T73S02 Tutorial Session 21 – Quantifying Constraint Effects in Fracture

Last Update: 1/1/16

Beyond the dominant crack tip field: quantifying constraint; T and Q stresses; Examples of high and low constraint geometries/loadings; The local approach(es)

This is a brief & simplified description of constraint and local approach methodologies – but it is not a substitute for reading the relevant Chapters of R6

Qu.: What is constraint again?

Constraint is the degree of stress triaxiality.

Qu.: Why is constraint important?

Constraint is important in fracture because high constraint (plane strain) leads to all stress components near the crack tip being higher than for low constraint (plane stress) – except the equivalent stress which is smaller. This is true for both LEFM and PYFM (HRR fields). Consequently we would expect high constraint conditions to increase the likelihood of fracture – and indeed they do.

Qu.: Don't we always want high constraint, plane strain, conditions?

We usually want high constraint, plane strain, conditions when carrying out fracture toughness tests in order to generate valid toughness data. But in plant components the degree of constraint is….whatever it is! Applying valid toughness data, obtained from fully constrained specimens, to any plant component will therefore be either appropriate or conservative.

Of interest in this session are those cases where the use of valid, fully constrained, toughness data to the assessment of a plant component is overly conservative due to the low constraint of the component.

Qu.: So, can benefit be claimed in assessments when constraint is low?

Yes.

The relevant part of R6 is Chapter III.7.

But there is likely to be a benefit only when the potential fracture is brittle, or includes a brittle component (transition region). Fully ductile initiation toughness is less likely to be enhanced by low constraint, although the tearing modulus may get steeper.

But the local approach may provide alleviation in ductile assessments, as well as brittle (see later).

Qu.: Is it common to appeal to low constraint in plant assessments?

No.

Off hand I can't think of any real plant assessments within EDF Energy which have deployed constraint arguments. Does anyone know of any? (I believe appeal has been made to low constraint in gas & oil pipeline assessments in the USA & Canada).

Qu.: Why not?

R6 Chapter III.7 provides a *procedure*, but it does not provide the crucial parameters which you require in order to *apply* the procedure. These parameters (α, β, m) depend upon the geometry and loading and also upon the material properties. These parameters are defined below. To find these parameters will generally require finite element analysis (β) and also bespoke low-constraint toughness tests for the material of interest (α, m) – or very sophisticated damage models. This is a great deal of work – and would require a long lead time (years rather than months, in practice).

In addition, most of our assessments are initiation assessments in ductile materials. So the range of plant problems that could benefit from constraint assessments is limited.

We shall see that R6 Chapter III.9 (local approach) provides an alternative means of addressing constraint which also offers other alleviations, in both the brittle and ductile regimes. And I can think of at least one important plant application of the local approach (SZB reactor coolant pump bowl).

Qu.: How does the constraint procedure work?

The first step is to quantify the degree of constraint. As previously remarked, there is no unique measure of constraint applicable in all situations. Here we are concerned about constraint near the crack tip. So let's take a closer look at the crack tip fields – and for this we must distinguish between LEFM and PYFM (HRR) cases…

Qu.: How are the LEFM Crack Tip Fields Modified in Low Constraint?

Sufficiently close to a Mode I crack tip the elastic stresses are necessarily of the form,

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} g_{ij}(\theta) \tag{1}$$

where the angular functions, $g_{ij}(\theta)$, were given explicitly in Session 13. But (1) is really just the leading term in a power series[1] in $\sqrt{r}$. So the next term is a constant wrt r, the next proportional to $\sqrt{r}$, etc. Confining attention to Mode I it turns out that we can write the elastic crack tip stresses to second order as,

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} g_{ij}(\theta) + T\delta_{xi}\delta_{xj} \tag{2}$$

The Kronecker deltas mean that the new T-stress term acts only in the x-direction, i.e., parallel to the crack faces. Consequently it does not cause any crack opening directly, but it does affect the hydrostatic stress near the crack tip, i.e., the constraint.

Turning this around, the degree of constraint of the geometry/loading determines the size of the T-stress.

If $T > 0$ then the T-stress increases the hydrostatic stress, i.e., increases the constraint. High constraint items have $T > 0$.

If $T < 0$ then the T-stress decreases the hydrostatic stress, i.e., decreases the constraint. Low constraint items have $T < 0$.

Qu.: Ah! So now LEFM cracks are characterised by two quantities: K and T?

Yes.

T, like K, depends upon the geometry and the type of loading.

Whatever mechanisms are responsible for damage accumulation near the crack tip, leading ultimately to fracture, they will depend upon both K and T.

[1] A Laurent expansion, to be technically accurate.

So this means that the effective toughness, which is the value of K at fracture (or initiation of tearing), will be a function of T. So the effective toughness will depend upon constraint.

Elastic finite element analysis will give us T (and K too) for our plant component and any given lab specimen.

Qu.: Can we now carry out a constraint-based fracture assessment?

No.

We still need to know how a given T changes the effective toughness. This obviously depends upon the material, and hence requires experimental input and cannot merely be calculated[2].

Qu.: What else do we need?

Assuming that you are *not* lucky enough to have a mechanistic model which tells you how to calculate the relevant 'fracture damage', you will need to carry out fracture tests on your material using specimens with a similar degree of constraint to the plant component. This is the problematical bit which usually kills the use of the constraint procedure in practice.

Qu.: How does the LEFM constraint procedure then work?

The β parameter is just a dimensionless form of the T-stress, as follows,

$$\beta = \frac{T}{L_r \sigma_y} \tag{3}$$

Hence β does not depend upon the material (the yield strength cancels). Also, if there are only primary loads acting, β does not depend upon the *magnitude* of the load either. However, β will depend upon the *type* of loading, and upon the geometry (including the crack size).

R6 Chapter III.7 then tells you to use a modified toughness in your assessment. If the usual (unmodified) toughness is K_{mat}, the modified toughness, K_{mat}^c, is given by,

If $\beta L_r > 0$ (high constraint items): $\quad K_{mat}^c = K_{mat}$ (4a)

If $\beta L_r < 0$ (low constraint items): $\quad K_{mat}^c = K_{mat}\left[1 + \alpha(-\beta L_r)^m\right]$ (4b)

But this requires the parameters α and m to be known. To find these parameters requires toughness tests on your material using a range of geometries with differing levels of constraint (i.e., differing T).

Recall that for the initiation of tearing in ductile materials it is unlikely that (4b) will actually provide much benefit. However, the parameters α and m are tearing dependent and hence a benefit will often be obtained as regards the torn toughness.

The references in R6 Chapter III.7 may be able to advise on the T-stress, and hence β, for your structural geometry/loading. This might avoid having to do the elastic FEA.

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[2] Unless you are lucky enough to have a theory which tells you how to calculate the relevant 'fracture damage', e.g. some form of local approach, in terms of T and K.

However, the parameters α and m are material dependent, so specific fracture tests are generally unavoidable to find them.

Qu.: Isn't there an alternative constraint procedure in R6?

Instead of increasing the toughness you can use the usual toughness and factor the FAD upwards in the K_r direction by the factor $\left[1+\alpha(-\beta L_r)^m\right]$, provided $\beta L_r < 0$. But of course this amounts to the same thing. (Warning: I haven't examined these procedures in detail, so there may be some subtle difference between the two approaches).

Warning: Constraint does not only depend upon the Mode I loading.

Whilst K does not depend upon the stress components in the plane of the crack, the constraint does depend upon these components. Clearly this must be the case since the stress components in the plane of the crack influence the hydrostatic stress, and hence the T-stress in (2).

Qu.: When is the LEFM (T-Stress) Constraint Procedure Valid?

R6 is not definitive on this, but the broad guidance is that the LEFM (T-stress) constraint procedure is recommended for $L_r < 1$.

Qu.: What is the procedure when $L_r > 1$?

When $L_r > 1$ it is probably better to use the PYFM procedure based on including a second order term with the HRR fields.

Like the LEFM crack tip fields, the HRR fields are really just the leading term in a series expansion. The second order expression can be written, approximately,

$$\sigma_{ij} = \sigma_{ij}^{HRR} + \tilde{\sigma}_H \delta_{ij} \tag{5}$$

The first term is the usual HRR field which applies in small scale yielding. The second term is an additional hydrostatic stress (over and above the hydrostatic part of the HRR field). The dimensionless Q and β parameters are defined by,

$$Q = \frac{\tilde{\sigma}_H}{\sigma_y} \qquad \beta = \frac{Q}{L_r} = \frac{\tilde{\sigma}_H}{\sigma_{ref}} \tag{6}$$

In particular, low constraint is again characterised by $\beta < 0$, and high constraint by $\beta > 0$. Note that this is a different β from that applying in the LEFM case.

Q and hence β can be found from elastic-plastic FEA.

In the case of mixed primary plus secondary loading, if Q is known individually for the primary and secondary loads, R6 Chapter III.7 gives a prescription for finding the combined load Q.

The effective toughness is then modified using Equs.(4a,b) as before.

Qu.: Are there some examples of T-stress solutions for simple geometries?

Yes – one source is reference [III.7.14] in R6. Some results from this paper by Sherry, France & Goldthorpe are given in the Appendix at the end of these notes. The normalisation used by Sherry et al differs from that of R6, though the definition of the T-stress is the same. Hence their B is $T\sqrt{\pi a}/K = T/Y\sigma$, where σ is the applied remote stress, as opposed from the reference stress used by R6. Two graphs are given for each geometry, one for B and one for T/σ. The latter equals B times the compliance factor Y.

Note the following,

- The single edge cracked bend specimen and the CTS are both highly constrained (positive constraint parameter), and their constraint *increases* with crack depth.
- The centre cracked plate in tension (CCPT) is low constraint (negative constraint parameter), and the constraint *reduces* as the crack gets bigger.
- Applying an additional load parallel to the crack face increases the constraint of a CCPT. The constraint parameter is still negative, but smaller in magnitude. This is of intermediate constraint.
- A fully circumferential crack in a cylinder under axial tension is a low constraint geometry (negative constraint parameter comparable to that of a CCPT).
- Although not given in the Appendix, the double edge-cracked plate in tension is of intermediate constraint

Qu.: What are "local approaches"?

Local approaches are an alternative means of assessing fracture, i.e., alternative to assessment methodologies based on K or J. They are based on micromechanical models of damage near the crack tip.

Guidance of local approaches is given in R6 Chapter III.9

Qu.: What local approaches does R6 consider?

R6 discusses four models,

- Beremin cleavage;
- Gurson-Tvergaard-Needleman ductile;
- Rousselier ductile;
- Beremin ductile.

We shall only discuss the last of these here.

The models all assume that the failure mode does not change after tearing, e.g., an initially ductile tearing mechanism might end in a cleavage fracture (transition region). Such cases are not covered. So, to employ (say) the Beremin ductile model you must be confident that the material is fully ductile.

Brittle fracture can occur in an intergranular manner (e.g., for some irradiated CMn steels). I believe that none of the models are valid for this mechanism.

Qu.: What is the Beremin ductile model?

The Beremin ductile model is chosen for illustration because it is the simplest to describe and the simplest to implement. Also it has been used in a plant application. It is based on the Rice & Tracy model of micro-void growth. A damage parameter at some point within the plastic zone near the crack tip is defined as,

$$D_{rt} = 0.283 \int \exp\left\{ \frac{3\sigma_H}{2\bar{\sigma}} \right\} d\bar{\varepsilon}^p \qquad (7)$$

where $\sigma_H, \bar{\sigma}, \bar{\varepsilon}^p$ are the hydrostatic stress, the Mises stress and the Mises equivalent plastic strain respectively. The damage must be integrated from zero load to the load of interest.

The absolute magnitude of D_{rt} is of little importance (and so the factor of 0.283 in (7) is not crucial). Of greater importance is how D_{rt} differs between the plant component and a fracture specimen, at the same distance from the crack tip. If the curve of D_{rt} against r for the plant component at the assessed load lies below the curve derived for the toughness specimen at the lower bound initiation load, then the component will not initiate tearing (according to this model, at least).

Qu.: Does this mean that elastic-plastic FEA is required for both the plant component and the toughness test specimen?

Yes.

Qu.: Does the Beremin ductile model give a specific initiation criterion?

Yes.

A specific criterion for 'fracture' (initiation) can be based on the volume-average of the damage over some microstructurally relevant region. This region is related to the spacing of inclusions or nucleation sites and is typically 100μm-500μm. Hence, the initiation criterion is that,

$$\left\langle D_{rt} \right\rangle = \frac{1}{V_{nuc}} \int_{V_{nuc}} \left(\int \exp\left\{ \frac{3\sigma_H}{2\bar{\sigma}} \right\} d\bar{\varepsilon}^p \right) dV \qquad (8)$$

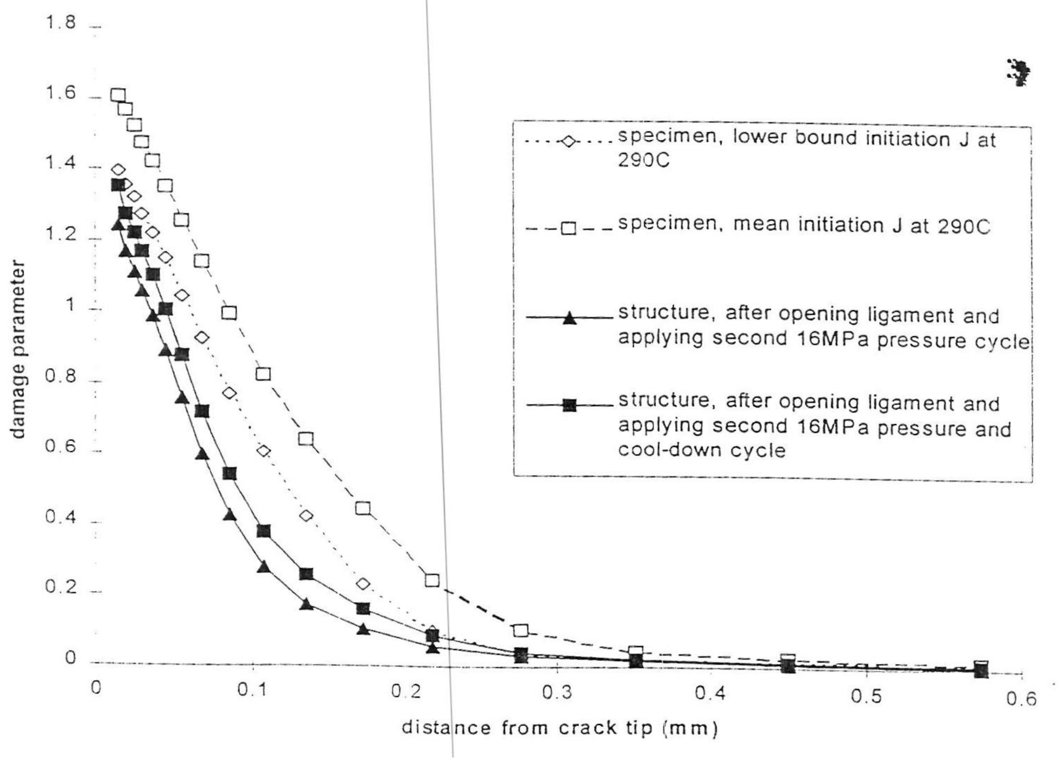
attains a 'critical' (initiation) value, which is found by evaluating the RHS of (8) for the specimen used in your fracture toughness test. Using the lower bound initiation load to determine $\left\langle D_{rt} \right\rangle_{init}$ is equivalent to using a lower bound toughness.

Qu.: How can the local approach be used to address constraint?

If you suspect that constraint is an issue for your structure, then carry out a toughness test on your material using a specimen and loading which has a similar level of constraint. This should be the specimen used in determining your initiation criterion, $\left\langle D_{rt} \right\rangle_{init}$, from (8). This will implicitly account for constraint.

Plant Example of Applying Beremin Ductile Model
SZB Reactor Coolant Pump Bowl
Paula Howarth, EPD/SXB/REP/0227/97

Variation of Damage Parameter for Embedded 10mm Deep Crack Suddenly Introduced into RCP Repair Weld Residual Stress Field, with Ligament Opened after First Plant Cool-down Cycle

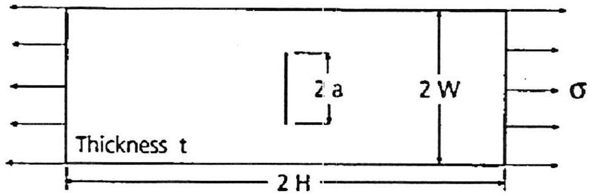


Appendix:

Example T-Stress Solutions from Sherry, France & Goldthorpe

Table 1. Table of polynomial coefficients for variation of the biaxiality parameter, B ($B0$ to $B4$) and T/σ ($T0$ to $T4$) with a/W for $0.1 \leqslant a/W \leqslant 0.6$ for a centre-cracked tension specimen (CCT) in the form:

$$Y = X0 + X1(a/W) + X2(a/W)^2 + X3(a/W)^3 + X4(a/W)^4$$



Ref.	B0	B1	B2	B3	B4	T0	T1	T2	T3	T4	Correlation
[15]	-1.004	0.248	-2.39	5.532	-4.069	-1.062	1.019	-6.493	12.129	-9.283	1.000
[20]	-0.991	0.163	-1.866	4.579	-3.542	-0.997	0.283	-3.268	6.622	-5.995	1.000
[19]	-1.044	0.085	-0.150	-	-	-1.174	0.860	-1.964	-	-	1.000

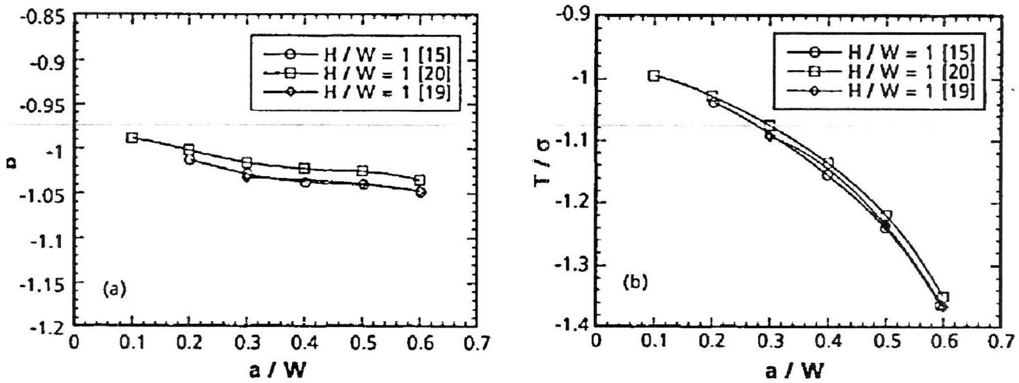
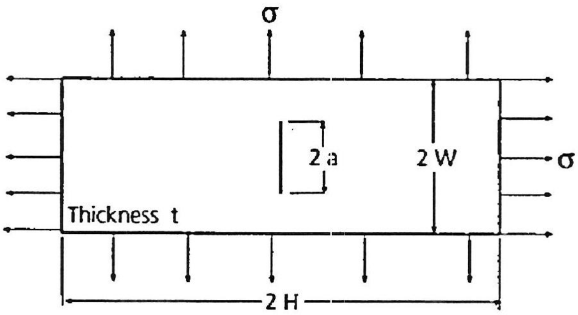


Fig. 1. Centre-cracked tension specimen (CCT): effect of a/W on (a) B and (b) T/σ.

Table 2. Table of polynomial coefficients for variation of the biaxiality parameter, B ($B0$ to $B4$) and T/σ ($T0$ to $T4$) with a/W for $0.1 \leqslant a/W \leqslant 0.6$ for a centre-cracked tension biaxial loading specimen (CCT) in the form:

$$Y = X0 + X1(a/W) + X2(a/W)^2 + X3(a/W)^3 + X4(a/W)^4$$



Ref.	B0	B1	B2	B3	B4	T0	T1	T2	T3	T4	Correlation
[20]	0.029	0.030	-2.870	4.829	-3.125	0.025	0.112	-3.354	5.917	-4.761	1.000

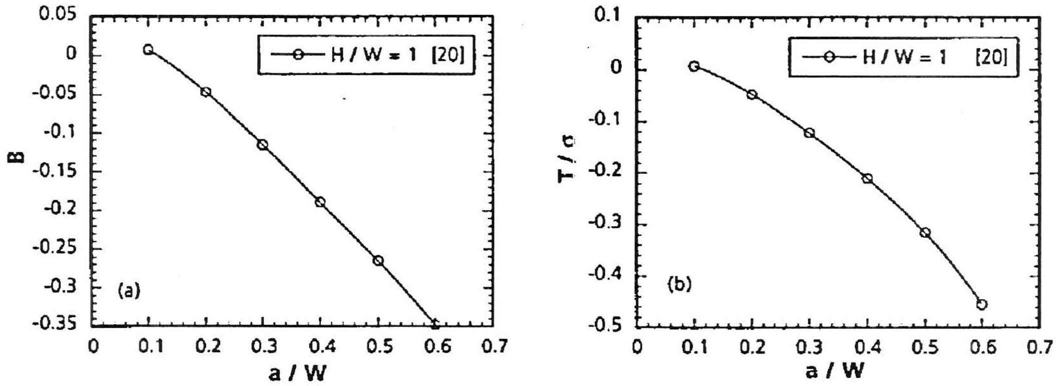
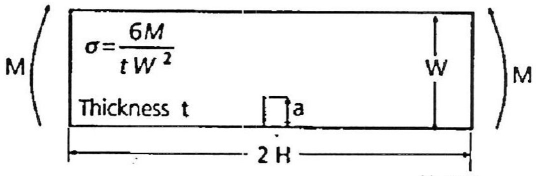


Fig. 2. Centre-cracked tension biaxial loading specimen (CCT): effect of a/W on (a) B and (b) T/σ.

Table 5. Table of polynomial coefficients for variation of the biaxiality parameter, B ($B0$ to $B4$) and T/σ ($T0$ to $T4$) with a/W for $0.1 \leqslant a/W \leqslant 0.8$ for a single edge cracked pure bending specimen (SEB) in the form:

$$Y = X0 + X1(a/W) + X2(a/W)^2 + X3(a/W)^3 + X4(a/W)^4 \ldots$$



Ref.	B0	B1	B2	B3	B4	T0	T1	T2	T3	T4	T5	Correlation
[21]	-0.437	0.339	4.818	-7.653	4.490	-1.036	12.063	-75.816	239.30	-343.07	188.65	1.000

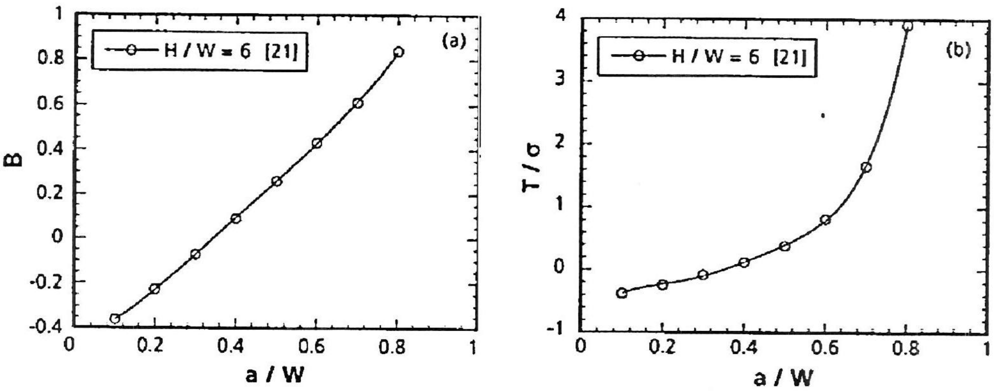
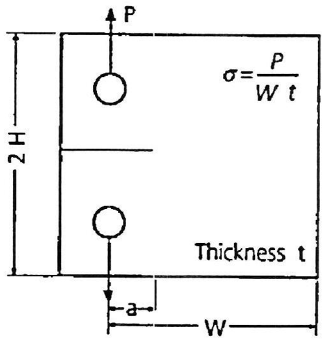


Fig. 5. Single edge cracked pure bending specimen (SEB): effect of a/W on (a) B and (b) T/σ.

Table 7. Table of polynomial coefficients for variation of the biaxiality parameter, B ($B0$ to $B4$) and T/σ ($T0$ to $T4$) with a/W for $0.2 \leqslant a/W \leqslant 0.7$ for a compact-tension specimen (CT) in the form: $Y = X0 + X1(a/W) + X2(a/W)^2 + X3(a/W)^3 + X4(a/W)^4$



Ref.	B0	B1	B2	B3	B4	Correlation	T0	T1	T2	T3	T4	Correlation
[15]	-0.513	1.708	13.404	-39.750	29.583	1.000	-1.996	10.169	10.546	-	-	0.999
[37]	-0.058	-0.276	12.790	-27.875	17.292	1.000	6.063	-78.987	380.46	-661.79	428.45	1.000
[20]	-0.353	-1.702	23.667	-47.33	28.333	1.000	-2.616	8.019	16.421	-	-	1.000

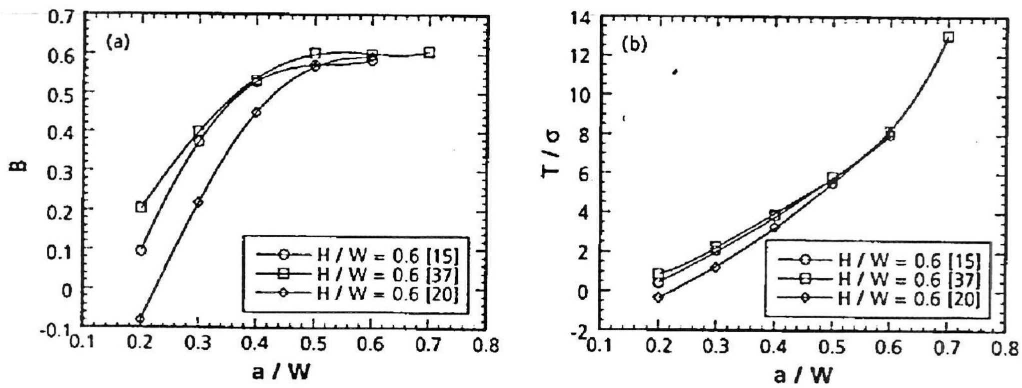
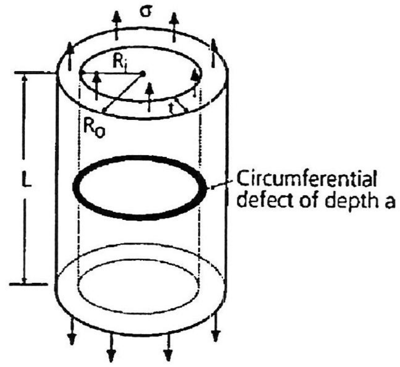


Fig. 7. Compact-tension specimen (CT): effect of a/W on (a) B and (b) T/σ.

Table 10. Table of polynomial coefficients for variation of the biaxiality parameter, B ($B0$ to $B4$) and T/σ ($T0$ to $T4$) with a/W for $0.1 \leqslant a/W \leqslant 0.8$ for an internal circumferentially cracked cylinder under tensile stress in the form: $Y = X0 + X1(a/W) + X2(a/W)^2 + X3(a/W)^3 + X4(a/W)^4 \ldots$



Ri/t	B0	B1	B2	B3	B4	B5	Corr.	T0	T1	T2	T3	T4	Corr.
5	-0.3253	-5.84E-01	1.62E+00	-5.50E-01	-1.81E+00	-	0.999	-0.4450	7.49E-01	-6.88E+00	1.68E+01	-1.50E+01	1.000
10	-0.3254	-4.10E-01	7.20E-01	1.74E+00	-3.20E+00	-	0.999	-0.4609	1.22E+00	-9.90E+00	2.33E+01	-1.89E+01	1.000
20	-0.1098	-4.28E+00	2.38E+01	-5.71E+01	6.48E+01	-28.814	0.994	-0.4726	1.58E+00	-1.25E+01	2.85E+01	-2.15E+01	0.999

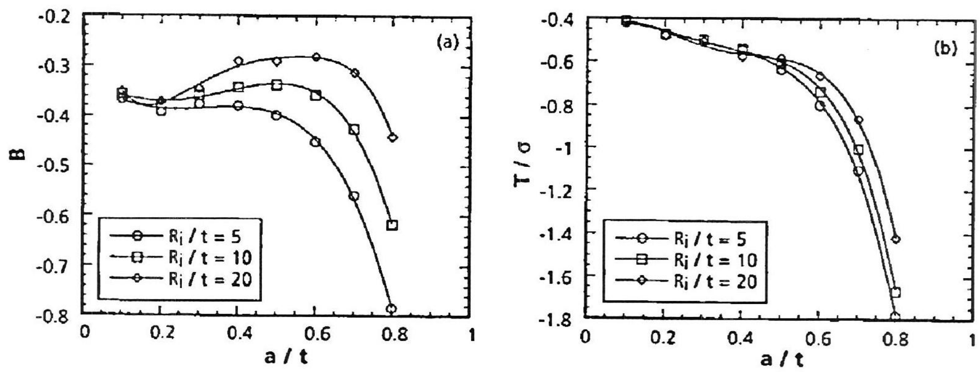


Fig. 10. Circumferentially cracked cylinder under tensile stress: effect of a/W on (a) B and (b) T/σ.