T73S02
Low Temperature Fracture
Tutorial Session 1
LEFM Crack Tip Fields

Notch Stress Concentration

- Qu.: What is the stress at the tip of a notch with root radius ρ?
- The stress concentration factor (SCF) of a notch increases as the radius, ρ, decreases.
- Qu.: So what does the notch tip stress become as the notch becomes sharp?
- The notch tip stress diverges.
- The Inglis solution for an elliptical hole an infinite plate subject to uniaxial tension gives the SCF at the notch tip to be…

Notch Stress Concentration Factor

$$1 + 2\sqrt{\frac{a}{\rho}}$$

Notch Stress Concentration Factor

- *a* is the semi-major axis
- ρ is the radius of curvature at the pointy end of the ellipse
- *b* is the semi-minor axis
- $\rho = b^2/a$
- The ellipse tends to an embedded crack of length 2*a* in the limit that b shrinks to zero. So the elastic stress at the tip of a crack is infinite.

$$1 + 2\sqrt{\frac{a}{\rho}}$$

Crack Tip Stress

- Qu.: If the remotely applied load is very, very small, what is the crack tip stress?
- Infinite.

- Qu.: So why doesn't anything with a crack in it break immediately?
- Because the assumptions of a perfect continuum which is isotropic, homogeneous, with only small-strains, and only elastic behaviour break down – and so the stress isn't really infinite.

- Qu.: Which of the assumptions breaks down: (a)continuum, (b)isotropic, (c)homogeneous, (d)small-strain, (e)elastic?
- All of them.

- Qu.: So should we just give up now, then?
- Don't be so defeatist! We'll return to this later.

- At least this makes it clear why we're concerned about cracks. They have a tendency to grow or to fast fracture, because of their attendant high crack tip stress fields.

Crack Tip Stress

- Qu.: How do we describe the stresses near to the crack tip?

- In terms of the stress intensity factor (SIF), *K*, the stress ahead of a crack loaded in Mode I is,

$$\sigma_y(r) = \frac{K_I}{\sqrt{2\pi r}}$$

The Archetypal SIF

- Qu.: What is the magnitude of the SIF in a simple case?

- For an embedded crack of length 2*a* in an infinite plate subject to uniform uniaxial tension the SIF is given by,

$$K_I = \sigma_\infty \sqrt{\pi a}$$

Why is crack tip stress proportional to $1/\sqrt{r}$?

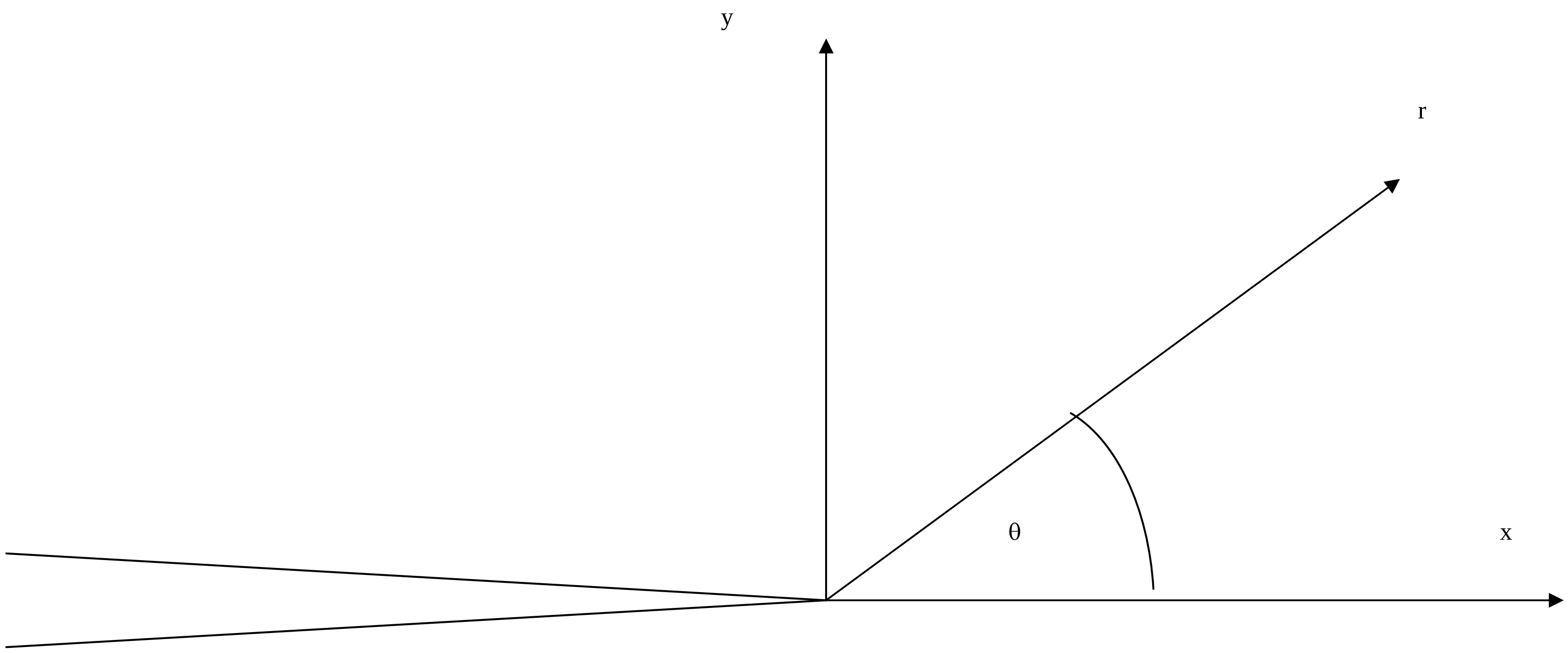
- It follows from the contour independence of the J-integral – but we haven’t covered that yet (Session 15).
- It also follows from dimensional analysis if we can assume that the SIF is proportional to $\sqrt{a}$,
- …but that begs the question, “where does $K = \sigma\sqrt{\pi a}$ come from?”
- So we’ll just have to derive these expressions from elastic analysis.

What do those factors of π mean?

- The π in the expression $K = \sigma_\infty \sqrt{\pi a}$ is just a convention.
- Since the stress is $\sigma(r) = K/\sqrt{2\pi r}$, the π cancels to give $\sigma(r) = \sigma_\infty \sqrt{(a/2r)}$.
- So we might just as well have defined the SIF as $K = \sigma_\infty \sqrt{a}$, and used $\sigma(r) = K/\sqrt{2r}$.
- ..and watch out because some papers do just that.

The Three Modes of K

- Conventional axes in fracture…



The Three Modes of K

- With reference to the above standard coordinate system, the three modes are defined by,
- Mode I: The remotely applied stress is σ_{yy} (opening mode)
- Mode II: The remotely applied stress is σ_{yx} (in-plane shear mode)
- Mode III: The remotely applied stress is σ_{yz} (out-of-plane shear mode)

The Three Modes of K

- What about the other three possible applied stresses?
- σ_{xx} , σ_{zz} and σ_{xz} all produce K = 0.

- A shaft under torsion has a part-penetrating, fully circumferential crack: what is the Mode?
- Mode III
- A shaft under torsion has a part-penetrating, long semi-elliptic axial crack: what is the Mode at the deepest point?
- Mode III
- A shaft under torsion has a fully-penetrating, axial crack: what is the Mode at the deepest point?
- Mode II
- A shaft has a part-penetrating, fully circumferential crack: how could it be loaded to produce Mode II?
- Apply a temperature difference to the shaft on either side of the crack.

But why are we interested in K anyway?

- Because in place of a failure criterion based on a critical failure stress, we base the criterion for brittle fracture on reaching a critical value of K, called “fracture toughness” or K_{Ic}.

How does K get around the problem of the crack tip stresses being infinite?

- Most obviously, K is finite.

- The logic is that it no longer matters that the LEFM crack tip fields become fictional as the tip is approached, so long as there is a region within which the LEFM fields are a reasonable approximation, i.e., at a finite distance.

- If the processes which lead to fracture occur within this LEFM zone, then K must control fracture since K controls the LEFM fields.

Derivation of $K = \sigma_\infty \sqrt{\pi a}$ and $\sigma(r) = K/\sqrt{2\pi r}$

- The challenge is to avoid being terminally boring.
- Derivation carried out for Mode III, because it's algebraically far simpler and illustrates all the essential features.

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