The Physical Limits of Information and Computation

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Given a finite amount of mass-energy, what is the maximum amount of information that can be represented by it? What is the greatest number of elementary computational steps that can be carried out? These questions are answered below. It turns out that the answers depend upon either how much time you have available or upon how much space you have available.

(A) Maximum Number of Computations

An elementary computation is a single logic gate such as AND, NOT, OR or XOR, etc. One means of ensuring that the number of elementary computations is uniquely defined is to reduce a computation to a sequence of Toffoli (or Fredkin) gates. Any computation can be carried out using only these gates, and hence must have a unique minimum. Consider the following statements,

- In principle, a computation can be carried out with no expenditure of energy.
- The rate at which computations can be carried out is limited by the available energy.

(Throughout, both ‘mass’ and ‘energy’ can be read interchangeably as ‘mass-energy’).

Both are true. That they are not contradictory is due to the second statement referring to the computational *rate*, and the first referring to the *expenditure* of energy. The latter is distinct from energy being involved in a conservative manner. It is trivially obvious that a computation cannot be performed unless some energy is available. This is because a computation is a physical process. Any physical process must involve some ‘stuff’ upon which the process is being carried out. According to the physical view of the world, this ‘stuff’ equates to mass-energy.

The first statement means that computation can be carried out, in principle, in a manner which does not ***dissipate*** any of the energy involved. That this is true derives from the observation that computation can be carried out in a reversible manner (Bennett, 1979 & 1982). Indeed, the meaning of ‘reversible’ in thermodynamics is ‘without energy dissipation’. The content of Bennett’s argument is essentially that logical reversibility implies the existence of a process which is also thermodynamically reversible.

This suggests that it should be possible, in principle, to solve the quantum-chromodynamic equations for the mass spectrum of the hadrons using just a handful of electrons and a few eV of energy. (Construction of a working device is left as an exercise for the reader). The snag is that, as well as being stupendously ingenious, you would have to wait a very long time for the answer. This is where the second statement comes in.

It is rather obvious that we will be able to compute faster with more mass-energy available – simply because we will now have enough material at our disposal to make a proper computer! However, for a given mass of material, how fast can we compute? The answer lies in recognising that the fundamental nature of a computation is to change 0 into 1 or vice-versa. Our computer must manifest these binary states physically. Thus, to compute quickly, we need to be able to change the binary physical states of the system as fast as possible. Clearly, this will be facilitated by making the physical states which represent 0 and 1 very similar – so there is less changing to be done. But there is a limit to how similar two physical states can be whilst remaining distinguishable. This is the quantum limit. To be distinguishable, the states must be orthogonal quantum states. Our problem therefore becomes, "how quickly can a quantum state be changed into an orthogonal quantum state?" The answer is provided by the uncertainty principle,

$$\Delta t \geq \pi\hbar / 2\Delta E \qquad (1a)$$

This defines the minimum time required to change from a state whose uncertainty in energy is ΔE to an orthogonal state. [NB: A state with precisely defined energy, E, can never undergo change, since its time dependence is just a phase factor $e^{iEt/\hbar}$ with no overlap to any other state].

The uncertainty in the energy of the state cannot exceed the expectation value of its energy, E, since energy cannot be negative. Consequently, we can reduce the computation time given by (1a) to a minimum by taking $\Delta E = E$, and thus,

$$\Delta t_{Min} = \pi\hbar / 2E \qquad (1b)$$

These arguments follow Lloyd, 2000 and 2002. Since (1b) is the minimum time for an elementary computation, the maximum computation rate is,

$$\text{Maximum Computation Rate} = \frac{2E}{\pi\hbar} \qquad (1c)$$

By summation over its parts, the same relation must also hold for the maximum computation rate of a macroscopic system in terms of its total energy.

Minimum Power Requirement for an Irreversible (Dissipative) Computer

The power requirement for a *reversible* computer of any given speed is zero. However, we assume instead that our computer is *irreversible* but dissipates energy at the minimum rate consistent with its speed. Equ.(1c) says that the minimum energy that must be available if the computer is to calculate at a speed C bits per second is $E = \pi\hbar C / 2$. We are assuming that this amount of energy, i.e. the minimum required to support the stated computing speed, is dissipated at each computational step. But each such step takes a period of time $\tau = 1/C$. Hence, the power requirement is $P = E/\tau = \pi\hbar C / 2\tau = \pi\hbar C^2 / 2$. Hence, the power requirement of such a computer is proportional to the computing speed squared. Unless we have taken special precautions to build a reversible computer, in practice this is likely to be the minimum power requirement of a dissipative computer.

The demand is modest. A modern home computer runs at a few GHz, which may be crudely interpreted as the computing speed. Hence, C^2 is ~ 10^{19} per second^2. Since Planck's constant is of order 10^{-34} Js, the minimum power requirement is ~10^{-15} W. I guess the power of a standard CPU chip is in the order of tens of W, so there is room for improvement by some 16 orders of magnitude before the ultimate quantum limit is reached. Of course, that could be improved further, and without limit, by going reversible.

Computational Capacity of the Universe

Using (1c) it is simple to calculate the maximum number of computations that can have been carried out within the observable universe since the Big Bang. The age of the universe is 13.7 Byrs = 4.3 x 10^{17} s. Including dark matter and dark energy as well as ordinary (baryonic) matter, the mean density of the universe is within ~1% of flatness (Spergel 2006). So the mean density is the critical density, $\frac{3}{8\pi G t^2}$ = 9.6 x 10^{-27} kg/m^3 = 5.7 H atoms/m^3. Multiplying by c^2 gives an energy density of 8.6 x 10^{-10} J/m^3. Hence, the upper limit to the physically possible computational rate per unit volume, using (1c), is 5.2 x 10^{24}s^{-1}m^{-3}. A pretty impressive figure for just 5.7 hydrogen atoms!

To find the total number of computations over the whole observable universe requires the (observable) universe's volume. This is $V = \frac{4}{3}\pi R^3$ where R ~ 3.5ct ~ 4.5 x 10^{26} m, hence V = 4 x 10^{80} m^3. So, the rate of computation by the whole observable universe is 2 x 10^{105} s^{-1}.

[NB: The reason why the radius of the universe is ~3.5ct rather than just ct involves two different subtleties. A star on the horizon has been moving at speed c away from here for a time t, accounting for a distance ct. During that period, space has also been expanding. In a flat space with no dark energy this would account for another 2ct, bringing the total to (exactly) 3ct. However, dark energy appears to be causing an acceleration in the universe's rate of expansion. Based on observational evidence (Spergel 2006), the amount of dark energy can be shown to give rise to a further expansion of space of about ~0.5ct, bringing the total to ~3.5ct.]

Multiplying the current computational rate of the universe by the age of the universe yields an upper bound for the total number of computations that can have been carried out over the life of the universe, i.e. ~0.8 x 10^{123}.

This is an upper bound for the number of computations that can have been carried out by the mass-energy which is observable in the present epoch. This is not the same as the number of computations that could have observable consequences here and now. The latter is a smaller number. The reason is that only a fraction of the observable universe was previously in causal contact with us. At earlier times, the causally connected universe comprised smaller amounts of mass-energy, and hence would have had a smaller computational capacity. For material now appearing over the horizon, we can see the results of its computations carried out shortly after the Big Bang, but not more recent computations. A more restrictive bound on the causally relevant computational capacity of the universe can be found by integration over the varying size of the universe. Thus, the computational rate at time t is found to be,

$$\text{Universe's Computational Rate} = \frac{f^3 c^5}{\pi \hbar G} t \qquad (2a)$$

Integrating over time gives the total number of computations by time t as,

$$\text{Universe's Total Computations} = \frac{f^3 c^5}{2\pi \hbar G} t^2 \qquad (2b)$$

where f ~3.5 is the factor by which ct is multiplied to get the radius of the universe, R (see discussion above). This integration makes a difference of a factor of two only, the number of computations causally connected to us being 0.4 x 10^{123}.

Ordinary (baryonic) matter comprises just 4.2% of the universe's density (Spergel 2006). The rest is dark matter (~20%) and dark energy (~76%). Hence, the computational capacity of the ordinary, baryonic matter alone is ~3.3 x 10^{121}.

Finally, we note that there is also a bound based purely on space-time limitations, without reference to the actual mass-energy content of the universe. This is derived as follows: One basis for quantum states is to use spatial localisation. States are distinguished because they are at different places. The limit to spatial resolution is provided by the Planck length,

$$L_P = \sqrt{\frac{\hbar G}{c^3}} = 1.6 \text{x} 10^{-35} \text{m} \qquad (3a)$$

Similarly, the limit to temporal resolution is provided by the Planck time,

$$t_P = \sqrt{\frac{\hbar G}{c^5}} = 5.4 \text{x} 10^{-44} \text{s} \qquad (3b)$$

These imply that the maximum computational rate density would be achieved if one bit of information in every volume L_P^3 were changing every t_P seconds. The computational capacity of the universe would then be,

$$\frac{4\pi}{3}\left(\frac{R}{L_P}\right)^3\left(\frac{t}{t_P}\right) = \frac{4\pi}{3}\left(\frac{4.5 \text{x} 10^{26}}{L_P}\right)^3\left(\frac{4.3 \text{x} 10^{17}}{t_P}\right) = 0.74 \text{x} 10^{246} \qquad (4a)$$

This is vastly greater than the true bound, 1.25 x 10^{123}, based on the mass-energy content of the universe [in fact it equals its square, to a good approximation – see (5)]. Thus, it is the mass-energy content of the universe which limits its computational capacity, not its size. This is readily apparent from the very low mean density of the universe.

Speculations Regarding Computational Restrictions on the Early Universe
The space-time limited number of computations, written algebraically, is,

$$\text{Universe's Total Computations} = \frac{4\pi f^3 c^{10}}{3\hbar^2 G^2} t^4 \qquad (4b)$$

Dividing the square-root of this by (2b) gives a constant,

$$\frac{\sqrt{\text{Computations(space - time)}}}{\text{Computations(mass - energy)}} = \frac{4\pi}{f}\sqrt{\frac{\pi}{3f}} \approx 2 \qquad (5)$$

Where we have used f ~ 3.5 to derive the value.

Note that the space-time limit for the universe's computational capacity, (4b), varies as t^4 whereas the mass-energy limit, (2b) varies as t^2. This suggests that we can find a time at which the two limits become the same. This time is,

$$t = \frac{1}{\pi}\sqrt{\frac{3}{8}} t_P \approx 0.2 t_P \qquad (6)$$

Since the earliest time with any physical meaning is $\sim t_P$, Equ.(6) implies that the space-time limit on the computational capacity of the universe is *always* more generous than the mass-energy based limit – even at the first instant after the Big Bang (zero plus t_P). This is not a trivial observation in that the divergent energy density at the Big Bang would lead to the mass-energy bound on the computational rate density to be divergently fast. If we ignored the (probable) physical limitation on times less than t_P, we would conclude that space-time would be unable to support the maximum computation rate which could be delivered by the energy content of the universe at times before $\sim 10^{-44}$ s. This would be rather paradoxical. We are saved from this conundrum by virtue of the coefficient in (6) being < 1. This result depends upon the assumption that the universe's mean density is the critical density, $\frac{3}{8\pi G t^2}$. If we assumed a mean density a factor X larger, then (6) would become,

$$t = \frac{1}{\pi}\sqrt{\frac{3X}{8}} t_P \approx 0.2\sqrt{X} t_P \qquad (6b)$$

This could be used as an argument for why the universe's mean density could not exceed X = 25 times the critical density. Spacetime would be unable to support the computational needs of the universe's energy inventory at the earliest epochs in such a universe. Of course, such universes would long since have collapsed.

(B) Maximum Amount of Information

B.1 Black Bodies

We now return to the question of how much information can be 'written' using a given finite amount of mass-energy. This is equivalent to asking what the maximum entropy of a given amount of mass-energy can be. Now the entropy of a gas of particles in thermodynamic equilibrium is essentially just the number of particles (providing that it is not so dense as to be nearing the liquid or solid state). Strictly, the number of particles must be multiplied by a number of order unity. For example, the entropy of a volume V of black body radiation (photons) is simply,

$$S/k = 3.6nV = 3.6N \qquad (7a)$$

where N is the number of photons in the volume V and n = N/V is the number of photons per unit volume, which is,

$$n = 0.2436\left(\frac{kT}{\hbar c}\right)^3 \qquad (7b)$$

For a monotonic ideal gas, the Sackur-Tetrode equation gives the entropy as,

$$S/k = N\left\{\frac{5}{2} + \log\left[\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2}\frac{V}{N}\right]\right\} \qquad (8)$$

where N is the number of particles of mass m in volume V, and S is their entropy. Note that the *lower* the particle density, the *larger* is the second term in the {....}. This is because the greater amount of room available per particle leads to a greater number of accessible quantum states. For example, if we consider the mean number density of protons in the universe ($0.24/m^3$), and assume the temperature of the CMB, i.e. 2.7 K, then the second term in {…} is 63.4, and so the dimensionless entropy is 65.9N. On the other hand if we consider hydrogen gas at STP, for which the molecular number density is 2.7×10^{25} m^{-3}, the second term in {…} is 11.5 and hence the entropy is 14.0N. Consequently, even this extreme difference in conditions only makes a difference in the entropy per particle of around a factor of 5.

So, how can we contrive to maximize the entropy for a given amount of mass-energy? It is clear from the above discussion that we need simply to maximize the number of particles. Subject to conservation of mass-energy this obviously means producing more particles of smaller mass. Since photons have zero rest mass, we need to convert everything to photons (or possibly to gravitons, these being the only other quanta with zero rest mass). The number of photons we get depends on their energy.

A spurious argument goes like this: We need very low energy photons to maximize their number. If conservation of mass-energy were the only limitation then we would conclude that there is no finite upper bound to the amount of information that may be represented by a finite amount of energy. We merely have to convert all our mass-energy to photons of sufficiently low energy. Thus, the amount of information obtainable from a mass M using photons of energy E_γ would be,

$$S/k = 3.6\frac{Mc^2}{E_\gamma} \tag{9}$$

and this is unbounded as $E_\gamma \to 0$. However, this is an incorrect argument because the entropy of the photons is only given by (7a) if we assume a black body spectrum of photon energies. In fact, for a given energy density, the black body spectrum of photon energies is exactly that which maximises the entropy, and hence is the answer to our question.

For a given amount of mass-energy (M) in a given volume V, the temperature, T_{bb}, of the equivalent black body radiation is that which equates with its energy density, i.e.,

$$\frac{Mc^2}{V} = 0.658\frac{(kT_{bb})^4}{(\hbar c)^3} \tag{10}$$

Consequently, substituting (10) into (7a,b) gives the maximum amount of information which can be represented by a mass M in a volume V to be,

$$\text{Maximum Information} = 1.2V^{1/4}\left(\frac{Mc}{\hbar}\right)^{3/4} \tag{11}$$

Using the total mass and volume of the universe in the current epoch, as derived above (i.e., $V = 4 \times 10^{80}$ m^3 and $M = 3.8 \times 10^{54}$ kg) gives the maximum information content of the universe to be 10^{93}. If we restrict attention to the information possible with the normal baryonic matter alone, then this scales by $0.042^{3/4} = 0.093$ to 10^{92}.

To achieve the computation rate bound derived above (3.3×10^{121}) each ‘baryonic bit’ would have to have been involved in 3×10^{29} computations over the life of the universe, an average of $\sim 8 \times 10^{11}$ computations per baryonic bit per second. There are $\sim 10^{80}$ baryons in the observable universe, so this equates to $\sim 3 \times 10^{41}$ computations per baryon, or an average of $\sim 7 \times 10^{23}$ computations per baryon per second. [NB: There are $\sim 10^{12}$ times more baryonic bits than baryons – because the baryons have been assumed to be converted to photons]. Hence, the maximum possible mean computation time per baryon is of order 10^{-23} s.

This is a striking result because $\sim 10^{-23}$ s is just the characteristic time of the strong nuclear interactions! Since we have used nothing in our derivation relating to the strength of the nuclear force, this appears at first sight to be a remarkable Cosmic Coincidence. But it is not. This characteristic time derives from the size of a typical nucleus (~3 fm), which, on dividing by c gives 10^{-23} s. The size of a nucleus, or proton, also defines the order of magnitude of its mass through $M_p \sim \hbar / rc$. Hence a size of order 1 fm sets the mass scale of nucleons to be in the order of hundreds of MeV, quite correctly. Now, the computation rate per baryon can be derived algebraically using the above arguments to be,

$$\frac{M_p c^2}{\pi\hbar} \sim 5x10^{23} \tag{12}$$

i.e. the dependence on G and t cancel out. Since $M_p \sim \hbar / rc$ the characteristic computation time per baryon is inevitably ~r/c, i.e. the strong interaction time scale. There is no coincidence; it is just algebraic self-consistency. Nevertheless, it provides an interesting alternative interpretation of the maximum computation rate, namely that it is the maximum possible number of strong nuclear interactions.

B.2 Black Holes

There is, however, a loop hole in the above derivation of the maximum information representable by a fixed amount of mass-energy, M. It is the assumption that the volume, V, is also specified and finite. This is explicit in Equ.(11). If the volume is allowed to diverge, then so does the available information. Physically this is because the temperature of the black body radiation reduces arbitrarily close to zero, with a divergent number of photons whose energy becomes arbitrarily small. However, this observation is not relevant to our *observable* universe, which has a finite size (due to its finite age).

A particular mechanism for generating large entropies for very large, but finite, volumes is as follows: Consider the whole mass-energy of the universe to collapse into a single enormous black hole! The entropy of a black hole is given by,

$$\frac{S}{k} = \frac{4\pi G M^2}{\hbar c} = \frac{A}{(2L_{Planck})^2} \tag{13}$$

where A is the surface area of the black hole's event horizon. This compares with the entropy of the cloud of gas of the same mass from which the black hole was formed of order $S_{gas} / k \approx 66 \frac{M}{M_p}$. The numerical factor depends insensitively on the density of the gas before collapse. We have used the low mean density of the present universe to get 66 (see above), but using typical terrestrial or stellar densities would give a factor of ~20 in place of ~66.

Because the black hole entropy depends upon mass-squared, whereas the entropy of a gas is proportional to its mass, the black hole has far greater entropy. For example, the entropy of a solar mass black hole exceeds that of the gas which formed it by about a factor of 5 x 10^{18}. In the case of the whole universe (M = 0.4 x 10^{55} kg), the factor is greater still – namely $2x10^{41}$. A black hole with the mass of the universe has an entropy of ~4 x 10^{125} (in k units). If we only allowed the baryonic matter to form the black hole the entropy would scale by 0.042^2 to 7 x 10^{122}. The fact that this approximately equals the maximum possible number of computations in the universe to date is again no accident, but rather a matter of algebraic equality. Equ.(2b) gives,

$$\text{Universe's Total Computations} = \frac{f^3 c^5}{2\pi\hbar G} t^2 \qquad (2b)$$

And in comparison the entropy of a black hole with the universe's mass at time t is,

$$\text{Universe-Baryonic-Mass Black Hole Entropy} = \pi f_b^2 \frac{f^3 c^5}{\hbar G} t^2 \qquad (14)$$

where f_b is the fraction of the universe's mass which is baryonic. Although I can think of no good reason for doing so, if these quantities are equated they imply,

$$f_b = \frac{1}{\sqrt{2\pi^2 f^3}} \qquad (15)$$

Thus, with f = 3.5 we find f_b = 3.4%, which is a reasonable estimate of the baryonic fraction (actually ~4%). Is this mere numerology or has it any significance? I don't know.

How can the black hole entropy given by (13) or (14) be interpreted as the information capacity of the universe? On the face of it, it would seem to be the amount of information locked up *within* the black hole and hence *inaccessible* to the universe! There are two ways around this conundrum. The first is to regard our universe as being the inside of a black hole. This is a respectable hypothesis (see for example Fahr, H.J., and Heyl, M, 2006a & 2006b). It rather neatly turns the inaccessible information into accessible information.

Intuition suggests that the universe cannot be a black hole because it is too large and has too low a density. But this is entirely wrong. The radius of a black hole's event horizon is,

$$R_{bh} = \frac{2GM}{c^2} \qquad (16)$$

Inserting the *baryonic* mass of the universe (4.2% of 0.4 x 10^{55} kg) gives a black hole radius of 2.4 x 10^{26} m. This compares with 3.5ct = 4.5 x 10^{26} m. These are remarkably close to equality. The reason can be elucidated by substituting for M in (16) in terms of the critical density and the size of the universe, R = fct. This gives,

$$R_{bh} = f_b f^3 ct \qquad (17)$$

Hence, equality of the black hole radius with the radius of the observable universe, R = fct, obtains for $f_b = 1/f^2$. For f = 3.5 this gives f_b = 8% - which is reasonably close to the best current estimate of 4.2%.

Returning now to the idea that a universe-mass black hole has formed and the information is trapped within it – how can we get it out again? Well, this will happen automatically through Hawking radiation. Admittedly you will have to wait a truly humungous length of time. The temperature of a black hole is,

$$kT = \frac{\hbar c}{8\pi m_G} = \frac{\hbar c^3}{8\pi GM} \qquad (18)$$

So our universe-mass black hole will have a temperature of only ~10^{-32} °K. At this temperature it's going to take quite a while to radiate away ~10^{55} kg of mass-energy. When it has done so, however, the entropy released into the universe will be at least that of the black hole – as you would expect. (In fact, I think it is greater by a factor of 4/3). Specifically, I estimate the entropy of the emerging radiation, integrated over time, to be roughly,

$$\frac{S^{\text{radiation}}_{\text{bh evaporation}}}{k} = 16.75\frac{GM^2}{\hbar c} = 16.75\left(\frac{M}{M_{\text{Planck}}}\right)^2 \qquad (19)$$

where the Planck mass is $M_{\text{Planck}} = \sqrt{\frac{\hbar c}{G}} = 2.2\text{x}10^{-8}\,\text{kg}$. Thus we retrieve the maximum information capacity of ~5 x 10^{125} (in k units), or ~10^{123} including only baryonic mass.

Note that the black hole has formed from material which had entropy not exceeding 2.8 x 10^{93} and yet, on evaporation, the entropy released is far greater, up to ~3 x 10^{126}. The black hole has achieved this trick because its temperature is so very low that its black body photons are of extremely low energy. It takes an awful lot of them to make up ~10^{55} kg of mass-energy. Thus, a black hole is a great way of chopping the mass-energy of the universe into really, really small pieces. But you have to be willing to wait ~10^{132} billion years for it to evaporate. (Evaporation time is about $5120\pi\frac{G^2M^3}{\hbar c^4}$).

CONCLUSIONS

[1] The minimum energy which must be dissipated to perform a computation is zero. This can be achieved only for a computer based on reversible logic, in which case the computer is also thermodynamically reversible.

[2] The maximum computing rate of a system with total mass-energy E is $\frac{2E}{\pi\hbar}$. A very fast reversible computer thus requires the loan of sufficient energy to drive its rate, but will return the energy unchanged at the end of the computation. ('Unchanged' means without entropy increase).

[3] An irreversible computer which dissipates energy at the minimum rate needed to support its computing speed (C bits per second) consumes power $P = \pi\hbar C^2 / 2$. This is likely to be the lower limit for computers which are not specifically devised to be reversible. Current computers consume about 10^{16} times this power.

[4] The upper limit to the physically possible computing rate that could be supported by the mass-energy within each cubic meter of interstellar space is 5.2 x 10^{24} bits per second, a pretty impressive figure for just 6 hydrogen atoms.

[5] This is based on the average universal density and implies a maximum computing rate by the whole inventory of mass-energy in the *observable* universe of 2 x 10^{105} bits per second.

[6] The maximum number of computations that could have been carried out by the total mass-energy which is observable at the present epoch is ~10^{123}. The maximum observable number of computations is about half this.

[7] At another epoch this would be $\frac{f^3 c^5}{2\pi\hbar G} t^2$, where f = R/ct and R is the radius of the observable universe and t its age.

[8] The maximum computing rate of the universe can also be interpreted as every baryon undergoing strong nuclear interactions at the maximum possible rate (~10^{23} per second per baryon).

[9] The limitation on the universe's computing capacity due to the Planck scale is vastly less restrictive than that above due to the finite mass-energy content of the universe.

[10] Spacetime can support more computations than the universe's mass-energy content even at the first instant of time ($t = t_P$).

[11] The maximum amount of information which can be represented by a mass M in a volume V is $1.2V^{1/4}\left(\frac{Mc}{\hbar}\right)^{3/4}$.

[12] The information capacity of the universe in the present epoch is ~10^{93}.

[13] The information capacity of the baryonic portion of the universe in the present epoch ~10^{92}.

[14] By regarding the universe as a black hole, the total information capacity of the baryonic material including gravitational degrees of freedom can be estimated to be ~7 x 10^{122}.

[15] Comparing [6] and [14], each bit of information has been changed the order of just one time during the life of the universe. *(Dubious? This is the ratio of two upper bounds – what does that mean?).*

[16] The hypothesis that the universe is the interior of a black hole, and that its horizon is determined by the baryonic mass alone, i.e., that $R_{bh} = f_b f^3 ct = R_{univ} = fct$, implies a baryon fraction of $f_b = 1/f^2 = 8\%$ (using f = 3.5). This compares with the best current estimate of 4.2%.

[17] The entropy of a black hole with the universe's mass at time t is $\pi f_b^2 \frac{f^3 c^5}{\hbar G} t^2$. Equating this with the universe's total computing capacity, $\frac{f^3 c^5}{2\pi\hbar G} t^2$, implies a baryon fraction $f_b = \frac{1}{\sqrt{2\pi^2 f^3}}$. Thus, with f = 3.5 we find f_b = 3.4%, which compares well with the best current estimate of 4.2%.

References

Bennett, C.H., (1979), "Logical Reversibility of Computation", IBM Journal of Research and Development, **6** (1979), 525-532.

Bennett, C.H., (1982), "Thermodynamics of Computation – A Review", Int.J.Theor.Phys. **21** (1982), 905-940.

Fahr, H.J., and Heyl, M., "About Universes With Scale-Related Total Masses and Their Abolition of Presently Outstanding Cosmological Problems", arXiv:astro-ph/0606048, 2 June 2006.

Fahr, H.J., and Heyl, M., “Concerning the Instantaneous Mass and the Extent of an Expanding Universe”, arXiv:astro-ph/0606448, 19 June 2006.

Lloyd, S., “Ultimate Physical Limits to Computation”, Nature, **406**, 31 August 2000, 1047-1054.

Lloyd, S., “Computational Capacity of the Universe”, Phys.Rev.Lett. 88, 10 June 2002, 237901-(1-4).

Spergel, D.N., et al, “Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Results: Implications for Cosmology”, arXiv:astro-ph/0603449 (14 May 2006).