SQEP Tutorial Session 8a: T72S01
Relates to Knowledge & Skills: 1.11, 1.12
Last Update: 1/2/14

What is plasticity? Yield criteria, Mises & Tresca; Theoretical strength versus reality (dislocations); Definition of proof stress and UTS; Engineering versus true stress and strain; Graphical construction for engineering stress and UTS from true stress curve; Typical stress-strain curve types; Ductility;

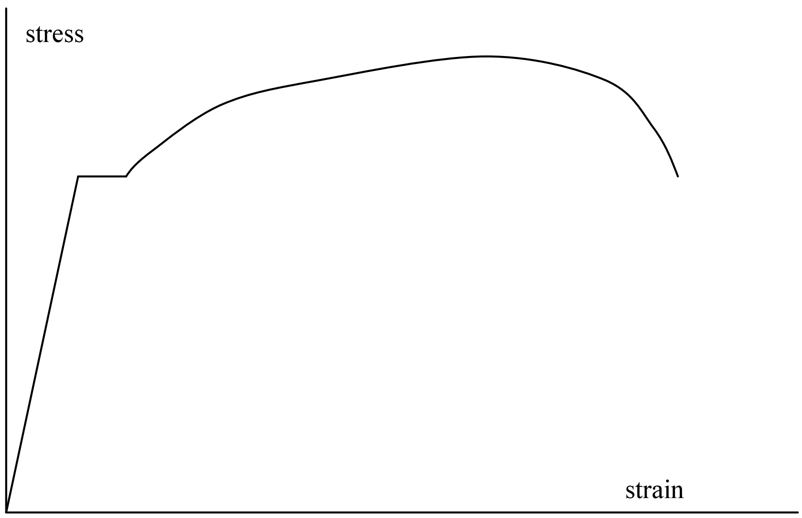
Qu.: What is plasticity?

The essence of plastic behaviour is the irreversibility of strain, i.e., permanent deformation. Conversely, the essence of elastic behaviour is reversible strains, i.e., fully recoverable deformation upon load removal.

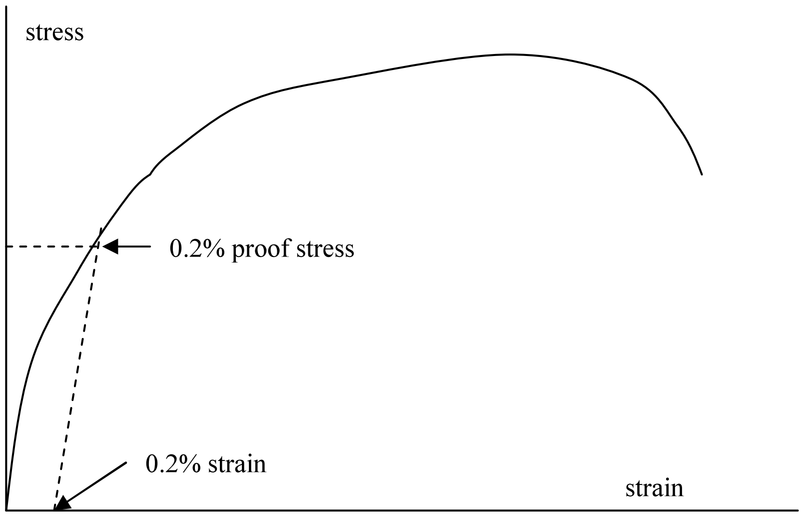
A non-linear stress-strain curve does not necessarily imply plasticity. Some elastic materials may have a non-linear stress-strain curve, e.g., rubber.

Qu.: What does a typical stress-strain curve look like in the plastic regime?

Some carbon steels exhibit a distinct yield point,



In this case it is clear how the yield stress is defined. Other steels, typically austenitics, do not have a distinct yield point but merely display a stress-strain curve of increasing curvature,



Qu.: How is a "yield" stress defined for such materials?

In place of a distinct yield, an arbitrary measure is defined by convention as the stress which produces a permanent (i.e., plastic) strain of 0.2%. Note that it is not the total strain which is 0.2%, but the plastic part alone. This defines the "0.2% proof strength".

The same idea can be used to define a proof strength at any desired strain. The 1% proof strength is a common alternative.

Qu.: What is the shape of the stress-strain curve?

There is no universal answer to this really. A common form to which data is fitted is the Ramberg-Osgood equation. This is just the sum of the linear elastic strain and a plastic strain expressed as a power law,

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{A}\right)^n \equiv \varepsilon_e + \varepsilon_p$$

[Aside: The usefulness of the material independent Option 1 FAD in R6 rests on the approximation that stress-strain curves tend to be similar when the stress axis is normalised by the 0.2% proof strength. That this is only a rough approximation is recognised by the provision of Options 2 and 3 in R6].

Qu.: What is the Ultimate Tensile Strength (UTS)?

The UTS is the largest engineering stress achieved in a standard tensile test.

Qu.: Is UTS a material property?

No.

Most engineers think that UTS is a material property, but it isn't. The reason is that the maximum load in a tensile test occurs due to a necking instability. This is when the cross section of the specimen reduces preferentially at one location. Once this starts, the reduction in cross section leads to an increased stress at that section, which concentrates the strain even more at the same position. The result is instability. But a structural instability depends upon the geometry and the nature of the loading, not just upon the material. So the UTS is uniquely defined only for the particular size and shape of a conventional tensile specimen, and standard loading details.

Qu.: Is the 0.2% proof stress a material property?

Yes.

No doubt the pedantic could quibble even with this. But to a good approximation at least, the characteristic stress at some small plastic strain, such as 0.2%, in a conventional uniaxial tensile test can be regarded as a material property.

Qu.: What are typical 0.2% proof stress values?

There is a wide range of proof strengths for common steels. For low alloy steels, or many austenitics, the 0.2% proof stress when in an annealed state might typically be of the order 250 MPa at room temperature, with a wide variation between steels and a typical spread for a given material type of $\pm$20%. This is representative of most steels used in pressure vessels, pipework or civil structures.

For low alloy ferritics with suitable heat treatment, the proof stresses can be raised to ~500 MPa.

High strength steels, e.g. spring steels, can have proof stresses of order 1000 MPa.

The chemical composition alone does not determine the proof strength – this depends crucially upon the heat treatment

Qu.: What is the theoretical strength of a flawless material?

The strength of a material is not usually determined by the requirement to break inter-atomic bonds. Instead, a material's strength is determined by its intrinsic flaws, i.e., its deviations from a perfect crystalline structure. It is instructive to estimate the order of magnitude of a material's strength if it *were* determined by inter-atomic forces…

Atomic spacings are of order one Angstrom (10^{-10} m). The binding energy of one atom in a crystal matrix is of order 1eV ~ 10^{-19} Joules. Dividing energy by distance gives an estimate of the inter-atomic bond force as, very crudely, ~10^{-9} N. Since there are 10^{10} atoms per metre of length, this means there are 10^{20} atoms in one square metre. The breaking strength would thus appear to be $10^{-9} \times 10^{20}$ N/m^2 = 10^{11} Pa = 10^5 MPa. But real materials do not have strengths of the order of 100,000 MPa. Their strengths are two to three orders of magnitude smaller than this.

Qu.: What are typical 0.2% proof stress values?

For the lower strength (most common) steels, the UTS generally exceeds the 0.2% proof stress by a factor of around 2 or 3. This means that the plastic strain required to reach the UTS may be very large in such steels (30%-60% is common). Such steels are said to be ductile, i.e., they will deform a great deal before failing.

Conversely, for the highest strength steels, the UTS may only be 10% or 20% above the 0.2% proof stress. This means that the plastic strain required to reach the UTS may be quite small, just a few percent or less (low ductility). If the plastic strain at failure is very small, comparable with the elastic strain or smaller, the steel is brittle (non-ductile).

Qu.: What flaws are most responsible for a steel's proof strength?

The chief culprit in lowering the strength from the theoretical level are dislocations. These effectively allow the crystal structure to unzip at far lower stresses.

Qu.: What is "work hardening"?

Work hardening just means that the stress-strain curve continues to rise above the proof stress. As well as this, the material actually does harden, in the sense that a hardness test will produce a higher hardness after yielding (e.g. Vicker's, H_V). This is because the load required to cause an indentation (which is the usual measure of "hardness") is related to the yield stress. We will derive relationships between indentation force and yield stress in a later session on slip line field theory.

Qu.: What is "perfect plasticity" ?

Perfect plasticity is the term for a material which yields but exhibits no work hardening. Instead the stress-strain curve is a horizontal line after a distinct 'yield' stress is reached. This is not at all typical of steels, though the assumption is often made in analyses for simplicity.

Qu.: What causes work hardening?

A material is soft when there are plenty of mobile dislocations. At a given stress (above yield) the straining stops (at least over short timescales) when there are no more dislocations which are able to move at that stress level. The dislocations which were mobile previously become immobilised for several reasons. One is that they hit obstacles like atoms of alloying elements, or much larger inclusions (e.g., carbides). Another is that dislocations get tangled together. Or the dislocation may hit the grain boundary and be annihilated. By raising the stress, some more dislocations are encouraged to move – until they too become immobilised. Thus, work hardening is due to dislocation immobilisation.

Further strain can accumulate over long timescales at high temperature, if there is sufficient thermal energy available to joggle dislocations free of their obstructions (creep).

Qu.: What are engineering stress and true stress?

Engineering stress uses the original area, $\sigma_{eng} = \frac{F}{A_0}$, whilst true stress uses the current area, $\sigma_{true} = \frac{F}{A}$.

Qu.: Which is larger, engineering or true stress?

In tension, $\sigma_{true} > \sigma_{eng}$ because the cross section is reduced, $A < A_0$.

But in compression, the *magnitude* of the true stress is smaller than that of the engineering stress.

Qu.: What about shear?

To first order, the shear area does not change upon deformation, so there is no distinction between engineering and true quantities in shear. The terms engineering stress and true stress tend only to be used for simple uniaxial tension or compression. In general states of 3D stress life gets more complicated (you need a large strain formulation of everything, which gets hard – the tensors become non-Cartesian).

Qu.: What are engineering strain and true strain?

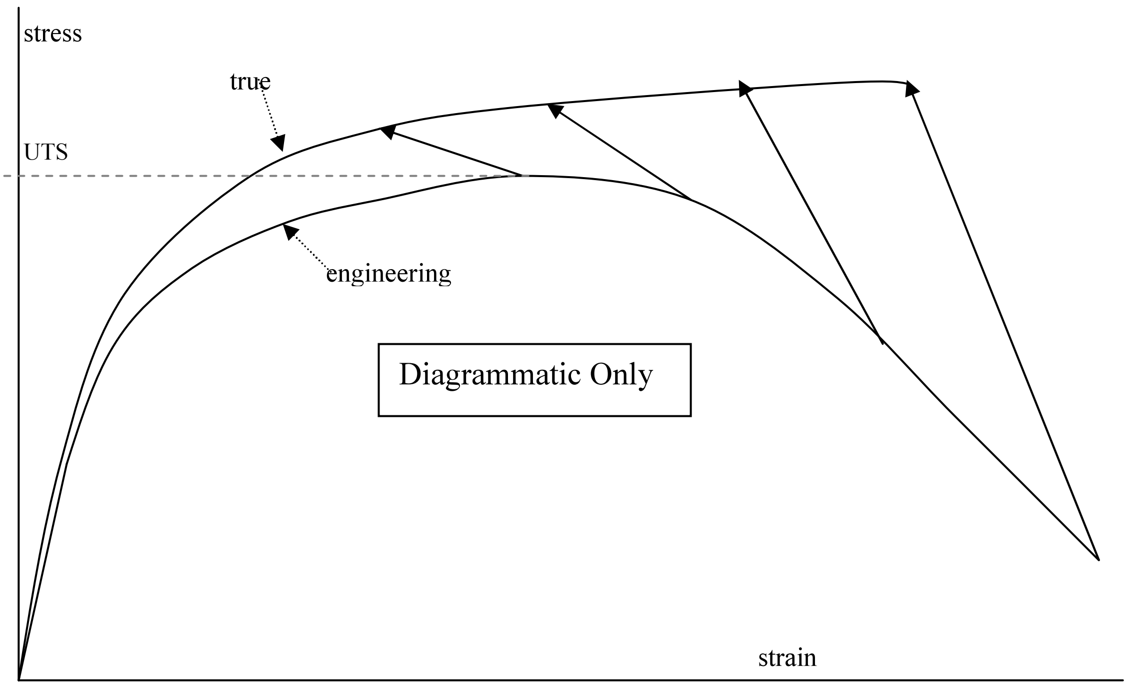
Engineering strain uses the original length, $\varepsilon_{eng} = \frac{\Delta L}{L_0}$. An increment of true strain is defined in the obvious manner using the instantaneous length, $\delta\varepsilon_{true} = \frac{\delta L}{L}$. Integrating then gives the true strain as $\varepsilon_{true} = \int_{L_0}^{L} \frac{dL}{L} = \log\frac{L}{L_0} = \log\left(1 + \frac{\Delta L}{L_0}\right) = \log(1 + \varepsilon_{eng})$. (NB: My 'log' is always the natural log unless indicated otherwise).

Qu.: Which is larger, engineering or true strain?

Recall that $\log(1+\varepsilon) = \varepsilon - \varepsilon^2/2 + \varepsilon^3/3 - \ldots$. Hence, $\varepsilon_{true} \approx \varepsilon_{eng} - \varepsilon_{eng}^2/2 + O(\varepsilon_{eng}^3)$. Hence, in tension the true strain is less than the engineering strain. This is also true in compression, though it is larger in *magnitude*.

Qu.: How does the use of true quantities affect the stress-strain curve?

By convention we assume we are talking about uniaxial tension. So the true stress is larger than the engineering stress, but the true strain is smaller than the engineering strain. So all points on the engineering stress-strain curve move upwards and to the left:-



True stress – true strain curves tend to be monotonic (i.e. never decreasing) in contrast with the maximum at the UTS displayed by the engineering stress-strain curve.

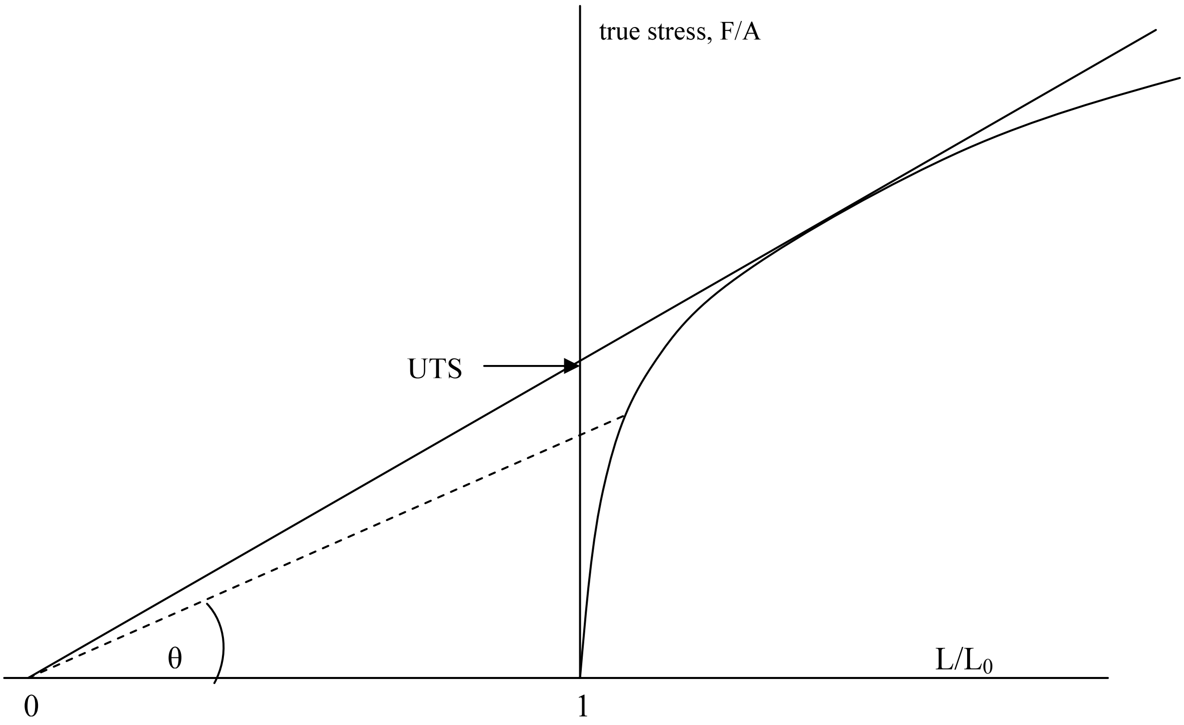
This shows that the UTS is a consequence of geometry change

Qu.: How does volume change during plastic deformation?

At the microscopic scale, plasticity involves sliding along planes (dislocation glide). This involves no change of volume. So $V = AL = A_o L_0$.

Qu.: How can the UTS be obtained from a graph of true stress?

There is a neat graphical interpretation for the UTS when true stress is plotted against engineering strain – or equivalently against length:-



$$\tan\theta = \frac{F/A}{L/L_0} = \frac{FL_0}{AL} = \frac{FL_0}{A_0 L_0} = \frac{F}{A_0} = \sigma_{eng}$$

So the engineering stress is maximum when θ is maximum, i.e. when the line draw from the origin (the point of zero length) is tangent to the curve. It follows that the point where this line crosses the y-axis is the UTS.

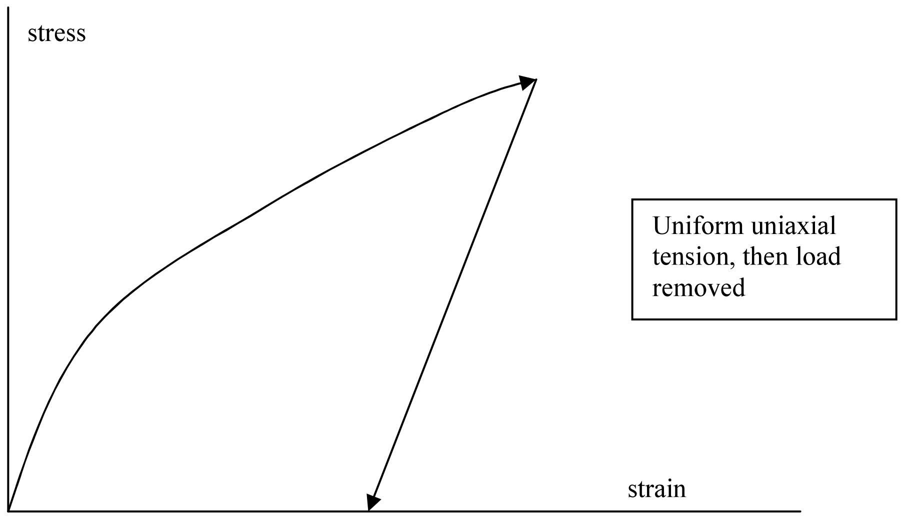
Qu.: What is ductility?

Loosely, ductility is the ability of a material to sustain substantial plastic deformation without breaking. It is quantified in standard tensile tests by one of two quite distinct quantities. The first is the strain at failure, measured from the extension over the gauge length of the specimen. The second is the fractional reduction of area. Since $A_0 L_0 = AL$ we have $\varepsilon = L/L_0 - 1 = A_0/A - 1 = \Delta A/A > \Delta A/A_0$, so the engineering strain at failure might be expected to exceed the fractional reduction of area. In practice, for materials which are sufficiently ductile to exhibit necking, the opposite is generally true. The reduction of area most often exceeds the engineering strain at failure, $\Delta A_{fail}/A_0 > \varepsilon_{eng}^{fail}$. The reason is that the engineering strain is measured from the extension over the whole gauge length, whereas the reduction of area is measured in the necked region – where the stress and strain are greater.

The reduction of area is really the better measure of ductility, since it is a local quantity. The strain based on the extension over the gauge length is rather arbitrary – it will reduce if you use a longer specimen.

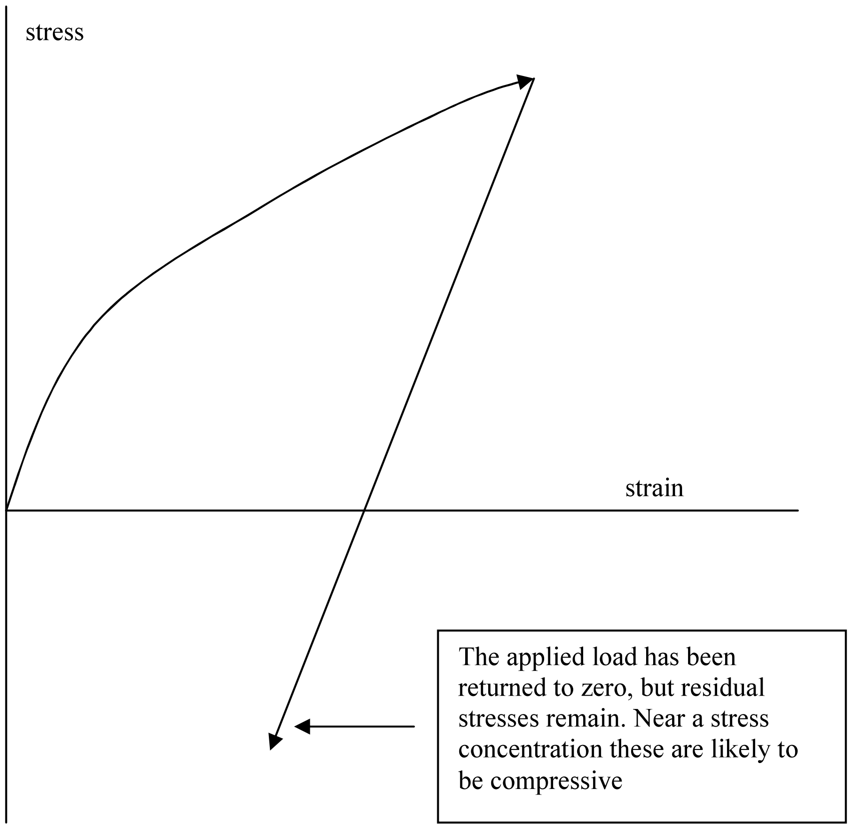
Qu.: What happens when a plastically deformed material is unloaded?

The stress-strain locus "unloads down the elastic line". So, under uniform uniaxial tension we get,



But note that we end up with zero stress only because the uniaxial specimen was assumed to be uniformly stressed. So, a return to zero load means a return to zero stress.

For more complex situations involving non-uniform stresses, the removal of load will not lead to zero stresses everywhere. Instead there will be "residual stresses". These will be in equilibrium everywhere. A simple example is the stress near a stress concentration. This tends to reverse when load is removed:-



Qu.: What determines yielding for a general 3D state of stress?

For isotropic materials, the yield condition is that one of the equivalent stresses reaches the yield (or proof) stress. The two most commonly deployed equivalent stresses for this purpose are the Tresca and Mises stresses.

Qu.: What is the Tresca stress?

The Tresca stress is simply twice the maximum shear stress for any orientation of the coordinate system. It is given in terms of the maximum and minimum principal stresses by,

$$\sigma_{Tresca} = \sigma_1 - \sigma_3 = \textit{twice maximum shear}$$

Qu.: What is the von Mises stress?

In terms of the principal stresses the Mises stress is given by,

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

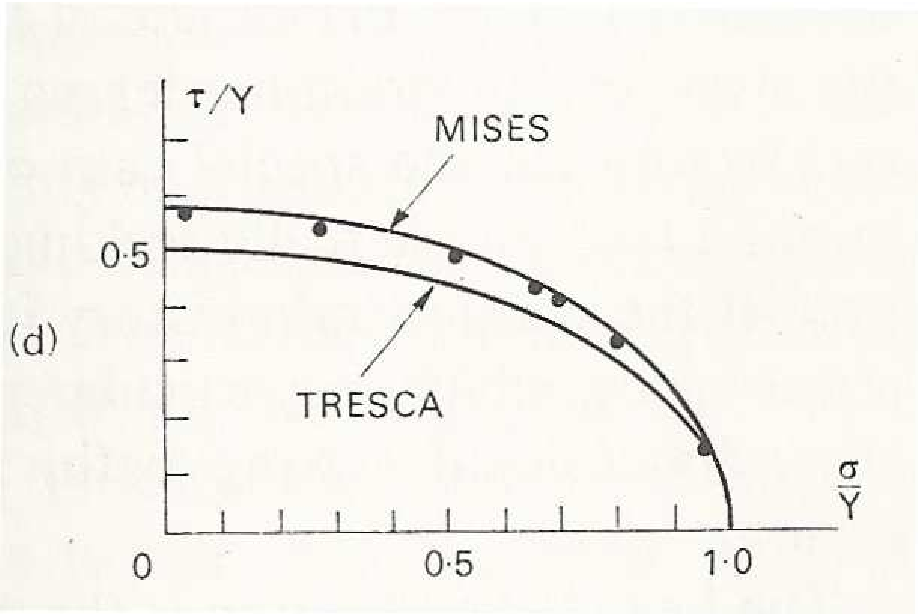
In an arbitrary coordinate system, the Mises stress is,

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{zx}^2)\right]^{1/2}$$

Qu.: Which 3D yield criterion is the best?

For structural steels, which are approximately isotropic due to being comprised of many randomly oriented crystals (grains), I have always advised that the Mises equivalent stress provides the better criterion. This is based on the classic test data of G.I. Taylor and H. Quinney, *Philos Trans R Soc* 230 (1931) 323.....

Taylor & Quinney (1931) Copper / Aluminium / Mild Steel



These tests considered cases of combined tension & shear - achieved using a tension-torsion machine. Note that such tests produce a *smaller* ratio of hydrostatic to Mises stress than simple tension (negative constraint). In such cases it was found that the Tresca criterion generally underestimates the load at yield whereas the Mises criterion is quite good.

However, the matter is not as clear cut as I used to believe. It is probably more common to be interested in cases for which the ratio of hydrostatic to Mises stress is elevated by the 3D stress state compared with simple tension (positive constraint). I have not done a literature search (which would be worth doing), but one paper based on tests applying tensile loading to specimens with local stress raisers is "*Yield criterions of metal plasticity in different stress states*", Acta Metall. Sin.(Engl. Lett.)Vol.22 No.2 pp123-130 Apr. 2009, Fengping Yang, Qin Sun and Wei Hu. This indicates that both Tresca and Mises criteria over-estimate the post-yield load compared with test data, and the Mises criterion is worst than Tresca in this respect. A proper literature survey needs doing.

Qu.: What are the deviatoric stresses?

The deviatoric stresses are the on-diagonal (direct) stresses minus the hydrostatic stress. They are denote by a 'hat' (caret): thus, $\hat{\sigma}_x = \sigma_x - \sigma_H$. The shear stresses are not affected. Hence,

$$\hat{\sigma}_{ij} = \sigma_{ij} - \sigma_H \delta_{ij} \qquad \text{or} \qquad \hat{\sigma} = \sigma - \sigma_H \mathrm{I}$$

where I is the unit matrix and the Kronecker symbol δ_{ij} are its components, i.e., $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$.

Qu.: What is the Mises stress in terms of the deviatoric stresses?

The answer is: $\bar{\sigma} = \left[\frac{3}{2} \hat{\sigma}_{ij} \hat{\sigma}_{ij} \right]^{1/2}$, noting that the summation convention applies to both indices.

Qu.: If a stress state is changed by adding a purely hydrostatic stress, do the equivalent stresses change?

No. Neither the Tresca stress nor the Mises stress change. This is because all three principal stresses increase by the same amount, i.e., by the hydrostatic stress added.

Consequently, hydrostatic stress does not cause plastic straining. Plastic straining is due only to the deviatoric stresses, which are just the stress components with the hydrostatic part subtracted off.

Qu.: Why does hydrostatic stress not cause plastic straining?

The reason is because of incompressibility and isotropy. Isotropy means that hydrostatic stress can only cause hydrostatic strain. But incompressibility says that the hydrostatic strain is zero. QED.