T73S02 Tutorial Session 15 Homework: Energy Release Rate

Mentor Guide K&S Questions:-

1.4 Define the energy release rate (G) and state the relationship between K and G in LEFM and for mixed mode loading.

2.1 Explain why the LEFM parameter K does not apply post-yield.

2.2 State the energy-based and the contour-integral definitions of the post-yield fracture parameter J. Explain the circumstances in which the contour integral is approximately path independent.

Numerical/Mathematical Questions:-

1) A shell whose in-plane dimensions are large compared with its thickness contains an extended surface crack running perpendicularly to the surface. The crack depth equals half the shell thickness of 20mm. If the Mode I, II and III stresses are all equal to 30 MPa and E = 200 GPa, ν = 0.3, what is the elastic energy release rate, G?

[Use normalised SIF solutions of your choice, e.g., from the session 14 notes].

2) A SIF solution can be used to find how a crack changes the elastic stiffness of a structure. Suppose a plate of length L, width w and thickness t has a tensile force F applied to it, causing a displacement D. (The force F acts parallel to the length dimension, L). The uncracked Mode I stress is thus,

$$\sigma_0 = F / wt \tag{1}$$

In terms of the normalised SIF, $k = K / K_0$, we can write,

$$K = k\sigma_0 \sqrt{\pi a} \tag{2}$$

where 'a' is the depth of an edge crack (running in the direction of the width dimension, w). The crack is of uniform depth through the thickness. Assume elastic behaviour.

The force acting when the displacement is D and the crack length is a is written,

$$F(a, D) = S(a)D \tag{3}$$

where $S(a)$ is the elastic stiffness when the plate is cracked to depth a. The strain energy of the body when the applied displacement is D is,

$$U(a, D) = \int_0^D F(a, D')dD' \tag{4}$$

We also have the standard relations for G in terms of U, and K in terms of G,

(*derivative at fixed D*) $$G = -\frac{1}{t}\frac{\partial U}{\partial a} \tag{5}$$

$$EG = \left(1 - \nu^2\right)K^2 \tag{6}$$

Use Equs.(1 to 6) to prove that the elastic stiffness of the cracked structure is,

$$S(a) = \left[\frac{1}{S(0)} + \frac{2\pi\left(1 - \nu^2\right)}{Etw^2}\int_0^a k^2 a' da'\right]^{-1} \tag{7}$$

where $S(0)$ is the uncracked elastic stiffness. Note that k^2 must be included within the integral since it varies with the crack size integrated over.

(PTO…

[Hint: Use (3) in (4) to find an expression for U, and then differentiate and use (5) to find a expression for G. Equate this to the expression you get for G from (6) after substituting (1,2). You'll then need to integrate with respect to crack depth].

3) If in Qu.2 the crack is shallow so that we can approximate the normalised SIF to be constant, $k \sim 1.12$, show that $S(0) = Etw/L$ and hence,

$$S(a) \approx \frac{Etw}{L}\left(1 - 3.6\frac{a^2}{wL}\right) \tag{8}$$

If a = 2mm, t = w = 10mm and L = 20mm, by what percentage is the elastic stiffness of the structure reduced by the crack?

An old fashioned means of checking for cracks is "wheel tapping", in the hope that the changed note will betray the presence of a crack. Given that natural frequencies are proportional to the square-root of the stiffness, by what percentage is the frequency of the note changed by the crack? Is wheel tapping likely to detect small cracks?

4) Consider the same plate as in Questions (2) & (3), but we now allow plastic behaviour. The elastic-plastic load-displacement behaviour of the plate can be approximated by the expressions,

For $D \le D_0 = 0.02mm$ $\quad F(a,D) = S(a)D$ (9a)

For $D \ge D_0 = 0.02mm$ $\quad F(a,D) = 43{,}734\left(1 - 0.36\frac{a}{w}\right)D^{0.2}$ (9b)

where the units are assumed to be N and mm. Equ.(9a) implies elastic behaviour for displacements less than $D_0 = 0.02mm$, whilst (9b) gives the plastic behaviour for larger displacements. Assuming E = 200 GPa the yield load of the uncracked plate is 20 kN, and that of the cracked plate is 18.56 kN [both (9a) and (9b) are consistent with these values, and (9b) is consistent with (8)].

By using Equs.(4,5) show that, for $D > D_0$,

$$G = 3.6\frac{Ea}{L^2}D_0^2 + \frac{13{,}120}{wt}\left(D^{1.2} - D_0^{1.2}\right) \tag{10}$$

In the following questions assume a = 2mm and k = 1.12:-

(a) When $D = D_0$ show that the first term in (10) corresponds, via (6), to the same K that can be derived from Equs.(1,2);

(b) Find the effective elastic-plastic K, defined as $K_{ep} = \sqrt{\frac{EG}{1-\nu^2}}$, for $D = 2D_0$;

(c) Compare the result of (b) with an elastic SIF derived in two different ways: (i)use the uncracked body elastic stress for $D = 2D_0$ together with Equ.(2); (ii)use the actual elastic-plastic load at $D = 2D_0$ together with Equs.(1,2). Why is (i) greater than the correct K_{ep} from (b), and why is (ii) smaller?

(d) Find the uncracked elastic load corresponding to $D = 2D_0$ and then find the value of K_{ep} at this load.

On a load-displacement diagram, plot the points corresponding to the above four SIFs. Note that the relative magnitudes of the SIFs is reflected by the relative magnitudes of U (the area under the relevant curves).