\sigma_{ref}} K^2_{TOT} f(\tau), \text{ where, } f(\tau) = \frac{(1+\tau)^{1+n}}{\phi\left[(1+\tau)^{1+n} - 1\right] + \dfrac{\varepsilon^p_{ref}}{\varepsilon^{ep}_{ref}}}, \quad \text{and, } \tau = \frac{\varepsilon^c_{ref}}{\varepsilon^{ep}_{ref}} \qquad (22)$$

<u>Warning</u>: this expression is valid only when Equ.(1) in E/REP/BDBB/0059/GEN/04 *Rev.003 (May 2010) by Ainsworth, Dean & Budden is a reasonable approximation for the pseudo-reference stress.*

The parameter τ is a dimensionless time. The short time expressions of (17-21) are valid only for $\tau \ll 1$.

The time dependence of C(t) is seen to consist of three physically distinct processes,

[1] **Redistribution** which is given by $f(\tau)$ and is generally on the shortest timescale. This gives rise to the "C(t) spike" near t = 0. This "spike" is strongly ameliorated by plastic straining;

[2] **Relaxation** of the pseudo-reference stress, σ_{ref}, which reduces both the terms $\frac{\dot{\varepsilon}^c_{ref}}{\sigma_{ref}}$ and K^2_{TOT} in (22). Note that $\frac{\dot{\varepsilon}^c_{ref}}{\sigma_{ref}}$ reduces because it is proportional to σ^{n-1}_{ref}, and K^2_{TOT} reduces because its secondary stress contribution is relaxing. In the limit σ_{ref} may relax as far as the (true) primary reference stress.

[3] **Primary Creep** which is causing the strain rate $\dot{\varepsilon}^c_{ref}$ to reduce (even if the stress were constant).

Which of [2] or [3] occurs on the faster timescale varies from case to case.

The limiting behaviours of the function $f(\tau)$ are, No Plasticity: $\tau \to 0: \; f(\tau) \to \frac{1}{\phi(1+n)\tau}$ (singular) (23)

No Plasticity: $\tau \to \infty: \; f(\tau) \to \frac{1}{\phi}$ (24)

With Plasticity: $\tau \to 0: \; f(\tau) \to \frac{\varepsilon_{ep}}{\varepsilon_p} > 1$ (finite) (25)

With Plasticity: $\tau \to \infty: \; f(\tau) \to \frac{1}{\phi}$ (26)

The following graphs illustrate the behaviour in various cases.

Qu.: What is the “redistribution time”?

The redistribution time, t_{red}, is defined as the time when the creep strain equals the elastic strain, $\varepsilon^c_{ref} = \varepsilon^e_{ref}$. If the plastic strain is zero, the redistribution time is $\tau = 1$, otherwise the redistribution time has $\tau < 1$.

The terminology implies that redistribution should be complete, or nearly complete, at the redistribution time. Note, however, that relaxation (if any) will probably be on a much longer timescale

Figure 1 No Plasticity

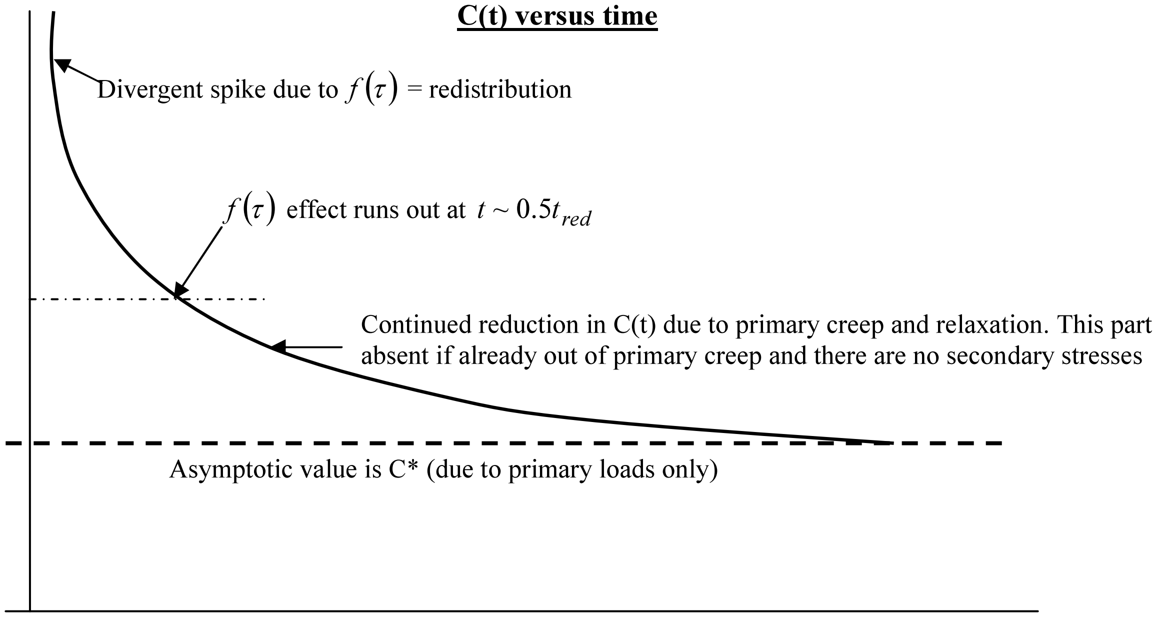


Figure 2 With Plasticity

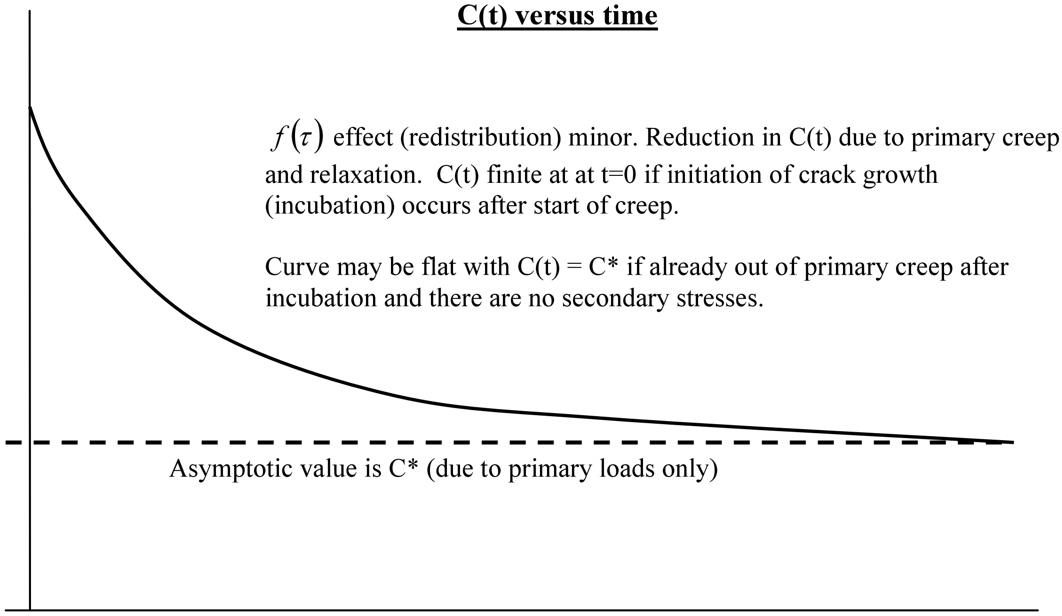
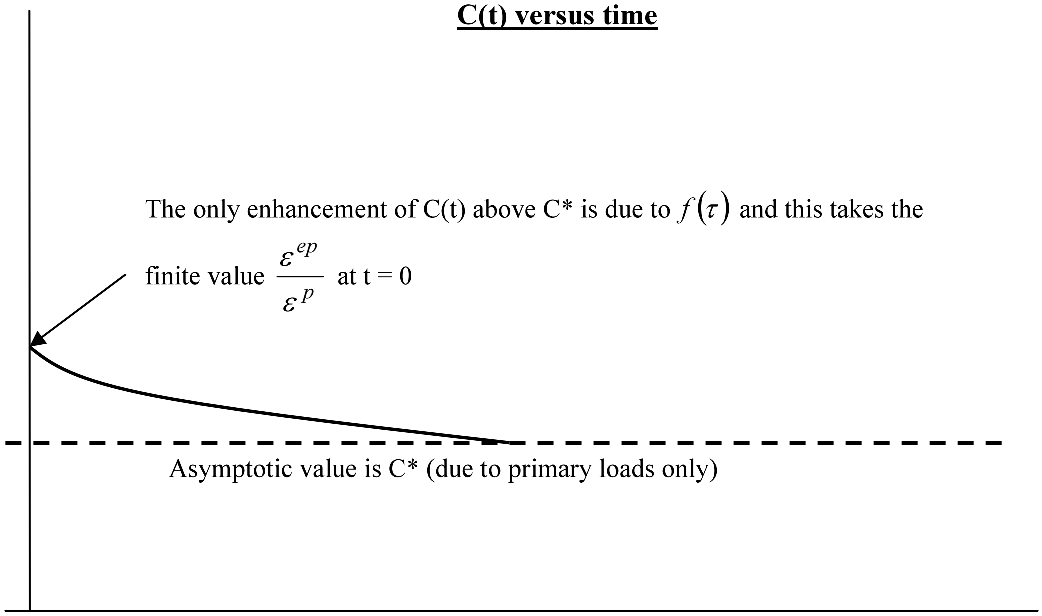


Figure 3 With Plasticity but No Primary Creep and No Secondary Stress



Hence we see that…

Plasticity is very beneficial in reducing the severity of the enhancement of C(t) above C* - so plastic strains are predicted to significantly ***reduce*** early creep crack growth rates

Qu.: Physically, why is plasticity beneficial in reducing C(t)?

Plastic strains reduce the early time C(t) due to reducing the $f(\tau)$ effect, which is due to redistribution. The reason is that the stress redistribution is already accomplished by plasticity and hence may be virtually complete before creep even starts.

There are subtleties here regarding the relative sizes of the plastic and creep stress indices which I have glossed over. If the plastic *n* is larger than the creep *n* then plasticity will cause full redistribution and eliminate the $f(\tau)$ effect on C(t) – see Figure 4(c) below. On the other hand, if the plastic *n* is smaller than the creep *n* then plasticity will not cause full redistribution and there will be a residual $f(\tau)$ effect on C(t) – see Figure 4(a) below. Hence C(t) reduces as the plastic index increases.

The graphs are based on Yuebao Lei's E/REP/BDBB/0083/GEN/05 results for an external circumferentially cracked cylinder:-

Fig.4(a) with *n(creep)* = 5 and *n(plastic)* = 3, residual stress only

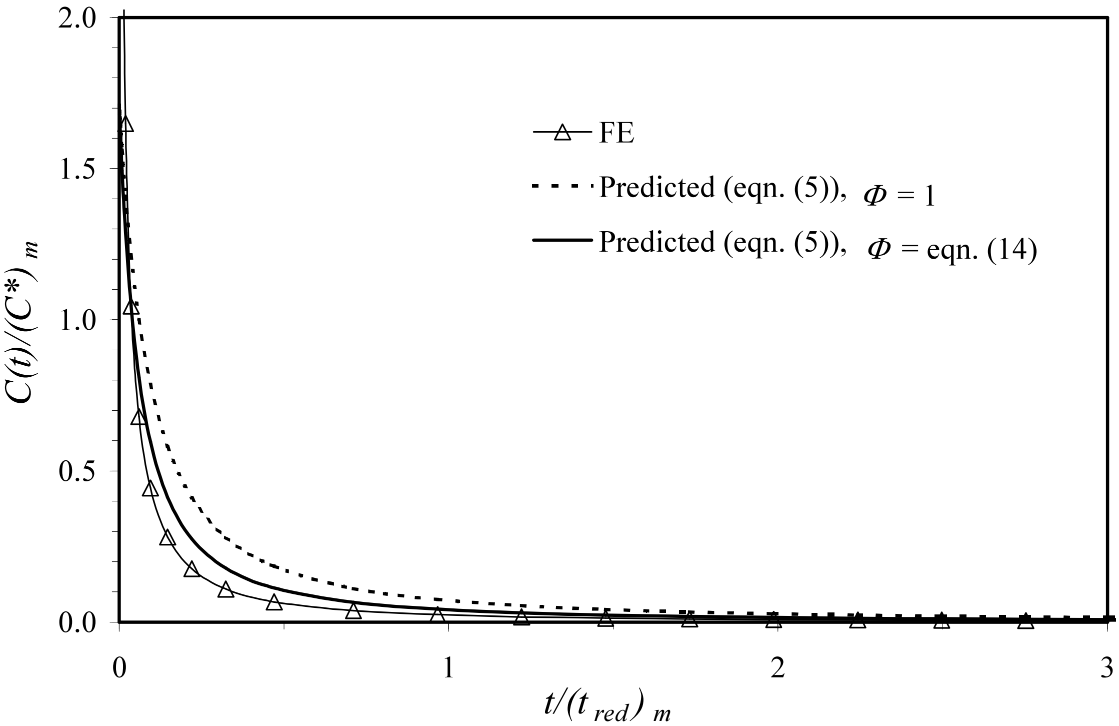


Fig.4(b) with *n(creep)* = 5 and *n(plastic)* = 5, residual stress only

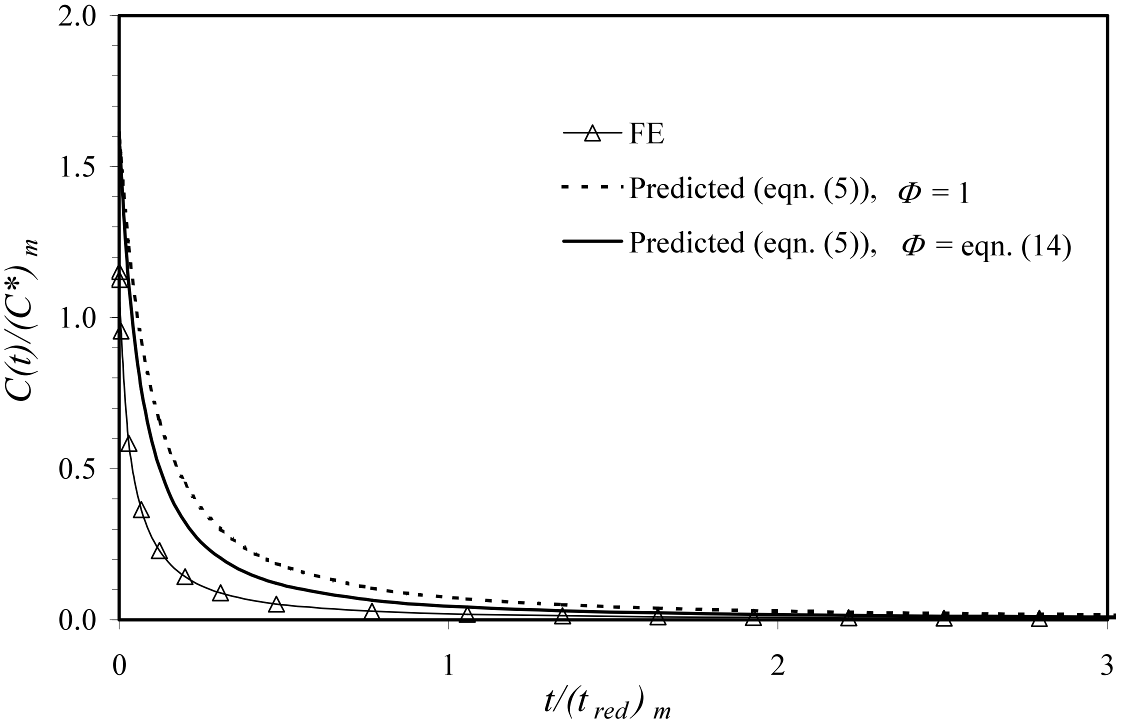
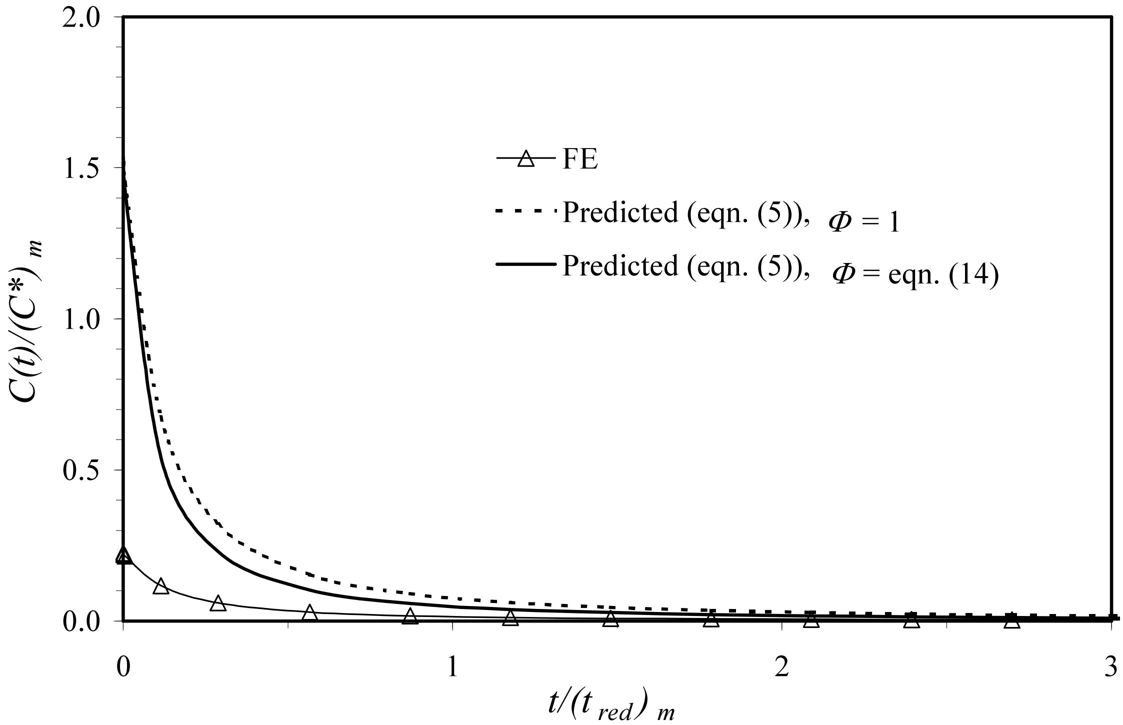


Fig.4(c) with *n(creep)* = 5 and *n(plastic)* = 10, residual stress only



This sequence of graphs shows that C(t)/C* reduces as the plastic index increases from being, (a) less than, (b)equal to, and , (c)greater than, the creep index

The formula for the Φ factor used by Yuebao was my Equ.(18b) above.