T73S02 Session 20

The Effects of Plastic Irreversibility on Low Temperature Fracture

Last update: 30/11/15

Contentious matters: What is the correct fracture parameter for thermal stresses, or residual stresses, or reversed plasticity or cyclic loading? Yuebao/JEDI v ABAQUS. What is stable tearing really? Does tearing really increase toughness, or does tearing decrease J? The plastic wake; Why is the lower shelf of toughness around 40 MPa√m or less? Warm pre-stressing.

The parts shaded yellow are beyond SQEP requirements

A Bestiary of Js

Qu.: How does the J-integral get modified for thermal stresses?

When a body has a temperature distribution which causes thermal expansions, part of the displacement of a given point is due to free expansion rather than related to stress. Following the derivation of the J-integral with this in mind provides a correction term,

$$J_1 = \int_\Gamma \left[W dy - \sigma_{jk} \hat{n}_k \cdot \frac{\partial u_j}{\partial x} ds \right] + \iint_A \sigma_{jk} \frac{\partial \varepsilon_{jk}^{th}}{\partial x} dA \qquad (1)$$

ε_{jk}^{th} are the free thermal strains ($\alpha \Delta T \delta_{jk}$ for an isotropic medium). The first integral in (1) is just the standard J-integral. The second integral in (1) is the correction term. It is an area integral over the region, A, within the contour Γ.

Like the standard J-integral, (1) is really only applicable for monotonic (proportional) loading.

ABAQUS will correctly evaluate (1), i.e., the ABAQUS J should be OK with thermal stresses.

Qu.: How does the J-integral get modified for body loads?

The derivation of the standard J-integral assumes no body forces (i.e., applied loads per unit volume, such as distributed weight or centrifugal loading). Following the derivation of the J-integral with this in mind provides another correction term.

ABAQUS uses a domain integral formulation. In this method a function of position, $q(\bar{r})$, is introduced which varies smoothly from 1 at the crack tip to 0 at the integration contour. This leads to an expression for J as an area integral only,

$$J_1 = \iint_A \left\{ \left[-W\delta_{1k} + \sigma_{jk} \frac{\partial u_j}{\partial x_1} \right] \frac{\partial q}{\partial x_k} + \left[\sigma_{jk} \frac{\partial \varepsilon_{jk}^{th}}{\partial x} - f_j \frac{\partial u_j}{\partial x_1} \right] q \right\} dA \qquad (2)$$

The first term is mathematically equivalent to the usual J-integral, but has been re-cast as an area integral rather than a contour integral. The second term is a correction factor for both thermal stresses and body forces. The latter are defined by $\sigma_{ij,j} + f_{,i} = 0$.

Like the standard J-integral, (2) is really only applicable for monotonic (proportional) loading.

ABAQUS will correctly evaluate (2), i.e., the ABAQUS J should be OK with both thermal stresses and body loads.

Qu.: What if tractions are applied directly to the crack faces?

In this case, none of the above expressions for J are correct. A further correction term is needed (see ABAQUS manual).

Qu.: What's the problem with using J when loading is not proportional?

The problem is that J has only been proved to be contour independent for non-linear elasticity – which is equivalent to plasticity only for proportional loading. And if J is contour dependent, then which contour should we use?

Qu.: What examples of non-proportional loading are there?

Non-proportional loading occurs in any case in which plasticity occurs and then the loading is reversed or a different type of load applied. A particularly important example is residual stress – because residual stress is, by definition, generated by plastic strains remaining on load removal.

Qu.: So how can J be redefined for non-proportional loading?

An approach that has been taken is to use the value of J on the limiting contour near the crack tip, that is on $\Gamma \to 0$. Evaluation over a finite domain can then be done by mathematically transforming the integral. This results in the domain integral,

$$J_1 = \iint_A \left\{ \left[-W\delta_{1k} + \sigma_{jk} \frac{\partial u_j}{\partial x_1} \right] \frac{\partial q}{\partial x_k} + \left[\sigma_{jk} \frac{\partial \varepsilon_{jk}}{\partial x} - \frac{\partial W}{\partial x} - f_j \frac{\partial u_j}{\partial x_1} \right] q \right\} dA \qquad (3)$$

Note that the strain appearing in the first term of the second [...] in (3) is the now the *total* strain (thermal plus elastic), not just the thermal strain. Hence, the second [...] is now a correction term for thermal stresses, body forces and non-proportional loading.

See Yuebao Lei's E/REP/ATEC/0063/GEN/02 for further details.

Qu.: So, that's pretty clear and simple then...

Not so fast. In E/REP/BDBB/0034/GEN/03 Yuebao Lei argues that, despite the generalised J defined by (3) being independent of the integration domain, it is not necessarily the quantity which controls the crack tip fields. This is possible because, with the introduction of non-proportional loading, the connection between the J integral and energy release rate has been lost.

In E/REP/BDBB/0034/GEN/03 Yuebao argues that it is better to use (3) with the proper elastic-plastic energy density, $W = \int \sigma_{ij} d\varepsilon_{ij}^m$, replaced by a pseudo linear value $W = \frac{1}{2}\sigma_{ij}\varepsilon_{ij}^m$. By FE evaluation for a simple geometry, Yeubao shows that (3) remains independent of the integration domain with this revised definition. Moreover, he also shows that it controls the crack tip stress and strain fields, even over many applied load-unload cycles.

The reason for the particular replacement $W = \frac{1}{2}\sigma_{ij}\varepsilon_{ij}^m$ escapes me. However, the problem which this obviates is that $W = \int \sigma_{ij} d\varepsilon_{ij}^m$ is monotonically increasing over successive cycles – despite the likelihood that the hysteresis cycles become stable.

Qu.: Can we use Yeubao's formulation of the J integral in FEA?

In principle, yes. Yeubao has a 2D/axisymmetric program (JMOD) to evaluate his modified J-integral for an ABAQUS model (apply to him if you want to use it). There is also a 3D equivalent, called JEDI. This is an MOD program, but we have access to it.

I believe Yeubao's 2D/axisymmetric program is robust – but consult him. However, my experience with JEDI is that it is very fragile. I would not recommend that it be used for safety critical applications in its present form.

Qu.: What else is there to evaluate J in the general case?

In extremis I have been driven to employ the CTOD from an FE model, and then use the HRR relation between CTOD and J to estimate the latter. However, this is a very bad method and can only give a crude order-of-magnitude estimate. (And strictly is available only for power law hardening).

The best way would be to use dP/dA. To do this you need to find P at a pair of slightly different crack lengths. The rest is simple – just find the difference in the two Ps and divide by the increase in crack area. QED. But it is necessary that your FE analyser gives you the total potential (or complementary) energy, P. Remember that $P = \sum_{\text{applied loads}} F_i D_i - U$, where U is the strain energy. I used to think that ABAQUS did not output P, but Chris Aird has put me right on this. It's worth spelling out how we think you can calculate dP/dA via ABAQUS…

Qu. How can dP/dA be calculated via ABAQUS?

This is by no means guaranteed, but what we think should work is…

[1] Start the analysis run with the body uncracked. In the same run apply a subsequent analysis step to release nodes to create the crack. It is essential that the whole crack is introduced in one step - not unzipped gradually (for reasons we will see shortly). Note the value of ETOTAL;

[2] Repeat [1], again starting from the uncracked body, but introducing a slightly longer crack in the subsequent step. Again note ETOTAL;

[3] The difference in ETOTAL between [2] and [1] is dP.

This should work with mixed loading types (load controlled and displacement controlled) and also with non-linear materials (plasticity), subject to the usual proviso that there be no unloading and no initial stresses/strains.

Stable Tearing

Qu.: How do we carry out a tearing assessment?

The methodology of an R6 tearing assessment is, roughly,

- Interpret the rising J_R curve against Δa as meaning that the toughness effectively increases with increasing tear length;
- Repeat the assessment with an incremented crack length, appropriately increasing Lr, increasing the SIF, and increasing the effective toughness.

Do not misunderstand the subsequent discussion…

The R6 tearing assessment methodology is perfectly fine

Qu.: So what's the concern?

My personal concern is that it does not make physical sense, despite being a perfectly good assessment methodology. Why? Well,…

Qu.: Does Stable Tearing Really Increase the Toughness?

No – not in my opinion.

How can it possibly do so? Why should the material grains ahead of the crack care whether the crack had recently extended or not?

The methodologies used for the engineering application of fracture mechanics can give a false impression regarding the nature of stable tearing. They encourage the belief that the material toughness really does increase in response to tearing. But why should the material immediately ahead of the crack tip (the process zone) care whether the material immediately behind the crack tip has recently torn, rather than having been cracked from the start? Why should the behaviour of the process zone material (specifically, whether it tears or not), depend upon the history of the material on the other side of the crack tip? But if its toughness does not *really* increase, why does its toughness *seem* to increase?

Qu.: What happens during stable tearing?

Whether or not the process zone material tears must be determined by the conditions within the process zone, together with the integrated history of the conditions within the same material element. Anything else is simply unscientific. In practice the "conditions" are highly inhomogeneous and anisotropic on the size scale of interest. In truth, these complications will generally play a crucial part in the fracture process. However, let us maintain the simplifying fiction that the behaviour can be understood in terms of continuum stresses and strains.

A material can tear and yet still be capable of sustaining a higher load than when the tearing first started. This experimental fact implies that the "conditions" immediately ahead of the new crack tip after tearing are *less* severe than was the case in the material which tore. If the J integral is indeed a reasonable measure of the proximity to tearing, then this implies that J evaluated on a path lying entirely within the process zone will have *reduced* after tearing. The reason is that,

Slow tearing of the crack reduces the severity of the strain singularity.

Qu.: How is this claim justified?

There are asymptotic solutions for the near-tip fields of a moving crack tip. The usual $1/r^{n+1}$ singularity in strain is replaced by a logarithmic singularity, which is far weaker. See, for example, Alan Zehnder's "Notes on Fracture Mechanics", January 2008, Cornell University. Such behaviour can also be observed in finite element models. Try it yourself – introduce a crack into a pre-loaded mesh one element at a time and discover that the elastic-plastic J is smaller than if you had introduced the crack in one go. (This result requires the J to be evaluated on a contour within the process zone, i.e., a small contour).

Qu.: And this means…what?

It means that stable tearing is not really due to toughness increasing, but actually due to the severity of the crack tip fields (and hence J) *reducing* due to tearing. From this perspective, it is not the toughness of the material which increases, but rather the near-tip J which reduces as a consequence of the amelioration of the strain singularity. That's my opinion, anyway. It may be contentious amongst experts.

Qu.: Are there any implications of this in practice?

Let me emphasise yet again – it is perfectly correct to apply the R6 methodology for stable tearing. However, there is one important implication if you use Option 3, i.e., if you evaluate J directly by FEA.

It is well known that very different values for J will result from elastic-plastic analyses which unzip a crack gradually compared with analyses which introduce a crack "suddenly", fully formed. Introducing the crack suddenly in one step will generally concur with expectations based on analytic methods, such as the reference stress approximation. This is what you should use for an R6 Option 3 assessment.

A slow unzipping of a crack will often produce values for J which are much smaller, especially when evaluated on contours close to the crack tip. My contention is that this phenomenon of reduced J near the tip of an extending crack provides the true mechanism which leads to stable tearing. However you should not use the reduced J derived in this manner for an assessment, unless part of a deliberately developmental approach with the involvement of an expert. And such a J should be assessed only against the *initiation* toughness - and despite this such an assessment will automatically include stable tearing. That's my opinion, anyway - but it may be contentious.

Qu.: What is the role of plasticity (irreversibility) in this phenomenon?

The phenomenon that J reduces due to tearing is possible only because plasticity is irreversible. It would not happen for a non-linear elastic material, even if its monotonic stress-strain curve were identical. Why? Because a non-linear elastic material has unique stress and strain fields for a given crack size. The stresses and strain would be identical however the crack was introduced. Hence we conclude that,

Stable tearing is intimately linked with plasticity, i.e., with non-conservative, irreversible, energy-absorbing behaviour

This is physically obvious, really, because tearing is clearly an irreversible phenomenon (hence increases entropy) - and hence requires an energy dissipation mechanism.

Qu.: Are the fields of HRR form after tearing?

No. As noted above the strain singularity is logarithmic and hence less severe than HRR form. But this gives us a theoretical problem…

Qu.: What happens to validity after tearing?

The approach to justifying the validity of measurements of initiation toughness is to demonstrate that the crack tip fields remain of HRR form at the initiation load. This provides a rational basis for the claim that the same crack tip fields would occur for the same J in a large engineering structure. However, once tearing has been permitted the reasoning becomes questionable. There is now no basis for the implicit claim that the same crack tip fields would occur for the same J_R (or the same tear length) in a large engineering structure. This theoretical underpinning has been challenged by the fact that J_R is empirically a global measure of toughness, and not related directly to the crack tip field after tearing. The validity of a torn toughness is therefore less secure than that of initiation toughness as regards its theoretical basis.

However, the likelihood is that the usual procedures do give a sound indication of the tearing behaviour of large structures, providing that the usual validity limits are respected. This has been validated experimentally by many large scale test programmes.

Qu.: What do the ONR think of stable tearing arguments?

The ONR (formerly the NII) have always been very resistant to the use of stable tearing arguments in AGR safety cases. However I believe they have been accepted in PWR safety cases for severe faults (LOCA). Whilst their scepticism has theoretical motivation, in practice their attitude is inconsistent.

Qu.: Why Is Stable Tearing Stable? – The Local Explanation

We have already answered this at one level – stable tearing is stable because tearing reduces the severity of the near crack tip fields.

But if the load is increased enough, tearing will become unstable. So why, and when, does tearing lead to less severe crack tip fields, and when do they become more severe and hence unstable?

Qu.: Why Is Stable Tearing Stable? – The Global Explanation

Irrespective of the details of what is happening within the process zone, there is a simple requirement which must be fulfilled if fast fracture is to occur: the energy released by an increment of crack advance must equal or exceed the energy absorbed by the structure. Conversely, tearing must be stable if the energy which would be absorbed by the structure in a notional crack advance exceeds the energy released in advancing the crack. In the case of yielding materials, the minimum energy absorbed by the structure is that required to push the plastic zone to the new crack tip position, leaving a plastic wake behind.

Consequently, quantifying the energy requirements of the plastic wake provides a lower bound to the energy absorption of the structure. (Other mechanisms may also absorb energy, such as the process of tearing itself and the creation of additional surface area). Hence, simple models based only on yield properties of the material can lead to sufficient conditions for crack stability.

Tearing must be stable if the energy absorbed by pushing the plastic zone ahead of the crack tip, and leaving behind a plastic wake, exceeds the energy released in advancing the crack.

The 'plastic wake' is illustrated, roughly, pictorially below,

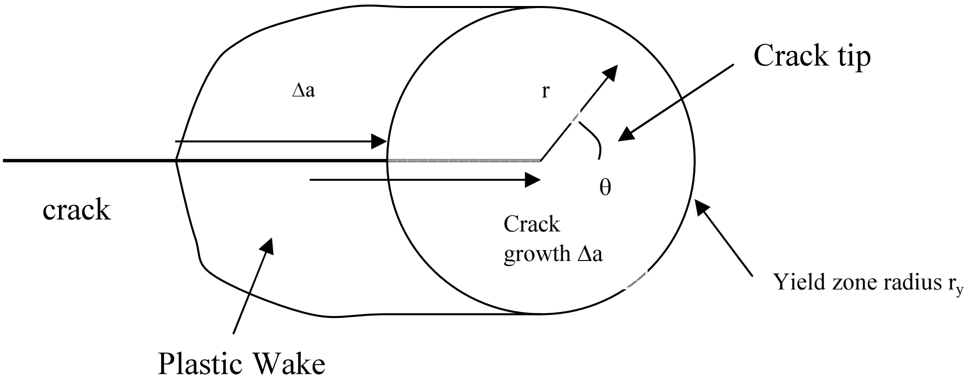


Figure 1

As the crack advances, the irreversibility of plastic deformation leaves large plastic strains in a 'wake' behind the 'active' crack tip field of the advancing crack.

Qu.: What is the algebraic expression for tearing stability?

When the crack advances by Δa the energy that is made available to drive the crack growth is $J\Delta a$. [For strain controlled loading, this energy originates from the strain energy of the body, which therefore decreases. In load control, the energy originates from the external agency that is maintaining the load constant. But the energy is $J\Delta a$ in either case].

Suppose the energy irreversibly absorbed by the plastic wake increases by $\Delta(W.E.)$ due to the crack advance of Δa. Then the tearing must be stable if $J\Delta a < \Delta(W.E.)$. This can be written,

$$\text{Tearing is stable if } \frac{1}{\mathrm{J}} \cdot \frac{\mathrm{d(W.E.)}}{\mathrm{da}} > 1$$

WARNING: This is my own theory and may not be generally accepted.

Qu.: Can we produce a simple model for the wake energy?

If we can make an estimate of the wake energy, the stability criterion $\frac{1}{\mathrm{J}} \cdot \frac{\mathrm{d(W.E.)}}{\mathrm{da}} > 1$ can be used to perform an assessment of tearing stability. I have produced a simple analytical model which gives the following result,

$$\frac{1}{J}\frac{d(W.E.)}{da} = \frac{2n}{(n+1)I_n} \cdot \log\left(\frac{r_y}{L_g}\right)$$

where n is the hardening index ($\varepsilon^p \propto \sigma^n$) and I_n is the parameter in the HRR field equations, r_y is the radius of the plastic zone and L_g is the grain size of the material. The plastic zone size is given by the HRR fields as,

$$r_y = \frac{J}{\varepsilon_0 \sigma_0 I_n}$$

The model is based on the assumption that the HRR fields provide a good enough approximation to the plastic energy density over the whole plastic zone. The accuracy of this approximation is unknown – and may be particularly questionable beyond general yield.

Qu.: What results do we get from this model?

The results for $\frac{1}{J}\frac{d(W.E.)}{da}$ are illustrated in the Table below for a material of strength $\sigma_0 = 200\text{MPa}$, $\varepsilon_0 = 0.125\%$ so that E = 160 GPa and assuming plane strain. Initiation toughness values between 10 and 120 MPa√m are considered, noting that J is given by $(1-\nu^2)K^2/E$.

For n = 5 these data give yield zone radii (r_y) of 0.45mm, 1.81, 7.3mm, 29mm and 65mm for the SIFs of 10, 20, 40, 80, 120 MPa√m respectively. The first three of these are likely to be contained yielding for typical specimen sizes, but not the larger two. However, we can equally imagine that we are dealing with a large structure.

For n = 3, 5, 7, 10 the HRR constant, I_n , is 5.51, 5.02, 4.77, 4.54 respectively.

Results are given for a grain size of 50 microns (or 200 microns, in brackets). Larger grained materials are predicted to have less propensity to tear stably.

n	K = 10	K = 20	K = 40	K = 80	K = 120
3	0.57 (0.20)*	0.95 (0.57)	**1.33** (0.95)	**1.71 (1.33)**	**1.93 (1.55)**
5	0.73 (0.27)	**1.19** (0.73)	**1.65 (1.19)**	**2.11 (1.65)**	**2.38 (1.92)**
7	0.83 (0.32)	**1.33** (0.83)	**1.84 (1.33)**	**2.35 (1.84)**	**2.65 (2.14)**

Bold black means $\frac{1}{J}.\frac{d(W.E.)}{da} > 1$, and hence tearing is predicted to be stable.

Red means $\frac{1}{J}.\frac{d(W.E.)}{da} < 1$, and hence tearing is predicted to be unstable.

Conclusion,

- For the assumed material strength and grain size, if the initiation toughness is greater than ~40MPa√m then tearing is likely to be stable.
- For the assumed material strength and grain size, if the initiation toughness is less than ~40MPa√m then tearing may be unstable, especially for the larger grain size.

Qu.: What does this imply for the lower shelf of fracture toughness?

The lower shelf of fracture toughness (for ferritic steels) is where cleavage dominates and no stable tearing is expected. For a steel with the strength and grain size illustrated above, we tentatively conclude that,

> The lower shelf of fracture toughness for ferritic steels is expected to be at or below ~40 MPa√m.

This is actually a good prediction – which might suggest my model is sound. But be warned – this model is unpublished and may be contentious.

Qu.: What does the model imply for a higher strength steel?

For a higher strength material with $\sigma_0 = 600MPa$, $\varepsilon_0 = 0.375\%$ (consistent with the same value of E, 160 GPa) we get,

n	K = 10	K = 20	K = 40	K = 80	K = 120
7	0.02 (0)	0.53 (0.02)	**1.04** (0.53)	**1.55 (1.04)**	**1.84 (1.34)**
10	0.05 (0)	0.60 (0.04)	**1.15** (0.60)	**1.71 (1.15)**	**2.03 (1.48)**

Attention has been confined here to larger n values since this is more likely for a high strength material.

Comparing with the results for $\sigma_0 = 200$MPa, this shows that $\frac{1}{J}.\frac{d(W.E.)}{da}$ reduces for the higher strength material, making stable tearing *less* likely and fast fracture more likely. This is what would be expected, since higher strength is correlated with reduced ductility. The onset of tearing stability is still around 40 MPa√m for the smaller grain size – but possibly as high as ~80 MPa√m for the larger grain size.

Warm Prestressing

Qu.: What is "warm prestressing" (WPS)?

Warm pre-stressing is the phenomenon whereby pre-stressing at an elevated temperature increases the fracture load at a lower temperature.

Qu.: To what materials and conditions does WPS apply?

WPS applies to ferritic steels with a ductile/brittle transition only.

Qu.: How is the WPS effect quantified?

R6 Chapter III.10 .4 provides a means of quantifying the benefit, subject to certain criteria.

Qu.: What are the criteria for the WPS advice in R6 to be valid?

The following are required,

- The failure mechanism at the reduced temperature must be brittle (either cleavage or brittle intergranular fracture);
- The flow strength should be greater at the reduced temperature (it generally is);
- Any sub-critical crack growth between the WPS condition and the reduced temperature fracture should be sufficiently small;
- The stress intensity factor during WPS exceeds the fracture toughness, K_{mat}, at the reduced temperature (where the latter includes the degrading effects of strain ageing, irradiation embrittlement, etc);
- Small scale yielding holds (yield zone < ligament size);
- The pre-stressing and the loading at the reduced temperature are in the same sense (generally both tensile).

In particular, note that compressive pre-loading may *reduce* the subsequent apparent toughness.

Qu. What's the simplest statement of the benefits of WPS?

If the above conditions are met, then,

Failure cannot be caused by reducing the temperature at constant stress

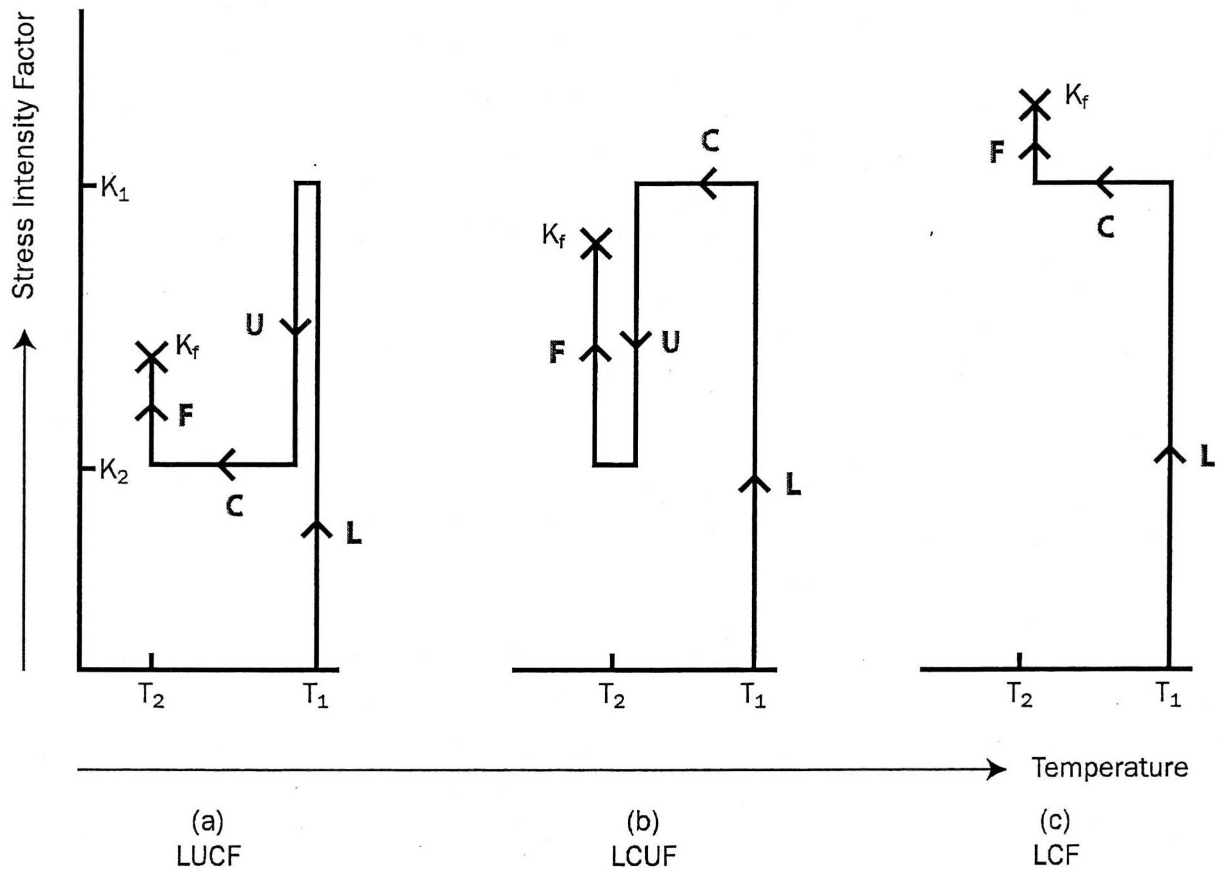
Qu.: But aren't there famous cases of fractures due to cooling?

Yes. During the fabrication of the thick ferritic pressure vessels for the early Magnox stations (e.g., Trawsfynydd) there were instances of brittle fracture occurring spontaneously when the vessel cooled following welding of the main seam butt welds. However, in these cases, the thermal (or residual) stresses were increasing due to the cool down as well as the material properties changing – so this does not contradict the above statement.

Qu.: What are the various possible WPS sequences?

They are as follows…

Figure 2 (From R6 Figure III.10.2)



L = Load

U = Unload

C = Cool

F = (Reload to) Fracture

Qu.: What is the R6 advice which quantifies the WPS benefit?

$K_1 = K_{WPS} =$ stress intensity applied during WPS;

$K_f =$ stress intensity at failure at the reduced temperature

$K_{mat} =$ fracture toughness at the reduced temperature, including the degrading effects of strain ageing, irradiation embrittlement, etc., if relevant;

$K_2 = K_{\min} =$ minimum stress intensity to which the structure is unloaded after WPS (noting that this may occur either at the WPS temperature OR at the reduced temperature);

The R6 formula for the effective toughness at the reduced temperature is,

$$K_f = K_{\min} + \sqrt{K_{mat}\left(K_{WPS} - K_{\min}\right)} + 0.15K_{mat}$$

Note that the preconditions include the requirement that $K_{WPS} > K_{mat}$.

Note that this simple model does not distinguish between LUCF and LCUF. In reality LUCF tends to be bounding, so the model is rather more conservative for the LCUF case.

Special cases:-

<u>LCF (Load-Cool-Fracture)</u>: In this case there is no unloading so that $K_{\min} = K_{WPS}$ and so $K_f = K_{WPS} + 0.15K_{mat}$, i.e., failure results only after the stress intensity is increased by an amount equal to 15% of K_{mat}. Note that the requirement that $K_{WPS} > K_{mat}$ means that the stress intensity at failure will exceed K_{mat} by more than 15%.

<u>Full Unloading</u>: In this case $K_{\min} = 0$ and $K_f = \sqrt{K_{mat}K_{WPS}} + 0.15K_{mat}$. Because $K_{WPS} > K_{mat}$ this means that failure results at a stress intensity which exceeds K_{mat} by more than 15%. However the failure SIF, K_f, is necessarily smaller than if there were no unloading.

<u>Partial Unloading</u>: It is an exercise for the reader to show that $K_{WPS} > K_{mat}$ implies that the failure SIF, $K_{\min} + \sqrt{K_{mat}\left(K_{WPS} - K_{\min}\right)} + 0.15K_{mat}$, is necessarily smaller than that for LCF, i.e., $K_{WPS} + 0.15K_{mat}$, for any $K_{\min}$ between 0 and K_{WPS}. So any degree of unloading reduces the failure SIF. Full unloading results in the smallest K_f.