SQEP Tutorial Session 4: T72S01

Basic formulation of 3D continuum stresses and strains

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Mentor Guide Questions

1.5 Describe the physical meaning of the six components of the stress tensor and explain why it is symmetric.

1.6 Describe the physical meaning of the six components of the strain tensor, their definitions in terms of the displacement derivatives, and the difference between engineering and tensorial shear strain.

Numerical Questions

1) By inversion of the matrix below, or otherwise,

$$E\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\end{pmatrix}=\begin{pmatrix}1 & -\nu & -\nu\\ -\nu & 1 & -\nu\\ -\nu & -\nu & 1\end{pmatrix}\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\end{pmatrix}$$

find the 3D isotropic elasticity expressions for stress in terms of strain. ("Otherwise" is probably the better bet).

2) A pipe with inner diameter 200mm and outer diameter 240mm has strain gauges fixed to the outer surface in the axial and hoop directions. Calculate the expected strains for an internal pressure of 20 MPa if E = 200 GPa and Poisson's ratio is 0.3.

Also calculate the strains on the inner surface. Why is it physically obvious that the axial strains must be the same on the two surfaces? How can the hoop strains be different?

[Hint: Use the expressions for the stresses in a thick cylinder under internal pressure, e.g., from Roark or elsewhere].

<u>Plant Example</u>:-

3) A gas circulator impeller is shrunk onto its shaft by heating the impeller in a furnace before fitting it onto the shaft. The shaft has diameter 150mm. The hub of the impeller when heated has a bore which exactly matches the shaft diameter at room temperature. The OD of the impeller hub is 250mm. Assume the shaft is infinitely stiff and that the stiffness of the impeller can be approximated by that of the hub alone. Poisson's ratio is 0.3, Young's modulus is 200 GPa and the coefficient of thermal expansion is 12 x 10^{-6} /^{o}C. Show that the interface pressure developed between the impeller hub bore and the shaft is numerically equal (in MPa) to the initial impeller temperature above ambient in ^{o}C, to an accuracy of ~1% assuming elastic conditions prevail.

(Hint: the shrink gives you the hoop strain, but both hoop and radial stresses are developed – assume the axial stress is small. I suggest you use the equation for the hoop stress in a thick cylinder due to internal pressure, e.g., from Roark).

How do you think that such an impeller may be removed from the shaft for maintenance? (The complete circulator assembly cannot be put in a furnace, of course, and local heating would be tricky and potentially damaging to nearby features). Hint: The method permits the calculated interface pressure to be confirmed.