Expectations Guide: Strength of Materials (T72S01)

Last Update: 5/9/13

Beam Theory

1.1 State or derive the algebraic relationships between applied load per unit length (*w*), shearing force (*F*) and bending moment (*M*).

This is treated in http://rickbradford.co.uk/T72S01TutorialNotes2.pdf.

The Mentee should be able to write down the relations $\frac{dF}{dx} = w$ and $\frac{dM}{dx} = F$. The Mentee should know that F and M are the shearing force and bending moment and understand why these relations are dimensionally correct. The Mentee should be able to derive these expressions (since all that is required is equilibrium of forces and moments respectively).

The Mentee should be able to write down the formula for the stress at a distance y from the neutral axis in a beam with moment of inertia I and with an applied bending moment M, i.e. $\sigma(y) = \frac{My}{I}$, and hence be able to define the 'bending stress' as $\sigma_b = \frac{My_{\max}}{I}$ where $y_{\max}$ is the greatest distance of the outer fibre from the neutral axis. S/he should be able to write down what this formula becomes in the special case of a rectangular section, i.e. $\sigma_b = \frac{6M}{t^2}$.

The Mentee should appreciate that the average shear stress across the section is F/A, but also know that the peak shear stress will be larger and located (typically) near the neutral axis. S/he should know that the shear stress is necessarily zero at the free surfaces (assuming only normal loads are applied). Ideally s/he should know that the shear distribution is parabolic for rectangular sections, and appreciate that more complex distributions occur for other section shapes.

The Mentee should appreciate that beam theory is just a simple approximation to the true elasticity problem. S/he should be able to describe under what conditions the approximation is likely to be accurate. (Large aspect ratio; plane sections remain plane; do not look too closely at the details around point loads, etc).

1.2 Sketch the shearing force and bending moment diagrams for an example beam problem (given the loading and end constraints).

This is treated in http://rickbradford.co.uk/T72S01TutorialNotes2.pdf.

Essentially this means integrating the equations $\frac{dF}{dx} = w$ and $\frac{dM}{dx} = F$ for a given w and plotting F and M against x. Qualitative features to watch for are: (a)appreciation that a point load means that F is discontinuous; (b)appreciation that the moment has a local maximum/minimum where the shearing force passes through zero. A simple test of understanding is to ask the Mentee to explain why the maximum moment is greater for a beam simply supported at both ends than for a beam encastre at both ends. The Mentee should appreciate that problems with displacement boundary conditions (e.g. encastre) often cannot be solved using $\frac{dF}{dx} = P$ and $\frac{dM}{dx} = F$ alone but also require 1.3, below.

1.3 State or derive the algebraic relations between beam rotation, transverse displacement and bending moment (θ, y and M).

This is treated in http://rickbradford.co.uk/T72S01TutorialNotes3.pdf.

The Mentee should be able to write down $\frac{d\theta}{dx} = \frac{M}{EI} = \frac{1}{\rho}$ and $\frac{dy}{dx} = \theta$, where ρ is the radius of curvature of the beam at the point in question. I would expect the Mentee to be able to derive $\frac{dy}{dx} = \theta$, and to appreciate that this applies only for small displacements.

The Mentee should be able to use the complete set of beam equations, i.e., $\frac{dF}{dx} = w$, $\frac{dM}{dx} = F$, $\frac{d\theta}{dx} = \frac{Mw}{EI}$ and $\frac{dy}{dx} = \theta$ to solve any soluble beam problem. A suitable example is a cantilever subject to a linearly varying load. Many Roark cases could be taken as exercises.

1.4 Define the moment of inertia and derive its value in simple cases.

This is treated in http://rickbradford.co.uk/T72S01TutorialNotes1.pdf

The following are basics which the Mentee should know. "Moment of inertia" is the term commonly used for the second moment of area: $I = \int y^2 dA$. It is defined with respect to a given axis lying in the plane of the section, and y is the perpendicular distance of the area element dA from that axis. I is a purely geometrical quantity and has units Length^4. A bending stress varies linearly across the section. The neutral axis is therefore placed so that $\int y dA = 0$, i.e., so that a pure bending stress produces no net force on the section. The Mentee should be able to derive I for simple sections, such as,

Rectangular (depth t, width B): $I = \frac{Bt^3}{12}$; Pipe: $I = \frac{\pi}{64}\left(D_o^4 - D_i^4\right)$

(Appreciation of the perpendicular axis theorem helps with the latter).

Formulation of Continuum Elasticity

1.5 Describe the physical meaning of the six components of the stress tensor and explain why it is symmetric

This is treated in http://rickbradford.co.uk/T72S01TutorialNotes4.pdf

and http://rickbradford.co.uk/T72S01TutorialNotes5.pdf

and http://rickbradford.co.uk/T72S01TutorialNotes6.pdf.

Three direct stresses and three shear stresses. The Mentee should be able to draw a picture of a small cubic element of material labelled with the forces which act on each face in terms of these stress components. The Mentee should be able to derive the symmetry of the stress tensor, $\sigma_{xy} = \sigma_{yx}$, from the requirement for equilibrium, specifically that there be no net moment.

Appreciation of the mathematical definition of a tensor quantity is desirable but not essential. However the Mentee should at least know that in 2D the stress tensor transforms under a rotation of the coordinate system as,

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \sigma_x & \tau \\ \tau & \sigma_y \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

and hence that,

$$\sigma'_x = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau\cos\theta\sin\theta$$
$$\sigma'_y = \sigma_y \cos^2\theta + \sigma_x \sin^2\theta - 2\tau\cos\theta\sin\theta$$
$$\tau' = \tau\cos 2\theta + (\sigma_y - \sigma_x)\cos\theta\sin\theta$$

The Mentee should appreciate that this means that the coordinate system can always be rotated to give zero shear $\tau' = 0$ (the principal axes), and that the direct stresses in this special coordinate system, σ'_x and σ'_y, are the principal stresses.

It is desirable that the Mentee appreciate the generalisation of this in 3D, which is most easily understood to be a consequence of the symmetry of the stress matrix. Any symmetric matrix has orthogonal eigenvectors. The matrix formed from these eigenvectors is an orthogonal transformation which rotates to the principal axes. The eigenvalues are the principal stresses. The existence of a principal coordinate system for stresses is therefore a consequence of equilibrium.

In 2D, the Mentee should be able to derive the maximum shear stress for given principle stresses, i.e. $|(\sigma_y - \sigma_x)|/2$.

The Mentee should be able to draw a Mohr's circle construction for the state of stress at a point, in both 2D and 3D, and explain the salient points. The regions representing the possible stress components should be understood, and where the principal stresses lie on the diagram, and the maximum shear stress.

The Mentee should understand that the force vector, $\delta\overline{F}$, acting on an element of area denoted by the vector $\delta\overline{A}$ is given by $\delta F_i = \sigma_{ij}\delta A_j$ (repeated indices summed).

1.6 Describe the physical meaning of the six components of the strain tensor, their definitions in terms of the displacement derivatives, and the difference between engineering and tensorial shear strain

This is treated in http://rickbradford.co.uk/T72S01TutorialNotes4.pdf.

The Mentee would be expected to appreciate the following features of strain. The tensorial properties of the strain tensor, i.e. how it transforms under rotations, are as per the stress tensor. Given that it is symmetric, the existence of principal axes of strain also follows. The definition of the strain tensor is $\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$. Hence the strain tensor is symmetric by definition. However, the reason why it is defined as symmetric is that the anti-symmetric components would be $\varepsilon_{ij} = \frac{1}{2}(u_{i,j} - u_{j,i})$ and, for small displacements, these represent rotations. A rotation of a small element of volume does not represent a deformation of that element, and hence would not be expected to correspond to a stress.

The factor of ½ in the definition is required so that the direct strains are just $u_{i,i}$ and hence conform to the usual definition as fractional change of length. To be a true tensor, this forces the tensorial shear strains to be $\frac{1}{2}(u_{i,j} + u_{j,i})$. Unfortunately, engineers historically defined shear strain as the angle of shear. This leads to a factor of 2 difference between the two definitions: $\gamma_{xy} = 2\varepsilon_{xy}$, etc.

1.7 State or derive the algebraic relationships between stress and strain for a linear elastic material, the definition of the elastic moduli and the relationship between the three elastic moduli for an isotropic material.

This is treated in http://rickbradford.co.uk/T72S01TutorialNotes4.pdf.

In general the relationship between stress and strain for a linear elastic material will be $\varepsilon_{ij} = C_{ijkl}\sigma_{kl}$, where this caters for the most general anisotropic medium. For an isotropic material the relations are,

$$E\begin{pmatrix}\varepsilon_x \\ \varepsilon_y \\ \varepsilon_z\end{pmatrix} = \begin{pmatrix}1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1\end{pmatrix}\begin{pmatrix}\sigma_x \\ \sigma_y \\ \sigma_z\end{pmatrix}$$

together with,

$$G\begin{pmatrix}\gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy}\end{pmatrix} = \begin{pmatrix}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\tau_{yz} \\ \tau_{zx} \\ \tau_{xy}\end{pmatrix}$$

where the latter gives the engineering shear strains, and the shear modulus is given in terms of Young's modulus and Poisson's ratio by $G = \frac{E}{2(1+\nu)}$. The Mentee should be able to derive this last expression (by starting with pure shear and then rotating to principal axes).

The Mentee should understand the interpretation of Poisson's ratio (ν) in terms of the contraction perpendicular to an applied stress. The Mentee should be able to derive the change of volume for a given state of strain (volumetric strain $\varepsilon_x + \varepsilon_y + \varepsilon_z$), and hence to derive the value of Poisson's ratio corresponding to incompressibility ($\nu = ½$).

From the above relations it follows that the principal axes for stress are also the principal axes for strain for isotropic materials. Is the same necessarily true for anisotropic materials?

The Mentee should understand that the strains defined as above are appropriate only when the strains are all <<1.

1.8 Define what is meant by: the principal stresses, the hydrostatic stress, the deviatoric stresses, the Tresca equivalent stress and the Mises equivalent stress.
This is treated in http://rickbradford.co.uk/T72S01TutorialNotes5.pdf.

The Mentee should be able to define these stresses as given here. Principal stresses and strains have been discussed above. They are the direct (on-diagonal) components when the stress or strain tensor is rotated to the principal axes in which the shear (off-diagonal) components are zero. This is always possible by virtue of the stress and strain tensors being symmetric.

Equivalent stresses are useful as a scalar with which to measure the proximity to yielding. They are required to reduce to the direct stress in the case of simple uniaxial stressing. The Tresca stress is defined as twice the maximum shear stress, and hence equals $|\sigma_1 - \sigma_3|$, where σ_1 and σ_3 are the algebraically largest and smallest principal stresses respectively. The von Mises stress is given in terms of the principal stresses by,

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

The concept of deviatoric stress is useful because the hydrostatic stress, defined by $\sigma_H = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z)$, does not usually contribute to plastic flow. Hence, the deviatoric stress tensor has the hydrostatic part subtracted off: $\hat{\sigma}_{ij} = \sigma_{ij} - \sigma_H \delta_{ij}$. The Mises stress is rationalised because it is, up to normalisation, the scalar formed from the deviatoric stress, i.e. $\bar{\sigma} = \left[\frac{3}{2}\hat{\sigma}_{ij}\hat{\sigma}_{ij}\right]^{1/2}$, where the factor of 3/2 is chosen so that the Mises stress reduces to the direct stress in the uniaxial case.

1.9 State or derive the algebraic expressions obeyed by the stresses to ensure equilibrium.
This is treated in http://rickbradford.co.uk/T72S01TutorialNotes6.pdf.

What is required is simply $\sigma_{ij,j} = -b_i$. The Mentee should be able to write this down. S/he should understand the notation and the physical meaning of the equation(s). It is reasonable to expect the Mentee to be able to derive the equation since only the static equilibrium of forces is required.

The Mentee should understand which three components of stress are necessarily zero at a free boundary, and which three components may not be.

1.10 Explain the role of displacement compatibility in imposing constraints on the strain fields, and hence state or derive a soluble system of linear elastic differential equations for the general case.

This is treated in http://rickbradford.co.uk/T72S01TutorialNotes6.pdf

and http://rickbradford.co.uk/T72S01TutorialNotes7.pdf.

This is a big question because it is really about setting up the complete system of equations for linear elasticity. This is the subject of three separate Notes on this web site,
http://rickbradford.co.uk/Elasticity3D.pdf
http://rickbradford.co.uk/Elasticity2D.pdf
http://rickbradford.co.uk/CompatibilityinPolars.pdf

However these Notes go beyond what a Mentee would necessarily be expected to know (especially the last!). The Mentee should be able to demonstrate having had some exposure to this mathematics, but it would be unreasonable to expect all of it to be reproducible cold. The issue here is understanding rather than memory. However, there are a few things the Mentee might be expected to recall, such as: (a)2D problems can be formulated neatly in terms of an Airy function which obeys the biharmonic equation, $\nabla^4\varphi = 0$; (b)3D problems can be formulated as second order differential equations in the displacements; (c)formulation of the elasticity problem without reference to displacements is possible but only if the compatibility equations are introduced.

Elementary Plasticity Theory and Collapse

1.11 Define what is meant by "plasticity" and describe a typical stress-strain curve for a metallic, elastic-plastic material, including defining the proof stress and the ultimate tensile strength. Explain the difference between engineering stress and strain and true stress and strain.

This is treated in http://rickbradford.co.uk/T72S01TutorialNotes8a.pdf.

Plasticity is the ability of a material to deform permanently without fracturing. For a stress analyst the defining characteristic of plasticity is that it is irreversible. Any Mentee who believes that the defining characteristic of plasticity is a non-linear stress-strain curve should (following the Zen masters) be given a sharp crack over the head. Elasticity is a material's ability to recover its original shape on unloading. A non-linear stress-strain curve is no barrier to reversibility. Materials like rubber have a grossly non-linear stress-strain curve whilst still being elastic. Thus, a plastic material stressed beyond some yield stress will unload "down the elastic line". The Mentee should be able to draw this on a stress-strain plot.

The Mentee should understand the mechanism of plastic flow in metals (dislocation glide). S/he should preferably have some familiarity with the huge disparity between theoretical material strength, based on individual atomic bond strengths, and the far lower flow stresses which pertain in practice – and how this is explained by dislocation movement.

The Mentee should be aware that proof stresses can be defined at any strain level, but that 0.2% is the most common, although 1% is also frequently used, especially for austenitic steels. The Mentee should know that this is the *plastic* strain (only), i.e. the permanent strain remaining on removal of load. The Mentee should be able to state the definition of UTS, as the maximum load sustainable by a standard uniaxial tensile test specimen divided by its original cross section. S/he should know that the load-displacement trace for ductile materials continues, on a downward trajectory, beyond the UTS. The Mentee should appreciate that the maximum load, which defines the UTS, occurs as the result of a necking instability. The UTS is therefore a specimen-geometry dependent quantity, rather than a material property (despite popular misconception).

The Mentee should be able to state the definition of true stress (using the instantaneous cross section), as opposed to engineering stress (using the original cross section). The Mentee should be able to describe qualitatively how the true stress varies along the length of a tensile specimen loaded to the UTS. The Mentee should be able to state the definition of true (uniaxial) strain, and also to derive it by integration. The Mentee should be able to draw and contrast graphs of engineering stress-strain and true stress-strain. The Mentee should appreciate that neither engineering nor true strain give a good measure of the state of strain in the neck of a specimen near UTS. S/he should appreciate that reduction of area is a better indication of the local strain in the neck. The Mentee should understand that 'ductility' is quantified by an associated strain measure. Strain at failure, or possibly strain at maximum load, are possibilities, but reduction of area is arguably the best measure for the above reasons. The Mentee should be able to draw the tangent construction for the identification of the UTS on a true-stress v true-strain graph.

The Mentee should appreciate the qualitative differences in yield behaviour between austenitic steels and ferritic steels, the latter often having a distinct yield stress, in contrast to the former. The Mentee should appreciate that some steels, e.g. some CMn steels, exhibit a large plateau on their stress-strain curve at the yield point, with a sudden large yield strain (Luders strain).

1.12 Define typical yield criteria for isotropic metals in a general 3D state of stress.
This is treated in http://rickbradford.co.uk/T72S01TutorialNotes8a.pdf

and http://rickbradford.co.uk/T72S01TutorialNotes8b.pdf

and http://rickbradford.co.uk/T72S01TutorialNotes8c.pdf.

First yielding occurs when the equivalent stress reaches some yield stress. The appropriate definition of equivalent stress depends upon the material. Either the Tresca stress or the Mises stress are generally regarded as the most appropriate for isotropic steels. The Tresca criterion is equivalent to assuming that plastic flow is controlled by the maximum shear stress. However, the Mises criterion is generally regarded as the more accurate for typical structural steels.

Given the uniaxial tensile yield stress (σ_y), the Mentee should be able to derive the shear yield stress, (a)according to the Tresce criterion ($\tau_y = \sigma_y / 2$), and, (b)according to the Mises criterion ($\tau_y = \sigma_y / \sqrt{3}$), and hence appreciate that they differ by ~15%.

For simple example cases of multi-axial stressing, the Mentee should realise which is the more likely to yield (i.e. to have the larger equivalent stress). An example might be a pipe under internal pressure and global bending.

1.13 Describe qualitatively the hardening behaviour of metals, the behaviour under load reversal, and the special cases of kinematic and isotropic hardening.
This is treated in http://rickbradford.co.uk/T72S01TutorialNotes8c.pdf

This is treated in http://rickbradford.co.uk/T72S01TutorialNotes8d.pdf.

The Mentee should know that, for continued monotonic loading beyond first yield, the equivalent stress-equivalent strain trajectory follows the uniaxial stress-strain curve. The Mentee should appreciate that the true stress-true strain curve may be most representative, but also understand its limitations. Thus, for example, the post-yield stress-strain behaviour in shear follows from the tensile behaviour for a given flow rule.

The Mentee should know that, on unloading, the stress at a point will tend to unload "down the elastic line". (The more perceptive will also realise that, for an arbitrary point in an arbitrary structure, this is really just a rule-of-thumb and exact unloading trajectories may differ, especially if reverse yielding takes place somewhere).

Given a simple enough example, the Mentee should be able to judge what residual stress is left at a given point in a structure after unloading. With Rev.001 of the Mentor Guide question 1.19 was added which requires the Mentee to be familiar with the derivation of the residual stress for the bent bar problem (see below). There is a tendency for people to believe that the stress at all points returns to zero on unloading, possibly because this is how a uniaxial tensile specimen behaves (pre-necking). Examples which might be used to explore residual stresses are the bent bar or a notch. The Mentee should understand that residual stresses are necessarily self-equilibrating.

The Mentee should know that, on re-loading, plastic straining recommences only when the previous highest stress is exceeded (i.e., the material hardens).

The Mentee should understand that differing behaviours are possible under load reversal. S/he should demonstrate understanding of the extreme cases: (a)reverse yielding occurs when the stress has been reduced by $2\sigma_y$ from its previous maximum (kinematic hardening); and, (b)reverse yielding occurs only when the compressive stress reaches (minus) the previous maximum stress (isotropic hardening). The Mentee should be able to generalise these extreme behaviours to an arbitrary stress trajectory. S/he should understand the concept of a "yield surface" in stress space, and that hardening in general involves a change of the size, shape and position of this yield surface as plasticity occurs. The Mentee should know that kinematic hardening involves the movement of a yield surface of unchanging size and shape. The Mentee should know that isotropic hardening involves the dilation of a yield surface of unchanging shape and fixed centre. The Mentee should know that the Mises yield surface is circular, and that the Tresca yield surface is a hexagon. S/he should know at what points the Mises and Tresca surfaces meet and at what points they are most different.

The Mentee should understand that the 'direction' of the plastic strain increment is defined by the shape of the yield surface, specifically that the direction is normal to the yield surface at the relevant stress. The Mentee should preferably be able to write down the Mises flow rule means that the 'direction' of the strain increment is $\delta\varepsilon_{ij}^{p} \propto \hat{\sigma}_{ij}$.

Armed with the above understanding, the Mentee should be able to construct simple stress-strain hysteresis loops qualitatively.

1.14 Describe qualitatively the influence of plastic yielding on the stress distribution in simple example cases, (i) for primary loads, (ii) for secondary loads.

This is treated in http://rickbradford.co.uk/T72S01TutorialNotes9.pdf.

In both cases the areas with largest equivalent stress will yield first and tend to shed load onto surrounding material. The key difference between primary and secondary loads is that the load resultants (F and M) do not decrease during yielding for primary loads. For secondary loads the load resultants generally reduce as a consequence of yielding. The Mentee should appreciate that real loads are frequently intermediate in character, often being "secondary-with-elastic-follow-up". The Mentee should therefore appreciate qualitatively the nature of elastic follow-up, and preferably be able to define a follow-up factor, Z. One possible example to examine is a beam, encastre at both ends, and subject to a point load in the middle. Is the moment at the end primary or secondary? Does it increase, decrease or remain unchanged during yielding? Is the moment at the centre primary or secondary? Does it increase, decrease or remain unchanged during yielding? [The yielding at the ends tends to increase the moment at the centre, because the beam moves closer to being simply supported].

The distinction between primary and secondary loads comes about because strains can affect the loads. Another example of strains causing changes to loads or stresses occurs if the strains or displacements are large enough. Thus, strains may reduce a cross-sectional area and hence increase the stress for a given load resultant. Geometry changes can also influence load resultants. Examples are: an out-of-round pipe will initially suffer large wall-bending stresses under internal pressurisation. But further increases in wall bending will cease once the pipe has become circular. An adverse effect of displacement occurs for a cantilever which is initially tilted upwards. Yielding at the built-in end causes the moment arm of the load to increase, potentially exacerbating the deformation.

1.15 Describe qualitatively why a structure under primary loading has an ultimate (plastic collapse) load.

This is treated in http://rickbradford.co.uk/T72S01TutorialNotes9.pdf

and http://rickbradford.co.uk/T72S01TutorialNotes10.pdf.

It follows from the definition in 1.14 that the load resultants do not reduce during yielding for a primary load. If the load is increased sufficiently it follows that the stress must become intolerable eventually.

1.16 State the lower bound theorem for the plastic limit load.

This is treated in http://rickbradford.co.uk/T72S01TutorialNotes9.pdf.

The Lower Bound Theorem applies for an idealised elastic-perfectly-plastic material with infinite ductility. It states, "A stress distribution such that every part of the structure is in equilibrium and such that the equivalent stress nowhere exceeds the yield stress will be in equilibrium with loads which do not exceed the plastic collapse loads". A proof of the lower bound theorem is provided in http://rickbradford.co.uk/BoundTheorems.pdf.

A corollary of the lower bound theorem is that the plastic collapse load of a structure cannot be reduced by adding more material, nor improved by removing material. This is in contrast to the situation for other failure modes. For example, a structure can be made more resistant to fracture by drilling a "crack stopper" hole at the tip of a crack or sharp corner, i.e. by removing material.

The Mentee should be aware of the important status of the lower bound theorem, i.e. that it underpins many of the collapse/reference stress solutions used in R6 and R5 assessments. The Mentee should also be aware of the principle weakness of the lower bound theorem when applied to structural steels, namely that these materials are not perfectly plastic but exhibit large amounts of strain hardening. The Mentee should be aware of the pragmatic attitude towards this difficulty adopted by assessment procedures (i.e. the use of a relevant proof stress or flow stress as an effective yield strength according to circumstance and interpretation).

The Mentee should ideally be aware also of the upper bound theorem. This is discussed (and proved) in http://rickbradford.co.uk/BoundTheorems.pdf

1.17 Define what is meant by "reference stress".
This is treated in http://rickbradford.co.uk/T72S01TutorialNotes9.pdf.

The Mentee should have an intuitive appreciation for the reference stress as (extremely crudely expressed) a sort of average stress across the weakest section. The Mentee should be able to state the definition of the reference stress as $\sigma_{ref} = \frac{P}{P_{collapse}}\sigma_{yield}$, where σ_{yield} is to be understood in the sense discussed in 1.16, and "P" symbolises whatever primary loads are applied. The Mentee should understand that only primary loads contribute to the reference stress. Hence, the ratio $\sigma_{ref} / \sigma_{yield}$ expresses the proximity to plastic collapse.

1.18 Derive the reference stress for a simple example case, e.g. a beam or pipe in bending.
This is treated in http://rickbradford.co.uk/T72S01TutorialNotes9.pdf.

and http://rickbradford.co.uk/T72S01TutorialNotes10.pdf.

The Mentee should be able to derive the famous factor of 3/2 by which the collapse moment of a rectangular section in bending exceeds the moment at first yield of the outer fibre. The Mentee should be able to derive the factor of 4/π by which the collapse moment of a thin-walled pipe in global bending exceeds the moment at first yield of the outer fibre. The Mentee should be able to derive the reference stress formula for a thick internally pressurised pipe. The Mentee should also be able to derive the interaction curve on an (F,M) diagram for a rectangular section subject to both a membrane and a bending load. Notes on these examples are available in http://rickbradford.co.uk/BoundTheorems.pdf. Further exercises on multiple load cases may be beneficial. A pipe under pressure, bending and end load may be suitable (see Bob Ainsworth's E/REP/GEN/0027/00).

1.19 Explain what is meant by "residual stress" and give examples of how residual stresses can arise. Derive the residual stress in a bar of rectangular section made of elastic-perfectly-plastic material and subject to an applied moment of collapse magnitude which is then removed.

This is treated in http://rickbradford.co.uk/T72S01TutorialNotes12B.pdf including the derivation of the residual stress distribution for the classic 'bent bar' problem.

Residual stresses are the stresses left behind in a structure when the original source of loading has been removed but stresses remain as a result of the yield stress having been exceeded when under load. Residual stresses occur only because plasticity is irreversible. Since residual stresses apply when the load has been removed (by definition) they are self-equilibrating. In this context, a self-equilibrating stress distribution is one which obeys $\sigma_{ij,j} = 0$ at every point within the structure, including on its surface. This does NOT necessarily mean that the load resultants over a section, N and M, are necessarily zero. Commonly they will not be. Any loading which causes yielding will, in general, leave behind a distribution of residual stresses when unloaded. The only exception I can think of is a uniaxial bar in uniform, homogeneous tension (and even then only if it does not neck). In particular residual stresses do not only arise from welding. Nor are residual stresses necessarily deleterious. Then can make the structure more load or defect tolerant. Examples of beneficial residual stress effects include, Warm pre-stressing; Proof testing; Peening; Weld overlay techniques.

1.20 Explain what is meant by "thermal stress" and give examples of how thermal stresses can arise. Derive the thermal stress in a flat plate restrained from bending and subject to a linear temperature difference through the thickness.

This is treated in http://rickbradford.co.uk/T72S01TutorialNotes12.pdf

1.21 Failure Mechanisms

Examples of failures against likely mechanisms can be found in http://rickbradford.co.uk/T72S01TutorialHomework12C.pdf.