SQEP Tutorial Session 17: T73S02

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The HRR fields; why stress and strain vary as $r^{-1/n+1}$ and $r^{-n/n+1}$; When are crack tip fields of HRR form; Loss of constraint (validity); Ductile fracture(tearing) criterion; The failure assessment diagram as a J based fracture criterion; R6 Options 1, 2 and 3; The J_R tearing resistance curve and how a tearing analysis is conducted; Tearing instability;

Qu.: For power law hardening, how do the crack tip stresses and strains depend upon the radial distance, r?

Recall the same question in LEFM, where it was found (after some ~~nasty~~ lovely analytic function arguments) that the stresses and strains both varied as $\propto 1/\sqrt{r}$. In power law hardening we have $\varepsilon \propto \sigma^n$, so it obviously cannot be true any more that both stress and strain vary as $\propto 1/\sqrt{r}$. But addressing the question now is easy, because we have the advantage of knowing that J is contour independent,

$$J = \int_{\Gamma} \left[W dy - \overline{T} \cdot \frac{\partial \overline{u}}{\partial x} ds \right] \tag{1}$$

where, $W = \int \sigma_{ij} d\varepsilon_{ij}$ and $\overline{T}$ is the traction acting over the boundary Γ. The traction is proportional to the stress, and the displacement gradient is proportional to strain. Consequently, both terms in (1) are proportional to the product of stress and strain. Integrating around a circle of radius *r* centred on the crack tip, both terms will also be proportional to *r* (because the integration variables, *dy* and *ds*, have dimensions of length). Hence, we have,

$$J \propto \sigma\varepsilon \cdot r = \text{independent of } r$$

So that $\sigma\varepsilon \propto 1/r$. But using also $\varepsilon \propto \sigma^n$ implies $\sigma^{n+1} \propto 1/r$, and hence,

$$\sigma \propto \frac{1}{r^{1/n+1}} \quad \text{and} \quad \varepsilon \propto \frac{1}{r^{n/n+1}} \tag{2}$$

For linear elastic behaviour we have $n = 1$ so (2) reduces to stress and strain both being proportional to $1/\sqrt{r}$ as it should.

For common structural steels, a typical value for *n* might be *n* ~ 5. Hence (2) shows that the singularity is much more severe in the strain than in the stress.

Perfect plasticity is the limit $n \to \infty$, for which the stress becomes a constant (within the region near the crack tip, that is – and hence within the plastic zone, so the equivalent stress is just the yield stress) and the strain takes up the full $1/r$ singularity.

Qu.: Are there "PYFM Crack Tip Fields" analogous to the LEFM Fields?

Yes and no. Under linear elastic conditions, the LEFM fields are completely general. They always apply sufficiently close to a sharp crack tip. There are no elastic-plastic fields of equivalent generality.

One reason is the irreversibility of plasticity – which means that (roughly speaking) for a given stress state the total plastic strain can be contrived to be almost anything. So clearly there is no unique plastic strain field.

But even if we consider only monotonically increasing load (so that irreversibility is irrelevant) then different structural geometries or different types of loading will often produce different crack tip fields once sufficiently large plastic strains have spread across the whole section. This is because the crack tip region becomes overwhelmed by the global structural response, i.e., the collapse mechanism. This is known as "loss of constraint" or "loss of validity".

However, there are a set of PYFM crack tip fields which apply in restricted circumstances, known as the HRR fields (for Hutchinson and Rice & Rosengren who developed the theory in 1968). They apply only for power-law hardening materials.

The HRR fields are the unique elastic-plastic crack tip fields which occur for power law hardening if the plastic zone is surrounded by an LEFM region.

Qu.: What is "Post-Yield Fracture Mechanics" (PYFM)?

PYFM is the fracture mechanics which applies when plasticity is significant and the conditions under which LEFM would be valid cease to hold. The logic of PYFM is underpinned by the existence of crack tip fields of HRR form (if power law hardening is assumed).

The definition of the HRR fields may seem to imply that PYFM is valid only when there is a region surrounding the HRR region within which the LEFM fields prevail. Actually this is not so. ..

The HRR fields may continue to apply near the crack tip even when there is no LEFM region – and perhaps even when plasticity has spread across the whole section.

Qu.: So, PYFM applies for all plastic states?

No.

PYFM is applicable only when HRR fields prevail near the crack tip. But, unlike the LEFM case, the crack tip fields cannot be assumed always to be of HRR form. There is no general guide, applicable to any geometry and loading , as to when HRR fields prevail. When sufficiently large plastic strains have built up across the whole section of the structure, the crack tip fields will deviate from HRR form.

Qu.: What is "validity"?

Just as LEFM is valid if there is a region surrounding the crack tip in which the LEFM fields prevail, so PYFM is said to be "valid" if there is a region surrounding the crack tip in which the HRR fields prevail (when power law hardening is assumed).

Conversely, if a structure, or test specimen, is subject to sufficiently large strains that the HRR fields no longer prevail, this is described as "loss of validity" or "loss of constraint".

Qu.: So what exactly are the HRR fields?

For a material with the power law stress-strain behaviour $\frac{\varepsilon}{\varepsilon_0} = \left(\frac{\sigma}{\sigma_0}\right)^n$ the HRR fields are given by,

$$\sigma_{ij}(r,\theta) = \sigma_0 \left[\frac{J}{\sigma_0 \varepsilon_0 I_n r}\right]^{1/n+1} \tilde{\sigma}_{ij}(\theta, n)$$

$$\varepsilon_{ij}(r,\theta) = \varepsilon_0 \left[\frac{J}{\sigma_0 \varepsilon_0 I_n r}\right]^{n/n+1} \tilde{\varepsilon}_{ij}(\theta, n)$$

$$u_i(r,\theta) = \varepsilon_0 r \left[\frac{J}{\sigma_0 \varepsilon_0 I_n r}\right]^{n/n+1} \tilde{u}_i(\theta, n)$$

In practice the angular functions $\tilde{\sigma}_{ij}(\theta,n), \tilde{\varepsilon}_{ij}(\theta,n)$ and $\tilde{u}_i(\theta,n)$ have to be found by numerical means. In the case $n = 1$ they reduce to the LEFM angular functions. The angular dependence varies with hardening index, n. However, since they are subject to the same boundary conditions, they have qualitatively similar forms. Tabulated values may be found in Fong Shih's 1983 paper (extracts of which for $n = 5$ are included at the end of these notes - you will need this for the homework).

The HRR fields depend upon the radial distance, r, only through the dimensionless ratio r/δ_0, where $\delta_0 = J/\sigma_0$. Moreover, their only dependence on the applied load is also via the ratio r/δ_0, since this is the only place in which J occurs.

Hence, conformance to HRR form can be tested by plotting stresses or strains against r/δ_0 for a range of increasing loads. All loads should produce the same unique curve, with stresses proportional to $\left(\frac{\sigma_0 r}{J}\right)^{-1/n+1}$ and strains proportional to $\left(\frac{\sigma_0 r}{J}\right)^{-n/n+1}$. See example plots at end from RAWB's 1984 paper on Mode II validity.

Qu.: What is "Crack Tip Opening Displacement (CTOD)"?

The conventional definition of CTOD uses lines drawn at 45° to the x-axis from the tip of the blunted crack. The points of intersection with the (deformed) crack faces define the position at which the CTOD is measured. The CTOD is roughly the same as twice the crack tip radius of curvature. The spatial scale $\delta_0 = J/\sigma_0$ is closely related to the CTOD. A common form of tabulation is in terms of a dimensionless D_n where,

$$\text{CTOD} = \varepsilon_0^{1/n} D_n \frac{J}{\sigma_0}$$

D_n is of order unity. For example, for n = 5 and $\varepsilon_0 = 0.1\%$, CTOD $\approx J/4\sigma_0$ (plane strain). More accurate values for D_n are tabulated below.

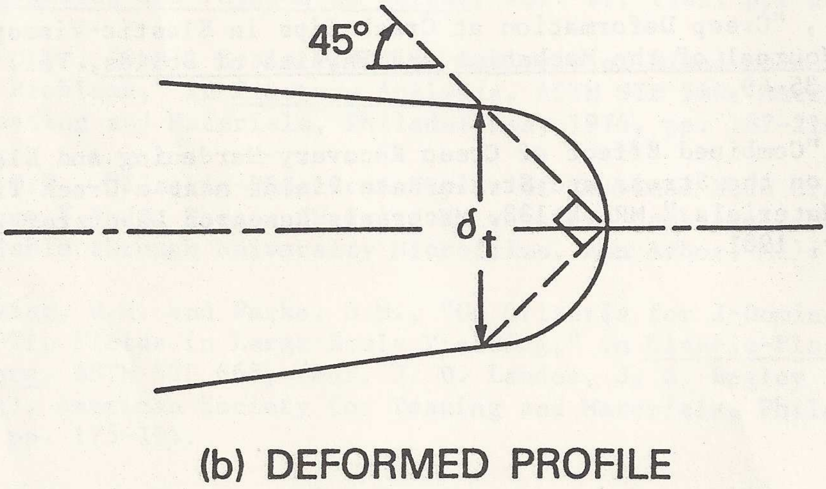


Figure 1 - Sharp crack and deformed profile illustrating the 45° procedure for defining crack-tip opening displacement.

Qu.: Are there examples of loss of validity?

Yes. The valid loading regime of test specimen geometries has been established by finite element modelling. The crack tip fields are found to be of HRR form below a certain load, but to depart from it at higher loads.

Qu.: How is the valid regime defined for test specimens?

A convenient way to express the valid loading regime is to use the quantity J/σ_y. This has the dimensions of length. Validity holds provided that the characteristic dimensions of the specimen, L (e.g, the uncracked ligament size or the thickness) are larger than some multiple of J/σ_y. In other words, valid toughness measurements result from the specimen if $L\sigma_y/J > N$, for some N characteristic of the specimen geometry and loading. The advantage of this formulation is that it is largely material independent, since the yield stress dependence is factored out. In principle, the numerical factor, *N*, might depend upon the plastic index (n), but in practice this dependence is slight.

The larger N, the bigger the test specimen must be to maintain validity. Hence, it is desirable to devise specimens with small N so that only small specimens are required. This requires less material, and also requires smaller testing machines (i.e., smaller applied loads).

For the most commonly used test geometry, the compact tension specimen (CTS), $N \approx 20$ (but note that some standards use the flow stress in place of the yield stress, i.e., the size requirement is $L\sigma_f/J > N$). For a centre-cracked plate (CCP) geometry, with applied tension, $N \approx 200$. The CTS is highly constrained, and this is reflected in a relatively small value for *N* (i.e., small specimens suffice). In contrast, the CCP is a low constraint geometry, and hence has a large *N*, i.e., a much larger specimen is required – which is why you rarely see a CCP geometry used, unless it's an experiment to prove this very point.

Size requirements of approximately these magnitudes may be found in a range of fracture toughness testing standards. It is important to recognise that these requirements originate primarily from finite element analyses and PYFM theory. The extent to which these size requirements have been confirmed as adequate by experiment would be worth exploring (off hand I don't know, though there were many comparisons of the results from different specimen geometries in the 70s and 80s).

Example: The requirements of BS7448: Part 4 (1997) Section 12

- CTS or a three-point bend specimen.
- J is not permitted to exceed $L\sigma_f / 20$, where the flow stress, σ_f, is the average of the 0.2% proof strength and the UTS, both at the test temperature, and L is the smaller of the out-of-plane specimen thickness and the uncracked ligament. To put that the other way around, the specimen dimensions, L, must exceed $20J / \sigma_f$.
- The $J < L\sigma_f / 20$ limits defines a maximum load, or equivalently a maximum amount of crack growth, which is valid in the test.
- There is also an explicit limit to the permitted crack extension, namely 10% of the uncracked ligament in the case of a J-test.

Qu.: But how do we know whether PYFM is valid for our plant component?

Good question! People rarely justify validity for the component being assessed. But actually this is OK. There are two situations:-

If the component is loaded in a similar manner to the test specimen used to obtain the toughness, but is just bigger, then validity of the component is ensured by the validity of the test specimen.

Alternatively, if the component is loaded in a different manner from the test specimen, or is smaller in some key dimension (e.g., thickness), then validity of the component cannot be assumed. However, loss of validity (low constraint) in the component will mean that its fracture load will be greater than it would be assuming full constraint prevailed. This is because high constraint promotes fracture. Complete loss of constraint will lead to the structure failing at the plastic collapse load, unaffected by fracture effects.

So, ignoring the issue of PYFM validity for the *structure* is conservative. If you wish to be less conservative, there are procedures within R6 for addressing less than full restraint – and the basis of these procedures will be discussed in a later session.

Qu.: What is the 'fracture' criterion in PYFM?

Fracture Criterion

$J = J_{init}$ or $J = J_{0.2}$

…but note that 'fracture' in PYFM may mean stable tearing rather than fast fracture.

Qu.: What does the subscript $_{0.2}$ mean?

The 0.2 refers to the amount of stable tearing, in mm, at which the toughness is measured.

More generally we refer (loosely) to the "initiation" value of J, where "initiation" refers to the initiation of stable tearing. Actually this is a misnomer because we never measure J at zero tear length. The various testing standards have differing definitions of the "initiation" toughness, but one is simply to adopt 0.2mm of tearing.

An alternative uses the blunting line, so that the initiation toughness is defined by the intersection of the given blunting line with the experimentally determined tearing resistance curve. So the amount of tearing at "initiation" varies (but is typically between ~0.2mm and ~0.5mm).

Qu.: This seems to imply that, when there is plasticity, we get stable tearing rather than catastrophic fracture?

Yes, it does. And this is often true providing that there is significant plasticity near the crack tip. The reason is essentially tautologous – if the material was brittle it would fracture before significant plasticity could occur. So, the occurrence of significant plasticity implies a ductile material. And ductile materials ***tend*** to exhibit stable tearing before fast fracture. But take care – ductile tearing is not always stable, and unstable tearing is functionally equivalent to brittle fracture. If you get hit by a flying fragment of exploding pressure vessel, you will not be worrying about whether the fracture surface displays ductile dimples.

Qu.: Why is tearing (sometimes) stable in ductile materials?

Tearing can be stable in ductile materials because the advance of the crack leaves behind it a 'wake' of plastically deformed material, as well as driving a new 'bow wave' of plasticity in front of it. So a substantial amount of energy is required to drive the crack forward. This energy is being supplied either by an external agency, or from the strain field of the body. We will show in a future session that the energy required to create a further increment of tearing can be larger than the energy which can be supplied – so that the increment of tearing cannot happen and the situation is stable.

Note that the energy permanently absorbed – and locked up in the plastic wake – is the central reason for tearing being stable. And note that this situation is possible only because of the irreversibility of plasticity. So it is really irreversibility - which is synonymous with the material absorbing energy - that permits tearing to be stable (a point rarely mentioned).

Qu.: Is tearing always stable?

No.

The above scenario does not apply for unlimited amounts of tearing. Eventually the tearing becomes unstable – and fast catastrophic failure ensues.

The limit of tearing stability results in just as nasty a failure as brittle fracture.

Qu.: When does tearing become unstable?

Eventually, because the crack length is increasing, the energy release rate (J) will get big enough to supply the energy requirements of both the plastic wake and the plastic bow wave….bang!

But there are other methods within R6 for assessing tearing and finding the limit of tearing stability (below).

Qu.: Have we now got three failure criteria: K_{mat}, G_{mat} and J_{init}?

No, they should all be equivalent. Recall that G and J are really the same quantity by another name, and by "K_{mat}" we mean $K_{mat} = \sqrt{\frac{EJ_{init}}{1-\nu^2}}$. (The latter assumes plane strain. This is implicit in the case of validity, i.e., full constraint). The conventional measure of toughness is K_{mat}, but often the value of J_{init} is quoted instead, the latter relationship being implicit.

Qu.: What is the R6 "Failure Assessment Diagram" (FAD)?

The failure assessment diagram is a means of implementing the 'failure' criterion $J = J_{init}$ as an engineering procedure. The FAD can be regarded as a graph which gives you an estimate of J for a given load.

Qu.: How is the FAD constructed?

Suppose we have some means of calculating J for any given load, P, on our structure, e.g., by FEA. We can express the result as a graph of J versus P. We can then read off the load at which tearing initiates, P_{init}, as the load when $J = J_{init}$. This is essentially an R6 Option 3 analysis.

But we may also be interested to know at what elastically calculated value of K our structure 'fails'. This is given by $K = \sqrt{EJ_{el}/(1-\nu^2)}$, where J_{el} is the elastic value of J at load P. And it may be convenient to normalise this elastic SIF by the toughness, K_{mat}, to express it as $K_r = K / K_{mat}$. But $K_{mat} = \sqrt{\frac{EJ_{init}}{1-\nu^2}}$, so we have $K_r = \sqrt{\frac{J_{el}}{J_{init}}}$.

But, because we are referring to the load at which tearing initiates, we have $J = J_{init}$, so we can write $K_r = \sqrt{\frac{J_{el}}{J}}$, where J is the elastic-plastic value of J at the same load, P_{init}, at which J_{el} is evaluated.

We can now place one point on a graph of $K_r = \sqrt{\frac{J_{el}}{J}}$ versus P_{init}. But we can notionally consider the material to have any toughness we like. So, as long as J and J_{el} are evaluated at the same load, P, we can generate a locus of K_r versus P, all points of which correspond to the initiation of tearing – but for different toughnesses.

This may seem an unduly laborious description of how an FAD is derived. However, note that the derivation of the FAD is a purely analytical affair, i.e., it can be done using FEA without reference to any level of toughness. This can seem paradoxical because, in applications, K_r is interpreted as $K_r = K / K_{mat}$, and hence requires the toughness. The reason that this is possible lies in the assumed ‘failure’ criterion, i.e., that $J = J_{init}$.

The final step in the construction of an FAD is to normalise the load axis by the load required to produce generally yield – that is the “collapse” load, $P_{0.2}$, referenced to a perfectly plastic material with yield strength equal to the material’s 0.2% proof strength. Defining $L_r = P / P_{0.2}$, we thus arrive at a plot of K_r versus L_r which is the FAD.

Qu.: What is the FAD good for?

By construction, the FAD corresponds to the initiation of stable tearing (or fast fracture if the material is brittle or tearing instability is reached). Points below the FAD are therefore safe.

If we know the shape of the FAD, then the only ingredients we need to carry out an assessment are,

- The elastic SIF;
- The collapse solution;
- The toughness and 0.2% proof strength (and perhaps the UTS).

So…

An R6 FAD is a means of applying the J-based ‘failure’ criterion $J = J_{init}$ without having to explicitly calculate *J*. The FAD itself is implicitly a *J* estimation procedure based on the above ingredients.

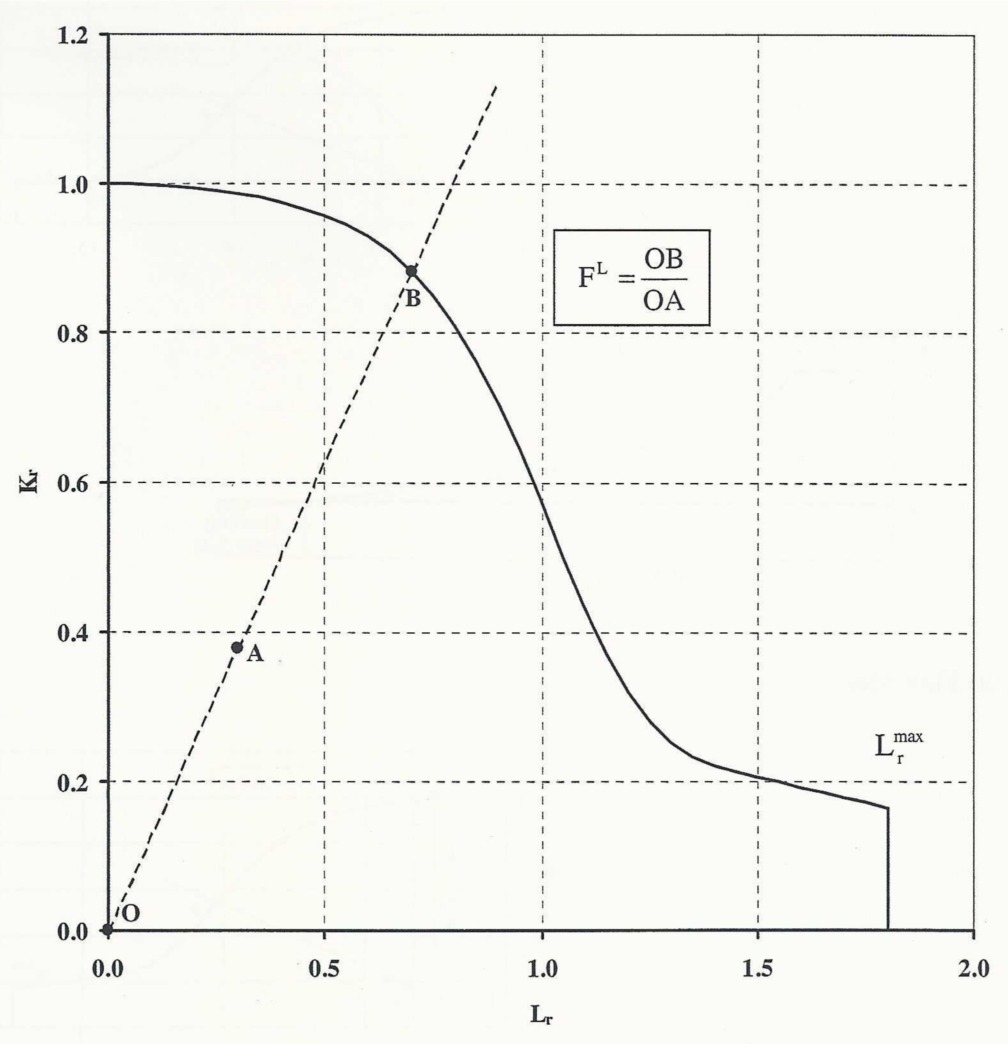
So the benefit is that we do not need an explicit elastic-plastic evaluation of J.

Qu.: What is the R6 Option 1 FAD?

The utility of the R6 approach lies in the fact that the FAD is not greatly sensitive to loading, geometry or material. Consequently it is possible to define a universal FAD – referred to as Option 1 - which can conservatively be applied in any situation. This universal FAD was derived originally by plotting the FAD for a wide range of geometries, loadings and materials, and adopting a curve which is towards the lower bound of the spread of results.

It is important to realise that the use of the universal (Option 1) FAD is equivalent to the assumption that all materials have the same stress-strain curve when normalised by their 0.2% values. The variation in such normalised stress-strain curves between materials gives a ready indication of the degree of approximation inherent in the use of the Option 1 FAD.

R6 Option 1 FAD



Qu.: Why is the 0.2% proof strength used to define L_r in R6?

It is almost true to say that this is just convention – but not quite. The proof strength chosen to define L_r has a bearing on how closely the Option 1 FAD represents all materials. For most structural steels the use of the 0.2% proof strength has been found to provide the least variation in the FAD between materials.

However, it is worth noting that some classes of materials can be better represented by other normalising strengths. For example, CMn steels display less scatter in the position of the FAD if the flow stress is used to define the normalised load axis. In earlier revisions of R6 this was recognised in a CMn specific FAD which used $S_r = P / P_f$ to define the x-axis, rather than L_r, where P_f was defined with respect to the flow stress $\sigma_f = (\sigma_{0.2} + UTS)/2$. Of course, this FAD has a different shape from that based on L_r. These days, if you think your material (e.g., a carbon steel) has a stress-strain curve which will cause it to deviate significantly from the Option 1 FAD then the recommendation is to use an Option 2 FAD…

Qu.: What is an R6 Option 2 FAD?

There is a method for estimating J based on the reference stress and the stress-strain curve. This is discussed in a later session. It permits an FAD to be derived, as described above, based on the specific material stress-strain curve. This is R6 Option 2.

There are a couple of variants of Option 2 which you can use if you do not have the full stress-strain curve, but the idea is the same.

Qu. …and Option 3?

Option 3 just means evaluation of J, almost certainly by finite element analysis. You may then wish to plot the results in the form of a geometry, load and material specific FAD – but you really don't need to in most cases.

Qu.: How is a stable tearing assessment carried out?

In a stable tearing assessment, the length of tearing is allowed to proceed beyond the nominal "initiation" value (probably 0.2mm). This requires some means of quantifying how the structure resists unstable crack extension. The standard approach, as given in R6, is described below. But note that although this is a perfectly good assessment procedure, it rather misleads as regards what is going on physically. A physically more reasonable description of stable tearing is the subject of a later session.

The standard approach uses the concept of a J-resistance curve. This is the idea that the effective toughness increases as the tear length increases. Thus the effective toughness might be described by (say) $J_{\Delta a} = J_{0.2} + \mu(\Delta a - 0.2)$, where Δa is the tear length in mm, and μ is a material constant, the tearing modulus, which measures how rapidly the toughness increases with tear length.

A stable tearing assessment can only be carried out if initiation is predicted under the assessed loading condition. Providing this is the case, a stable tearing assessment essentially consists of attempting to find a tear length, Δa, which is a solution to,

$$J(P, a + \Delta a) = J_{\Delta a} \tag{3}$$

The LHS of Equ.(3) is the value of J under the required loading condition (P) and with the crack size (a) increased by the tear length. The RHS of Equ.(3) is the value of the enhanced toughness at the same tear length.

If a solution to Equ.(3) for Δa exists, then this is the tear length which will arise under this loading, and the torn crack is stable.

If a solution to Equ.(3) for Δa does not exist, then the applied load results in tearing instability and catastrophic fast fracture.

Although this is mathematically correct, this is not the way in which a stable tearing assessment is generally carried out. There is another way using the FAD (below).

Qu.: How much tearing can be claimed?

In practice, it would not be valid to claim a very large tear length, even if this obeyed Equ.(3). This is because the validity limits of the toughness tests from which the J-resistance curve was derived would be violated beyond a certain amount of tearing $(\Delta a_{\max})$. Commonly the maximum valid tear length is around 0.8mm to 1.4mm for data obtained on standard specimens and for materials we commonly assess. The testing standards impose an upper limit on the valid tear length, typically 10% of the remaining ligament. However, the value of $\Delta a_{\max}$ is most often defined by the associated value of J after tearing reaches the valid limit, e.g., $J_{\max} = L\sigma_f / N$.

If the tearing analysis implies that the assessed load would require a tear length in excess of $\Delta a_{\max}$, then the assessment would not be able to validly confirm stability, i.e., the structure would fail the assessment.

Qu.: How is a tearing analysis done in R6 Options 1 or 2?

The above description of a tearing analysis requires an evaluation of J, i.e., an Option 3 assessment. In Options 1 or 2, where the FAD is being used in lieu of an explicit evaluation of J, there is a useful graphical construction.

Firstly the assessment point is plotted for the full load, and assuming initiation toughness. This point lies outside the FAD if tearing is to occur. The assessment is then repeated for a small tear length, increasing the crack length and increasing the toughness accordingly in the calculation of both K_r and L_r. This will lead to a new assessment point which is below and to the right of the initial point.

This process is repeated for a number of small increments of tearing, each producing an assessment point which is below and to the right of the last. If the resulting locus of points enters the FAD before a tear length of $\Delta a_{\max}$ is required, then a stable tearing assessment has been achieved. The actual predicted tear length is just that when the assessment locus reaches the FAD.

If a tear length of $\Delta a_{\max}$ is not sufficient to reach the FAD, then a valid stable tearing case cannot be made, and the structure fails the assessment.

If the locus of assessment points is veering to the right sufficiently that it is clearly turning away from the FAD, so that no amount of tearing would ever result in it reaching the FAD, then tearing is unstable. In this case, the structure does not merely fail the assessment, a definite prediction of failure is made (subject to the usual conservatisms).

The largest load for a given crack (or the largest crack for a given load) which just results in tearing stability can be found by the above construction when the locus of assessment points is tangential to the FAD. However this is valid only if the point of tangency in achieved with no more than a length $\Delta a_{\max}$ of tearing. Otherwise the limiting stable condition is defined by a tear length of $\Delta a_{\max}$ just reaching the FAD.

Qu.: How can an R6 assessment be used to find a reserve factor on load?

Illustrate by example…graphical construction useful when there are primary loads only, and no stable tearing. Otherwise, iterate!

Tabulated HRR Parameters (after Shih, 1983)

Dimensionless Constants Associated with HRR Singularity

$$I_n = \int_{-\pi}^{\pi} \left\{ \left(\frac{n}{n+1}\right) \tilde{\sigma}_e^{n+1} \cos\theta - \left[\left(\tilde{\sigma}_r(\tilde{u}_\theta - \dot{\tilde{u}}_r) - \tilde{\sigma}_{r\theta}(\tilde{u}_r + \dot{\tilde{u}}_\theta)\right)\sin\theta + \left(\frac{1}{n+1}\right)\left(\tilde{\sigma}_r \tilde{u}_r + \tilde{\sigma}_{r\theta}\tilde{u}_\theta\right)\cos\theta \right]^{*} \right\} d\theta$$

$$S_n = \frac{n}{n+1} \int_{-\pi}^{\pi} \tilde{\sigma}_e^{n+1} \, d\theta$$

$$D_n = (\tilde{u}_x + \tilde{u}_y)^{\frac{1}{n}} \, \frac{2\tilde{u}_y}{I_n} \; {}^{\dagger}$$

* $(\ )^{\cdot}$ denotes differentiation with respect to θ.

† $\tilde{u}_x$ and $\tilde{u}_y$ are evaluated at $\theta = \pi$.

	PLANE STRAIN			PLANE STRESS		
n	I_n	D_n	S_n	I_n	D_n	S_n
2	5.94	1.72	2.01	4.22	2.62	2.55
3	5.51	1.33	2.15	3.86	2.04	2.65
4	5.22	1.17	2.20	3.59	1.78	2.67
5	5.02	1.08	2.21	3.41	1.64	2.65
6	4.88	1.03	2.21	3.27	1.55	2.63
7	4.77	1.00	2.20	3.17	1.48	2.61
8	4.68	0.97	2.19	3.09	1.44	2.59
9	4.60	0.95	2.18	3.03	1.40	2.58
10	4.54	0.93	2.16	2.98	1.37	2.56
12	4.44	0.91	2.14	2.90	1.32	2.53
15	4.33	0.88	2.11	2.82	1.28	2.50
20	4.21	0.86	2.06	2.74	1.23	2.45
30	4.08	0.84	2.00	2.66	1.18	2.40
50	3.95	0.82	1.94	2.59	1.14	2.35
100	3.84	0.80	1.88	2.54	1.10	2.31
∞	3.72	0.79	1.81	2.49	1.07	2.26

Values for $n=\infty$ are obtained by extrapolation with respect to $1/n$

There follows the stress and strain angular functions for the case n = 5

PLANE STRAIN MODE 1 $n = 5$

θ^{o}	$\tilde{\sigma}_{e}$	$\tilde{\sigma}_{rr}$	$\tilde{\sigma}_{\theta\theta}$	$\tilde{\sigma}_{r\theta}$	$\tilde{\varepsilon}_{rr}$	$\tilde{\varepsilon}_{\theta\theta}$	$\tilde{\varepsilon}_{r\theta}$	$\tilde{u}_{r}$	$\tilde{u}_{\theta}$
0	0.4621	1.6836	2.2172	0.0000	-0.0183	0.0183	0.0000	-0.1095	0.0000
2	0.4625	1.6852	2.2163	0.0284	-0.0182	0.0182	0.0019	-0.1094	0.0045
4	0.4638	1.6901	2.2135	0.0566	-0.0182	0.0182	0.0039	-0.1090	0.0089
6	0.4659	1.6983	2.2090	0.0845	-0.0181	0.0181	0.0060	-0.1083	0.0133
8	0.4690	1.7097	2.2027	0.1120	-0.0179	0.0179	0.0081	-0.1074	0.0177
10	0.4732	1.7241	2.1947	0.1388	-0.0177	0.0177	0.0104	-0.1061	0.0221
12	0.4785	1.7416	2.1850	0.1649	-0.0174	0.0174	0.0130	-0.1046	0.0264
14	0.4853	1.7618	2.1736	0.1900	-0.0171	0.0171	0.0158	-0.1028	0.0306
16	0.4937	1.7843	2.1607	0.2141	-0.0168	0.0168	0.0191	-0.1006	0.0347
18	0.5040	1.8085	2.1463	0.2370	-0.0163	0.0163	0.0229	-0.0981	0.0388
20	0.5166	1.8335	2.1304	0.2587	-0.0159	0.0159	0.0276	-0.0951	0.0427
22	0.5315	1.8578	2.1132	0.2790	-0.0153	0.0153	0.0334	-0.0917	0.0465
24	0.5489	1.8798	2.0947	0.2981	-0.0146	0.0146	0.0406	-0.0877	0.0502
26	0.5683	1.8980	2.0750	0.3159	-0.0139	0.0139	0.0494	-0.0831	0.0537
28	0.5893	1.9111	2.0543	0.3326	-0.0129	0.0129	0.0602	-0.0777	0.0569
30	0.6111	1.9188	2.0325	0.3482	-0.0119	0.0119	0.0728	-0.0713	0.0600
32	0.6332	1.9213	2.0097	0.3629	-0.0107	0.0107	0.0875	-0.0640	0.0627
34	0.6552	1.9192	1.9861	0.3768	-0.0092	0.0092	0.1041	-0.0554	0.0652
36	0.6767	1.9132	1.9615	0.3899	-0.0076	0.0076	0.1226	-0.0456	0.0672
38	0.6976	1.9039	1.9362	0.4024	-0.0057	0.0057	0.1430	-0.0344	0.0689
40	0.7179	1.8920	1.9100	0.4144	-0.0036	0.0036	0.1650	-0.0216	0.0700
42	0.7374	1.8777	1.8831	0.4257	-0.0012	0.0012	0.1888	-0.0072	0.0706
44	0.7562	1.8616	1.8556	0.4366	0.0015	-0.0015	0.2142	0.0089	0.0706
46	0.7743	1.8439	1.8273	0.4470	0.0045	-0.0045	0.2410	0.0268	0.0698
48	0.7917	1.8247	1.7984	0.4569	0.0078	-0.0078	0.2692	0.0466	0.0683
50	0.8084	1.8044	1.7688	0.4664	0.0114	-0.0114	0.2987	0.0684	0.0660
52	0.8243	1.7831	1.7387	0.4754	0.0154	-0.0154	0.3293	0.0922	0.0627
54	0.8396	1.7608	1.7080	0.4840	0.0197	-0.0197	0.3608	0.1181	0.0585
56	0.8542	1.7377	1.6767	0.4922	0.0243	-0.0243	0.3930	0.1460	0.0531
58	0.8680	1.7139	1.6450	0.5000	0.0293	-0.0293	0.4258	0.1760	0.0465
60	0.8812	1.6894	1.6127	0.5073	0.0347	-0.0347	0.4590	0.2081	0.0387
62	0.8938	1.6645	1.5801	0.5143	0.0404	-0.0404	0.4922	0.2423	0.0296
64	0.9056	1.6390	1.5469	0.5208	0.0464	-0.0464	0.5254	0.2786	0.0190
66	0.9167	1.6131	1.5134	0.5269	0.0528	-0.0528	0.5583	0.3168	0.0068
68	0.9272	1.5868	1.4795	0.5326	0.0595	-0.0595	0.5905	0.3569	-0.0069
70	0.9370	1.5602	1.4452	0.5379	0.0665	-0.0665	0.6219	0.3988	-0.0223
72	0.9461	1.5334	1.4107	0.5428	0.0737	-0.0737	0.6522	0.4424	-0.0394
74	0.9545	1.5063	1.3758	0.5472	0.0813	-0.0813	0.6811	0.4875	-0.0583
76	0.9621	1.4791	1.3406	0.5512	0.0890	-0.0890	0.7085	0.5341	-0.0791
78	0.9691	1.4518	1.3053	0.5547	0.0970	-0.0970	0.7340	0.5818	-0.1018
80	0.9754	1.4244	1.2697	0.5578	0.1051	-0.1051	0.7575	0.6306	-0.1265
82	0.9810	1.3971	1.2339	0.5605	0.1134	-0.1134	0.7787	0.6801	-0.1532
84	0.9859	1.3697	1.1979	0.5627	0.1217	-0.1217	0.7974	0.7303	-0.1819
86	0.9900	1.3425	1.1619	0.5644	0.1301	-0.1301	0.8134	0.7808	-0.2127
88	0.9935	1.3154	1.1257	0.5657	0.1386	-0.1386	0.8266	0.8314	-0.2455
90	0.9962	1.2884	1.0895	0.5665	0.1470	-0.1470	0.8369	0.8818	-0.2804

PLANE STRAIN MODE 1 n = 5

θ^o	$\tilde{\sigma}_e$	$\tilde{\sigma}_{rr}$	$\tilde{\sigma}_{\theta\theta}$	$\tilde{\sigma}_{r\theta}$	$\tilde{\varepsilon}_{rr}$	$\tilde{\varepsilon}_{\theta\theta}$	$\tilde{\varepsilon}_{r\theta}$	$\tilde{u}_r$	$\tilde{u}_\theta$
92	0.9982	1.2618	1.0532	0.5668	0.1553	-0.1553	0.8440	0.9318	-0.3173
94	0.9995	1.2354	1.0169	0.5666	0.1635	-0.1635	0.8481	0.9811	-0.3563
96	1.0000	1.2095	0.9807	0.5659	0.1716	-0.1716	0.8489	1.0294	-0.3972
98	0.9998	1.1839	0.9445	0.5647	0.1794	-0.1794	0.8464	1.0765	-0.4401
100	0.9989	1.1588	0.9084	0.5630	0.1870	-0.1870	0.8408	1.1219	-0.4849
102	0.9973	1.1343	0.8725	0.5607	0.1943	-0.1943	0.8319	1.1656	-0.5315
104	0.9949	1.1104	0.8367	0.5579	0.2012	-0.2012	0.8200	1.2071	-0.5798
106	0.9919	1.0872	0.8011	0.5545	0.2077	-0.2077	0.8050	1.2462	-0.6297
108	0.9881	1.0647	0.7657	0.5505	0.2138	-0.2138	0.7873	1.2828	-0.6812
110	0.9837	1.0431	0.7306	0.5460	0.2194	-0.2194	0.7668	1.3164	-0.7342
112	0.9785	1.0223	0.6958	0.5409	0.2245	-0.2245	0.7439	1.3471	-0.7884
114	0.9728	1.0025	0.6614	0.5351	0.2291	-0.2291	0.7187	1.3744	-0.8438
116	0.9664	0.9837	0.6274	0.5287	0.2330	-0.2330	0.6916	1.3983	-0.9003
118	0.9593	0.9659	0.5938	0.5217	0.2364	-0.2364	0.6628	1.4185	-0.9577
120	0.9517	0.9493	0.5606	0.5140	0.2392	-0.2392	0.6325	1.4350	-1.0158
122	0.9436	0.9338	0.5280	0.5056	0.2413	-0.2413	0.6012	1.4477	-1.0745
124	0.9349	0.9195	0.4959	0.4965	0.2427	-0.2427	0.5690	1.4565	-1.1336
126	0.9258	0.9064	0.4645	0.4867	0.2435	-0.2435	0.5364	1.4612	-1.1931
128	0.9163	0.8945	0.4336	0.4762	0.2437	-0.2437	0.5035	1.4619	-1.2526
130	0.9064	0.8838	0.4035	0.4649	0.2431	-0.2431	0.4706	1.4586	-1.3121
132	0.8961	0.8743	0.3742	0.4529	0.2419	-0.2419	0.4381	1.4513	-1.3713
134	0.8856	0.8658	0.3456	0.4402	0.2400	-0.2400	0.4061	1.4400	-1.4302
136	0.8748	0.8584	0.3178	0.4267	0.2375	-0.2375	0.3749	1.4248	-1.4886
138	0.8639	0.8519	0.2910	0.4124	0.2343	-0.2343	0.3445	1.4058	-1.5462
140	0.8528	0.8462	0.2651	0.3974	0.2305	-0.2305	0.3153	1.3830	-1.6030
142	0.8415	0.8413	0.2401	0.3817	0.2261	-0.2261	0.2871	1.3566	-1.6588
144	0.8302	0.8369	0.2162	0.3653	0.2211	-0.2211	0.2602	1.3266	-1.7135
146	0.8187	0.8329	0.1934	0.3481	0.2155	-0.2155	0.2347	1.2933	-1.7668
148	0.8073	0.8293	0.1717	0.3303	0.2094	-0.2094	0.2104	1.2567	-1.8187
150	0.7957	0.8257	0.1511	0.3119	0.2028	-0.2028	0.1875	1.2169	-1.8691
152	0.7841	0.8222	0.1318	0.2928	0.1957	-0.1957	0.1660	1.1742	-1.9178
154	0.7724	0.8184	0.1137	0.2733	0.1881	-0.1881	0.1459	1.1286	-1.9647
156	0.7605	0.8143	0.0968	0.2532	0.1800	-0.1800	0.1271	1.0803	-2.0097
158	0.7486	0.8097	0.0813	0.2327	0.1716	-0.1716	0.1096	1.0294	-2.0526
160	0.7365	0.8045	0.0670	0.2118	0.1627	-0.1627	0.0934	0.9762	-2.0935
162	0.7241	0.7984	0.0542	0.1906	0.1535	-0.1535	0.0786	0.9207	-2.1321
164	0.7115	0.7913	0.0426	0.1691	0.1439	-0.1439	0.0650	0.8632	-2.1685
166	0.6984	0.7831	0.0325	0.1475	0.1339	-0.1339	0.0527	0.8037	-2.2024
168	0.6849	0.7735	0.0238	0.1259	0.1237	-0.1237	0.0415	0.7424	-2.2339
170	0.6708	0.7624	0.0164	0.1042	0.1133	-0.1133	0.0316	0.6796	-2.2628
172	0.6558	0.7494	0.0104	0.0827	0.1025	-0.1025	0.0229	0.6153	-2.2892
174	0.6399	0.7344	0.0058	0.0614	0.0916	-0.0916	0.0154	0.5497	-2.3129
176	0.6226	0.7169	0.0026	0.0404	0.0805	-0.0805	0.0091	0.4829	-2.3340
178	0.6035	0.6963	0.0006	0.0199	0.0692	-0.0692	0.0040	0.4152	-2.3523
180	0.5819	0.6719	0.0000	0.0000	0.0578	-0.0578	0.0000	0.3467	-2.3678

PLANE STRESS MODE 1 $n = 5$

θ^o	$\tilde{\sigma}_e$	$\tilde{\sigma}_{rr}$	$\tilde{\sigma}_{\theta\theta}$	$\tilde{\sigma}_{r\theta}$	$\tilde{\varepsilon}_{rr}$	$\tilde{\varepsilon}_{\theta\theta}$	$\tilde{\varepsilon}_{r\theta}$	$\tilde{u}_r$	$\tilde{u}_\theta$
0	0.9906	0.6907	1.1349	0.0000	0.1186	0.7602	0.0000	0.7117	0.0000
2	0.9906	0.6905	1.1343	0.0195	0.1188	0.7599	0.0282	0.7127	0.0017
4	0.9908	0.6900	1.1324	0.0390	0.1193	0.7589	0.0564	0.7157	0.0033
6	0.9911	0.6892	1.1293	0.0584	0.1201	0.7571	0.0846	0.7208	0.0047
8	0.9915	0.6880	1.1250	0.0777	0.1213	0.7547	0.1127	0.7278	0.0058
10	0.9920	0.6865	1.1194	0.0969	0.1228	0.7515	0.1408	0.7368	0.0065
12	0.9926	0.6847	1.1126	0.1159	0.1246	0.7475	0.1688	0.7478	0.0068
14	0.9932	0.6826	1.1045	0.1348	0.1268	0.7427	0.1967	0.7608	0.0064
16	0.9939	0.6801	1.0953	0.1533	0.1293	0.7370	0.2244	0.7757	0.0055
18	0.9947	0.6774	1.0849	0.1716	0.1321	0.7304	0.2520	0.7924	0.0037
20	0.9955	0.6743	1.0734	0.1897	0.1352	0.7229	0.2793	0.8111	0.0011
22	0.9962	0.6710	1.0607	0.2073	0.1386	0.7143	0.3063	0.8315	-0.0025
24	0.9970	0.6674	1.0468	0.2247	0.1423	0.7046	0.3330	0.8537	-0.0071
26	0.9977	0.6635	1.0319	0.2416	0.1463	0.6938	0.3591	0.8775	-0.0129
28	0.9984	0.6594	1.0159	0.2581	0.1505	0.6818	0.3847	0.9030	-0.0200
30	0.9990	0.6551	0.9989	0.2741	0.1550	0.6686	0.4095	0.9301	-0.0284
32	0.9995	0.6505	0.9808	0.2897	0.1598	0.6542	0.4336	0.9585	-0.0383
34	0.9998	0.6458	0.9618	0.3047	0.1647	0.6384	0.4568	0.9883	-0.0497
36	1.0000	0.6408	0.9418	0.3193	0.1699	0.6214	0.4789	1.0194	-0.0627
38	1.0000	0.6358	0.9210	0.3332	0.1753	0.6030	0.4998	1.0515	-0.0775
40	0.9997	0.6306	0.8992	0.3466	0.1808	0.5833	0.5193	1.0846	-0.0941
42	0.9993	0.6253	0.8766	0.3593	0.1864	0.5624	0.5374	1.1185	-0.1125
44	0.9986	0.6199	0.8532	0.3714	0.1922	0.5402	0.5539	1.1530	-0.1329
46	0.9976	0.6145	0.8291	0.3828	0.1980	0.5168	0.5686	1.1880	-0.1553
48	0.9962	0.6091	0.8043	0.3935	0.2039	0.4922	0.5814	1.2233	-0.1798
50	0.9946	0.6038	0.7788	0.4035	0.2098	0.4666	0.5923	1.2587	-0.2064
52	0.9926	0.5985	0.7526	0.4127	0.2157	0.4401	0.6010	1.2939	-0.2351
54	0.9902	0.5933	0.7259	0.4212	0.2215	0.4128	0.6075	1.3289	-0.2660
56	0.9875	0.5883	0.6987	0.4289	0.2272	0.3847	0.6117	1.3632	-0.2991
58	0.9843	0.5835	0.6711	0.4358	0.2328	0.3561	0.6136	1.3968	-0.3343
60	0.9808	0.5790	0.6430	0.4418	0.2382	0.3271	0.6131	1.4294	-0.3717
62	0.9768	0.5747	0.6145	0.4470	0.2435	0.2978	0.6103	1.4607	-0.4113
64	0.9724	0.5708	0.5858	0.4512	0.2484	0.2685	0.6051	1.4906	-0.4529
66	0.9675	0.5673	0.5568	0.4546	0.2531	0.2393	0.5975	1.5188	-0.4965
68	0.9622	0.5642	0.5276	0.4571	0.2575	0.2105	0.5878	1.5451	-0.5422
70	0.9565	0.5616	0.4983	0.4586	0.2615	0.1821	0.5759	1.5692	-0.5897
72	0.9504	0.5595	0.4689	0.4592	0.2652	0.1543	0.5620	1.5911	-0.6390
74	0.9439	0.5580	0.4396	0.4588	0.2684	0.1274	0.5462	1.6105	-0.6899
76	0.9369	0.5571	0.4102	0.4574	0.2712	0.1015	0.5287	1.6272	-0.7425
78	0.9296	0.5567	0.3810	0.4551	0.2735	0.0767	0.5098	1.6410	-0.7964
80	0.9220	0.5570	0.3520	0.4516	0.2753	0.0531	0.4895	1.6520	-0.8516
82	0.9140	0.5580	0.3233	0.4472	0.2766	0.0309	0.4682	1.6598	-0.9080
84	0.9058	0.5595	0.2948	0.4418	0.2774	0.0101	0.4460	1.6645	-0.9653
86	0.8973	0.5617	0.2667	0.4352	0.2777	-0.0091	0.4232	1.6659	-1.0234
88	0.8886	0.5644	0.2391	0.4277	0.2773	-0.0269	0.3999	1.6640	-1.0822
90	0.8797	0.5677	0.2120	0.4191	0.2765	-0.0430	0.3764	1.6588	-1.1414

PLANE STRESS MODE 1 $n = 5$

θ^o	$\tilde{\sigma}_e$	$\tilde{\sigma}_{rr}$	$\tilde{\sigma}_{\theta\theta}$	$\tilde{\sigma}_{r\theta}$	$\tilde{\varepsilon}_{rr}$	$\tilde{\varepsilon}_{\theta\theta}$	$\tilde{\varepsilon}_{r\theta}$	$\tilde{u}_r$	$\tilde{u}_\theta$
92	0.8706	0.5715	0.1855	0.4095	0.2750	-0.0576	0.3529	1.6502	-1.2009
94	0.8614	0.5756	0.1596	0.3988	0.2730	-0.0706	0.3294	1.6382	-1.2605
96	0.8522	0.5801	0.1345	0.3871	0.2705	-0.0820	0.3062	1.6229	-1.3201
98	0.8429	0.5848	0.1101	0.3745	0.2674	-0.0920	0.2835	1.6042	-1.3795
100	0.8335	0.5897	0.0866	0.3608	0.2637	-0.1005	0.2612	1.5822	-1.4385
102	0.8241	0.5946	0.0639	0.3462	0.2595	-0.1076	0.2395	1.5570	-1.4969
104	0.8147	0.5994	0.0423	0.3307	0.2548	-0.1134	0.2185	1.5286	-1.5546
106	0.8053	0.6040	0.0216	0.3143	0.2495	-0.1179	0.1983	1.4971	-1.6115
108	0.7960	0.6083	0.0021	0.2971	0.2438	-0.1213	0.1789	1.4625	-1.6673
110	0.7866	0.6122	-0.0164	0.2791	0.2375	-0.1235	0.1603	1.4251	-1.7220
112	0.7772	0.6156	-0.0336	0.2603	0.2308	-0.1246	0.1425	1.3848	-1.7754
114	0.7679	0.6183	-0.0497	0.2409	0.2236	-0.1248	0.1257	1.3417	-1.8273
116	0.7585	0.6203	-0.0645	0.2209	0.2160	-0.1240	0.1097	1.2960	-1.8777
118	0.7491	0.6215	-0.0779	0.2004	0.2080	-0.1224	0.0946	1.2478	-1.9264
120	0.7396	0.6217	-0.0901	0.1793	0.1995	-0.1200	0.0805	1.1972	-1.9733
122	0.7301	0.6208	-0.1009	0.1579	0.1907	-0.1169	0.0673	1.1443	-2.0183
124	0.7204	0.6188	-0.1103	0.1362	0.1815	-0.1131	0.0550	1.0892	-2.0613
126	0.7106	0.6156	-0.1183	0.1143	0.1720	-0.1086	0.0437	1.0321	-2.1022
128	0.7005	0.6110	-0.1249	0.0922	0.1622	-0.1036	0.0333	0.9731	-2.1410
130	0.6902	0.6050	-0.1301	0.0700	0.1520	-0.0982	0.0238	0.9122	-2.1774
132	0.6795	0.5974	-0.1339	0.0479	0.1416	-0.0922	0.0153	0.8497	-2.2115
134	0.6684	0.5882	-0.1362	0.0260	0.1310	-0.0859	0.0078	0.7858	-2.2431
136	0.6566	0.5772	-0.1372	0.0042	0.1201	-0.0792	0.0012	0.7204	-2.2723
138	0.6442	0.5642	-0.1368	-0.0172	0.1090	-0.0722	-0.0044	0.6538	-2.2989
140	0.6309	0.5490	-0.1350	-0.0381	0.0977	-0.0649	-0.0091	0.5861	-2.3230
142	0.6164	0.5313	-0.1319	-0.0585	0.0862	-0.0574	-0.0127	0.5174	-2.3443
144	0.6004	0.5107	-0.1275	-0.0782	0.0747	-0.0498	-0.0152	0.4479	-2.3631
146	0.5824	0.4865	-0.1219	-0.0971	0.0630	-0.0420	-0.0167	0.3778	-2.3791
148	0.5614	0.4578	-0.1151	-0.1149	0.0512	-0.0342	-0.0171	0.3072	-2.3924
150	0.5362	0.4227	-0.1072	-0.1317	0.0394	-0.0263	-0.0163	0.2363	-2.4029
152	0.5040	0.3775	-0.0983	-0.1469	0.0275	-0.0185	-0.0142	0.1652	-2.4107
154	0.4581	0.3120	-0.0885	-0.1603	0.0157	-0.0108	-0.0106	0.0941	-2.4157
156	0.3687	0.1710	-0.0779	-0.1706	0.0039	-0.0030	-0.0047	0.0233	-2.4180
158	0.4098	-0.3132	-0.0669	-0.1696	-0.0079	0.0025	-0.0072	-0.0474	-2.4176
160	0.4764	-0.4112	-0.0563	-0.1611	-0.0197	0.0077	-0.0124	-0.1183	-2.4145
162	0.5163	-0.4675	-0.0463	-0.1500	-0.0316	0.0133	-0.0160	-0.1895	-2.4088
164	0.5457	-0.5087	-0.0371	-0.1372	-0.0434	0.0193	-0.0182	-0.2607	-2.4004
166	0.5691	-0.5414	-0.0288	-0.1231	-0.0553	0.0254	-0.0194	-0.3317	-2.3893
168	0.5888	-0.5688	-0.0214	-0.1078	-0.0671	0.0316	-0.0194	-0.4023	-2.3755
170	0.6058	-0.5921	-0.0150	-0.0916	-0.0788	0.0379	-0.0185	-0.4725	-2.3590
172	0.6210	-0.6123	-0.0097	-0.0745	-0.0904	0.0441	-0.0166	-0.5421	-2.3398
174	0.6348	-0.6299	-0.0055	-0.0567	-0.1018	0.0502	-0.0138	-0.6109	-2.3181
176	0.6474	-0.6452	-0.0025	-0.0382	-0.1131	0.0562	-0.0101	-0.6788	-2.2937
178	0.6592	-0.6586	-0.0006	-0.0193	-0.1243	0.0621	-0.0055	-0.7457	-2.2668
180	0.6702	-0.6702	0.0000	0.0000	-0.1352	0.0676	0.0000	-0.8114	-2.2373

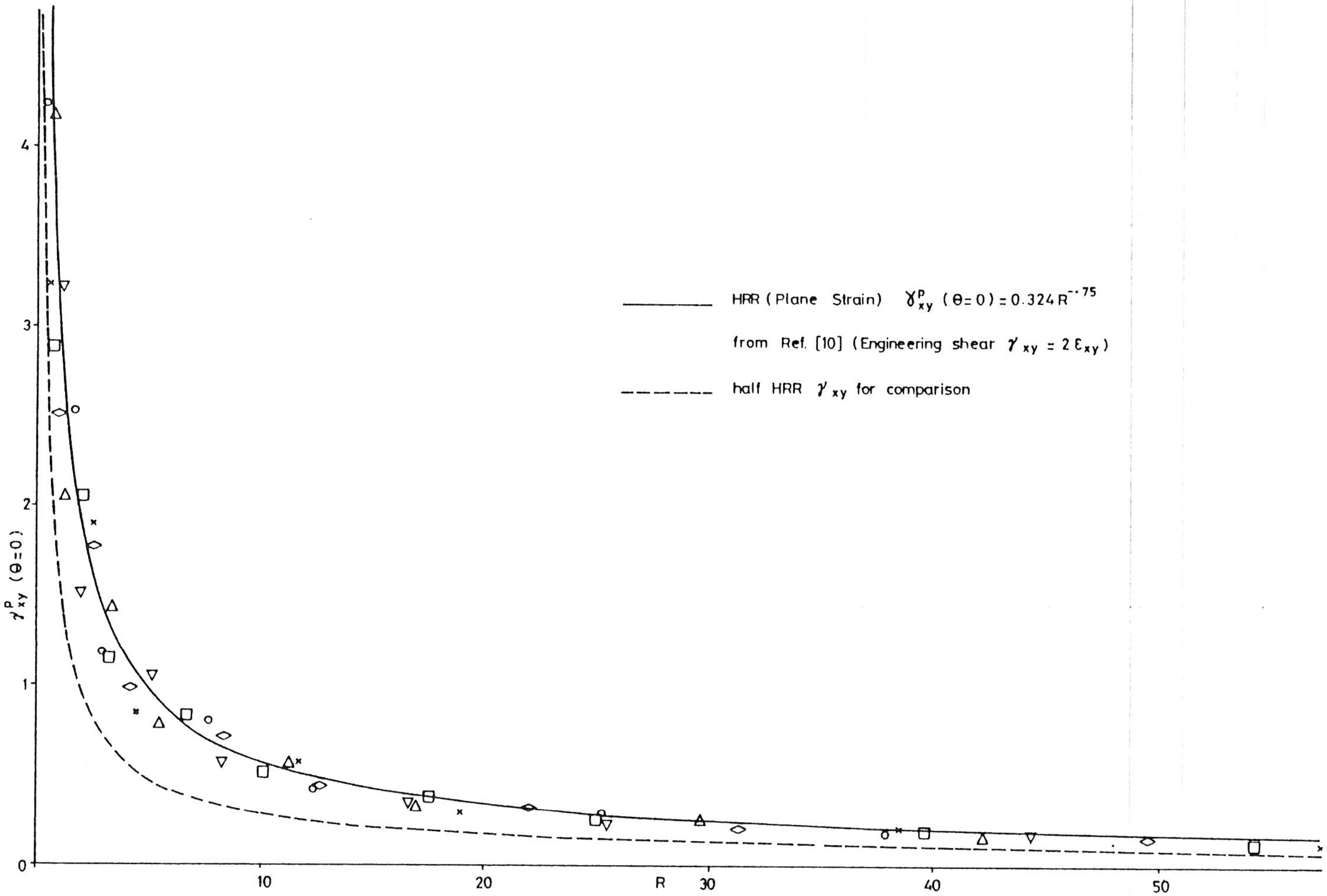


Figure 5. Engineering shear strain on the ligament versus dimensionless distance, R, up to $R = 55$.

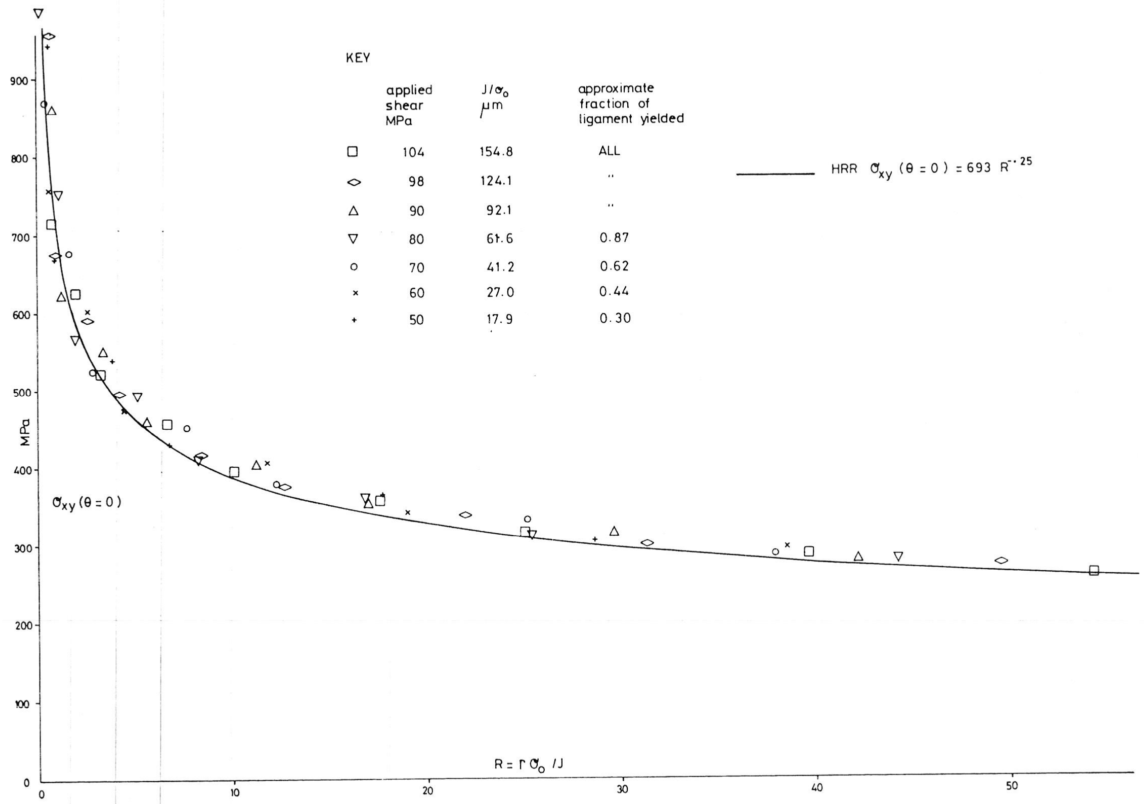


Figure 4. Shear stress on the ligament, $\theta = 0$, versus dimensionless distance from crack tip, R, up to $R = 55$.