Thick Pipe under Internal Pressure: Steady State Creep Stress Distributions

RAWB, 25/10/10

The elastic Lame solution gives,

$$\sigma_r = -P\frac{(b/r)^2 - 1}{(b/a)^2 - 1} \tag{1}$$

$$\sigma_h = P\frac{(b/r)^2 + 1}{(b/a)^2 - 1} \tag{2}$$

$$\sigma_a = P\frac{1}{(b/a)^2 - 1} \tag{3}$$

It can be shown that assuming the Tresca condition, that is, assuming that the Tresca creep strain rate, $\dot{\varepsilon}_T$, is given in terms of the Tresca stress, σ_T, by $\dot{\varepsilon}_T = B\sigma_T^n$, the stress distributions in steady state creep are,

$$\sigma_r = -P\frac{(b/r)^{2/n} - 1}{(b/a)^{2/n} - 1} \tag{4}$$

$$\sigma_h = P\frac{\left(\frac{2}{n} - 1\right)(b/r)^{2/n} + 1}{(b/a)^{2/n} - 1} \tag{5}$$

$$\sigma_a = P\frac{\left(\frac{1}{n} - 1\right)(b/r)^{2/n} + 1}{(b/a)^{2/n} - 1} \tag{6}$$

These expressions apply after a sufficiently long time so that transient creep has subsided and stresses have become constant.

The Tresca stress is obtained by subtracting (4) from (5) giving,

$$\sigma_T = P\frac{\frac{2}{n}(b/r)^{2/n}}{(b/a)^{2/n} - 1} \tag{7}$$

The limit $n \to \infty$ can be taken noting that, $LIM_{y\to 0}\left(x^y\right) = 1 + y\log x + O\left(y^2\right)$, so that,

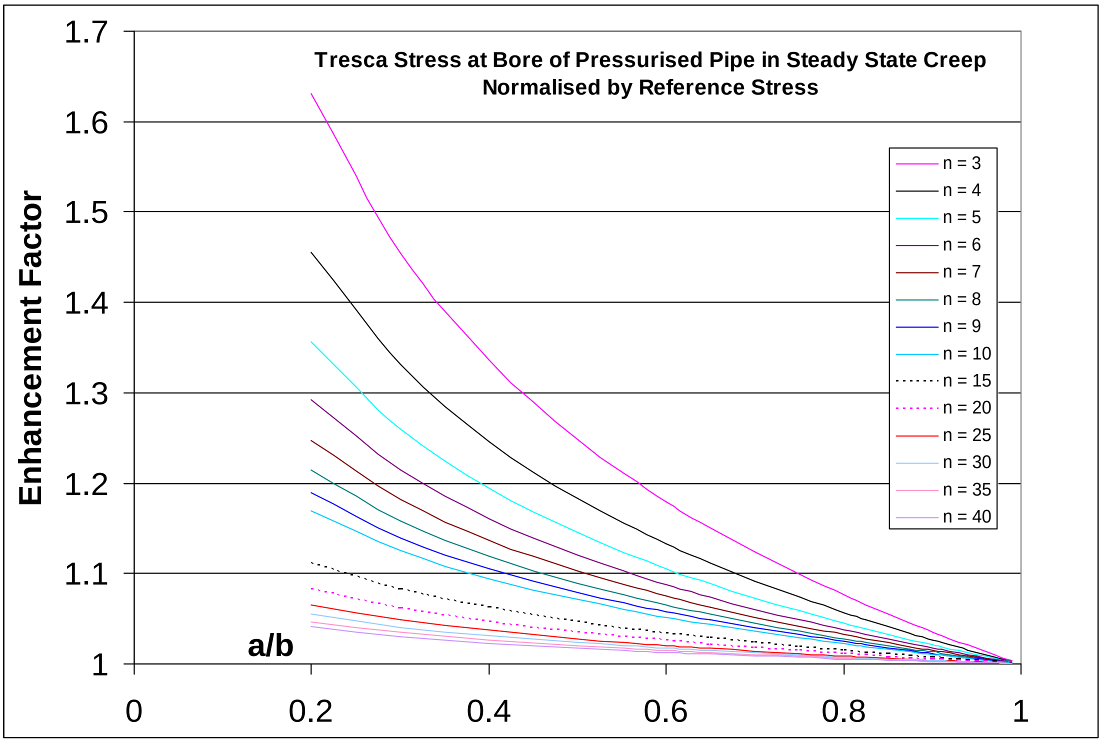
$$LIM_{n\to\infty}(\sigma_T) = P\frac{\frac{2}{n}\times 1}{\left(1 + \frac{2}{n}\log\frac{b}{a}\right) - 1} = \frac{P}{\log\frac{b}{a}} \tag{8}$$

So, in this limit, the Tresca stress is uniform across the section (i.e., independent of *r*) and equals the usual Tresca reference stress solution. **<u>QED</u>**.

However, this does not establish that no χ-based stress concentration enhancement is required to derive a rupture reference stress for *realistic* values of n. From (7) the Tresca stress is greatest at the bore when n is finite. Its value normalised by the usual reference stress, i.e., (7) divided by (8), is,

$$\frac{\sigma_T(ID)}{\sigma_T(n \to \infty)} = \frac{\frac{2}{n}\log\frac{b}{a}}{1-(a/b)^{2/n}} \qquad (9)$$

This is plotted for n values from 3 to 40 and for a/b values from 0.2 to 0.99 in the graph below,



Realistic n values are typically around 4 to 9, so the above graph implies that the usual reference stress is not valid without an enhancement factor. Aargh!

What may save us is that our CMV plant generally has a/b > 0.6. Confining attention to this regime, and assuming that n ≥ 4, the enhancement factor does not exceed 1.133. Consequently if we appeal to Mises, which would reduce the reference stress by a factor of $\sqrt{3}/2$ = 0.866, the usual methodology comes out about right. That's lucky!

Nevertheless, the above observation surprises me greatly. And <u>the usual methodology would not be right for either very thick cylinders or fairly thick cylinders in creep brittle materials (small n)</u>.