Normal Distribution

- Probably the most used distribution type.
- Can lead to problems when small or negative values
- Usually described by its mean and standard deviation (or coefficient of variation (CoV), which is the ratio of standard deviation to mean)

Log-Normal Distribution

- Often used when the variable cannot be negative (e.g. material properties).
- Usually described by either the mean and standard deviation of the log of the variable or the median and CoV of the variable itself.
- Very similar to Normal when the CoV is low.
- A simple way to generate a sample L_s from a Log-Normal distribution with median of m and a CoV of V is to convert a sample N_s from a Standard Normal Distribution (i.e. with a mean of zero and a standard deviation of 1) as follows.

$$L_s = m.\exp\left(N_s.\sqrt{\ell n\left(1+V^2\right)}\right)$$

Estimating a CoV from Best Estimate and Bounding Values

It is rare that complete information is available on the statistical distribution of any of the parameters in a structural integrity assessment. In some cases, however, best estimate (or median) and bounding (characteristic) values may be known. Often, this would be for data on material properties from handbooks, such as R51 and R66. Typically, in such handbooks, a variable may be described by its best estimate (or median) value, m, and an upper (or lower) bounding value, u (or ℓ), associated with some confidence level, p. This is sufficient information to estimate the Coefficient of Variation of the parameter's distribution, as follows.

The first step is to find the characteristic offset, o, associated with the confidence level, p, of the given bounding value, using the Table below. If the confidence level of the bound is unknown, it must be guesstimated. 95% upper and 5% lower confidence levels are common.

If it is known, or believed, that the bounding value(s) were derived assuming a Normal Distribution (a clue would be upper and lower bounds which are symmetrical about the best estimate), then the standard deviation, σ, and the Coefficient of Variation, V, are calculated as follows.

$$\sigma = \frac{u-m}{o} \qquad \text{or} \qquad \sigma = \frac{\ell - m}{o}$$

$$V = \frac{\sigma}{m}$$

If it is known, or believed, that the bounding value(s) were derived assuming a Log-Normal Distribution, then m would usually be the median value (a clue would be unsymmetrical upper and lower bounds, with the ratio of upper to median equal to the

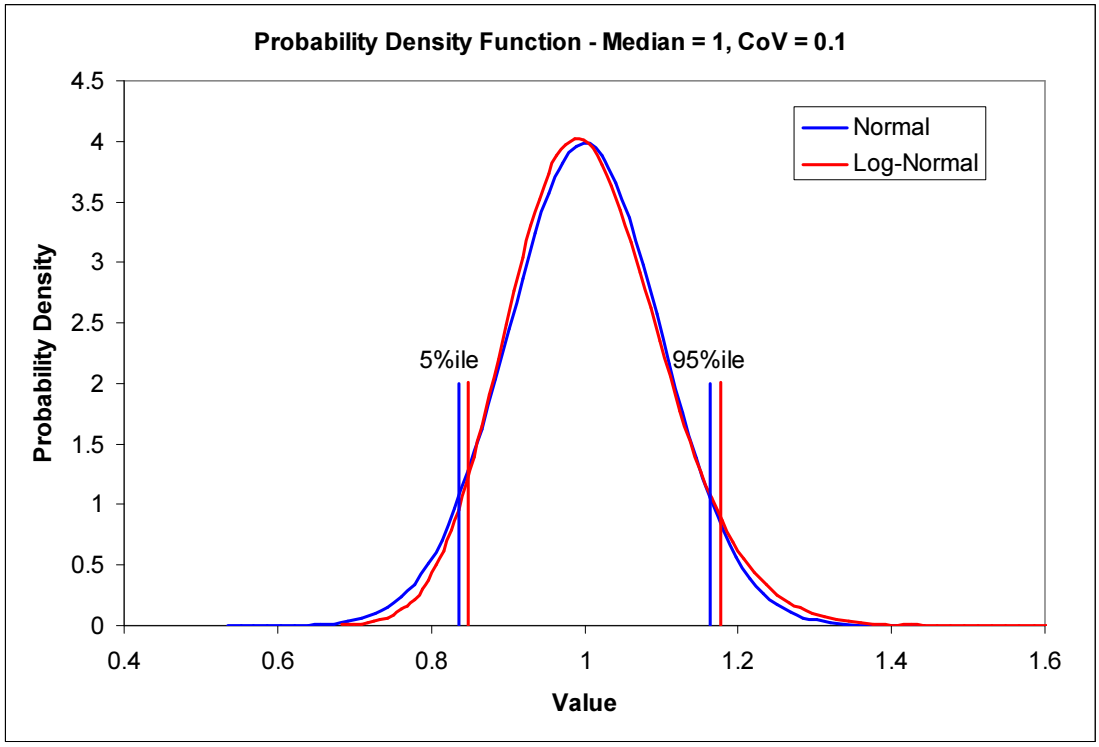
ratio of median to lower). The standard deviation of the log data, σ, and the Coefficient of Variation, V, are calculated as follows.

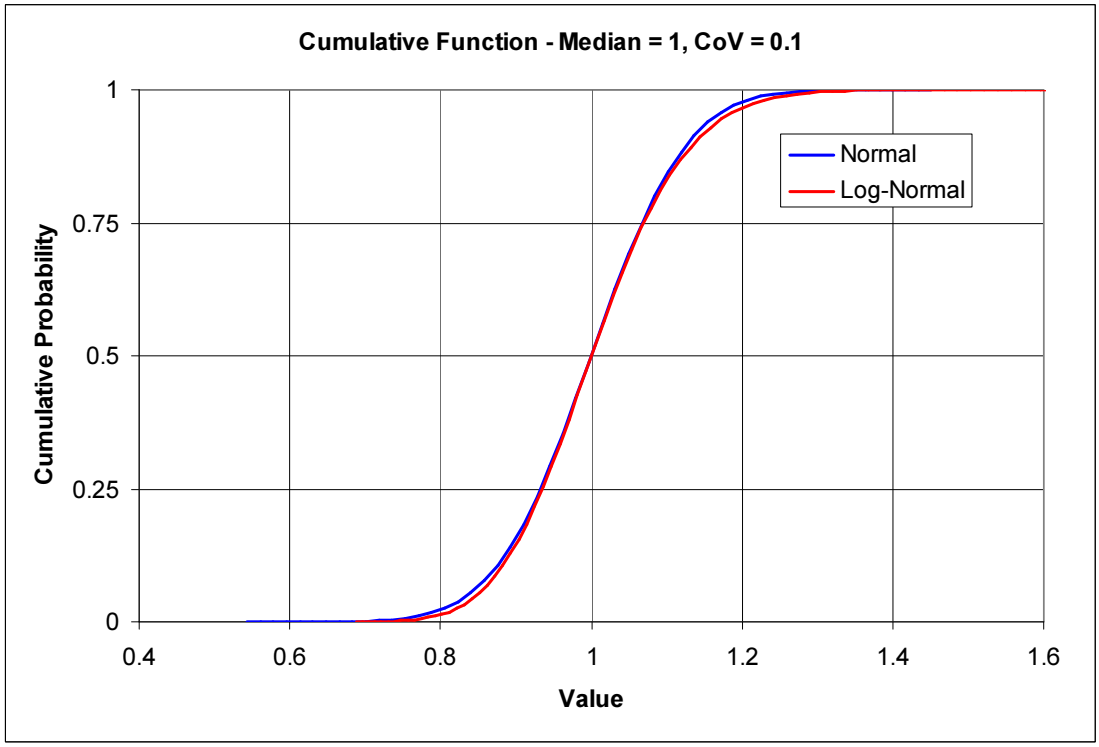
$$\sigma = \frac{1}{o}\ell n\left(\frac{u}{m}\right) \qquad \text{or} \qquad \sigma = \frac{1}{o}\ell n\left(\frac{\ell}{m}\right)$$

$$V = \sqrt{\exp\left(\sigma^2\right) - 1}$$

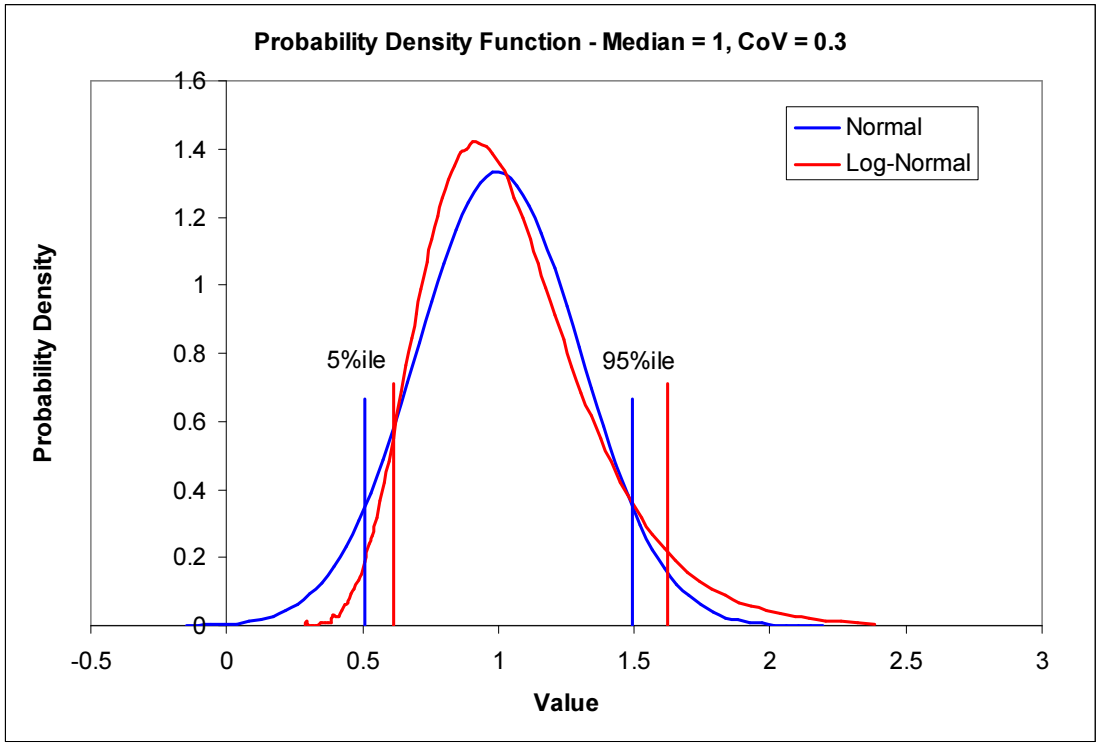
Fractile Level (Probability of Variable being Less Than Characteristic Value)	Confidence Level (p)	Characteristic Offset (o)
0.1 %	99.9% Lower	-3.090
1%	99% Lower	-2.326
5%	95% Lower	-1.645
10%	90% Lower	-1.282
20%	80% Lower	-0.842
50%	50%	0
80%	80% Upper	0.842
90%	90% Upper	1.282
95%	95% Upper	1.645
99%	99% Upper	2.326
99.9%	99.9% Upper	3.090

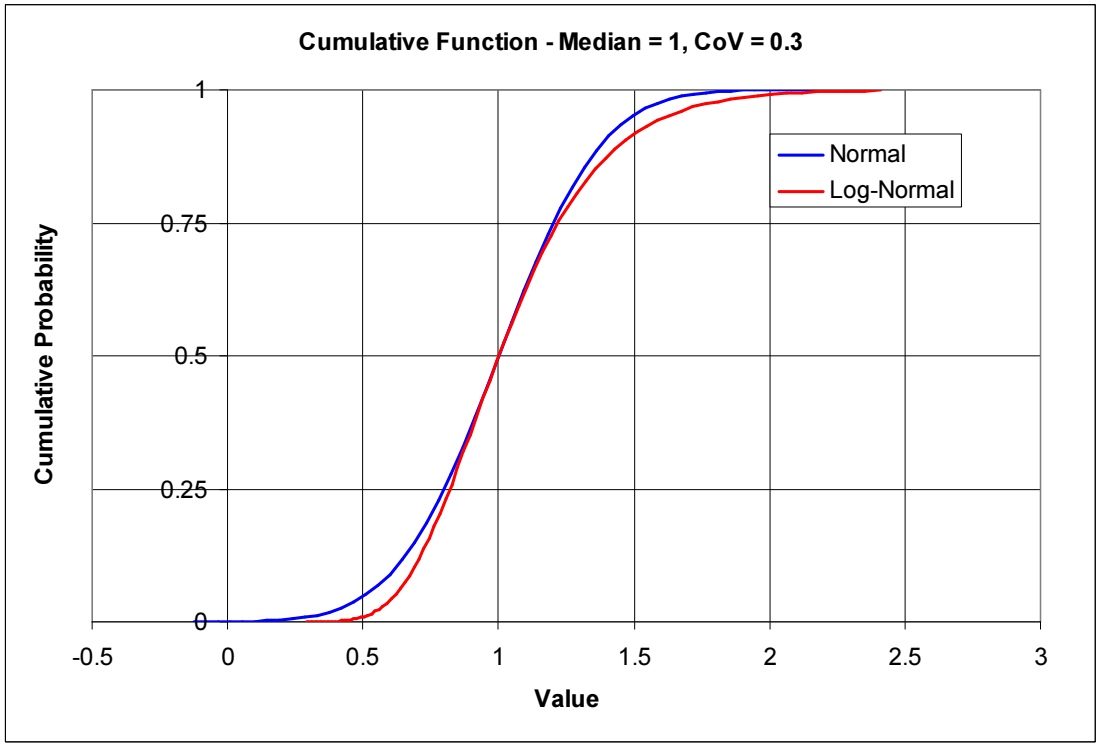
Comparison of Normal and Log-Normal for Low CoV



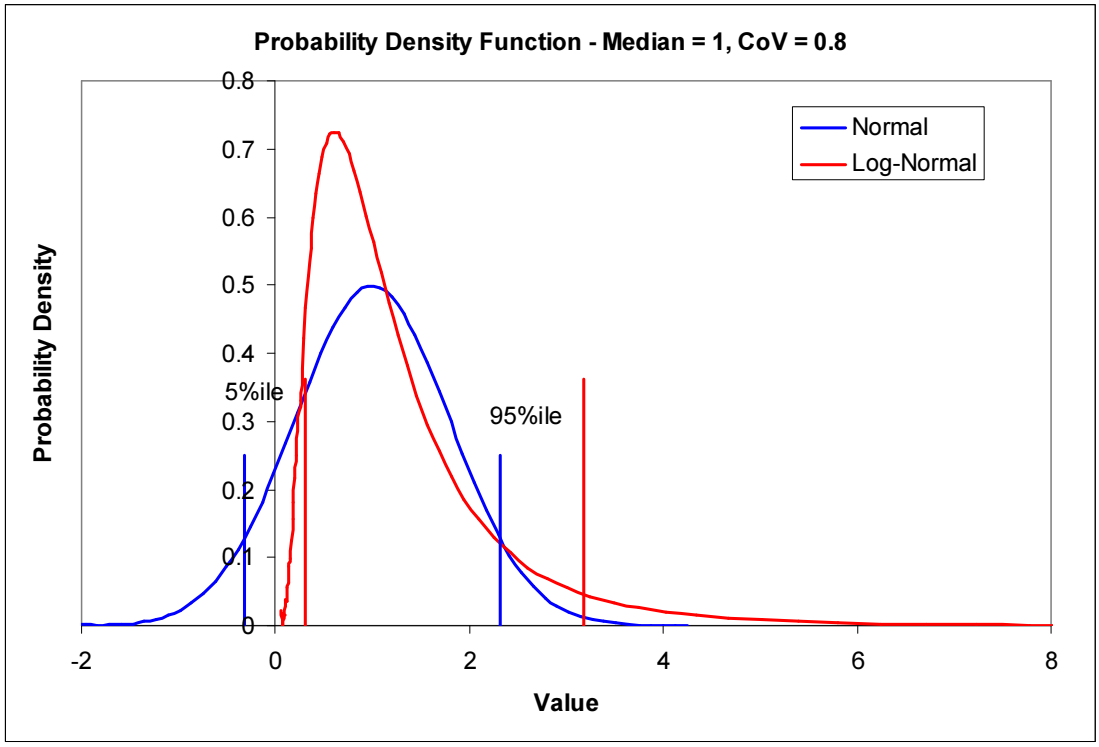


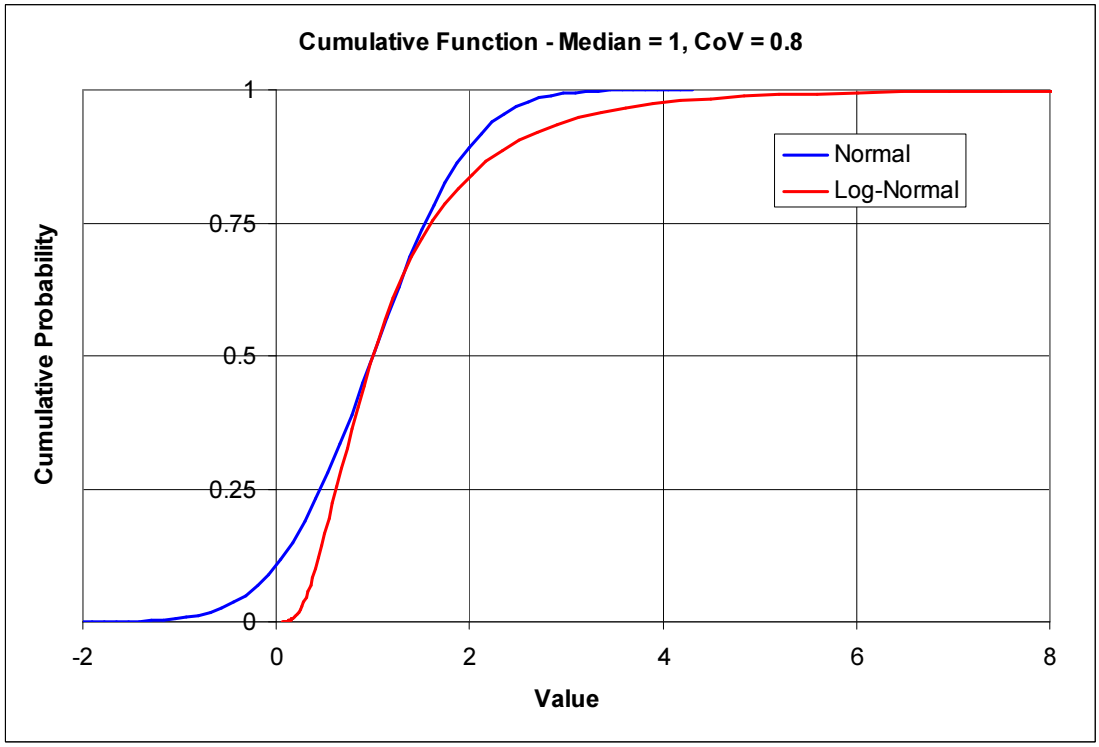
Comparison of Normal and Log-Normal for Intermediate CoV





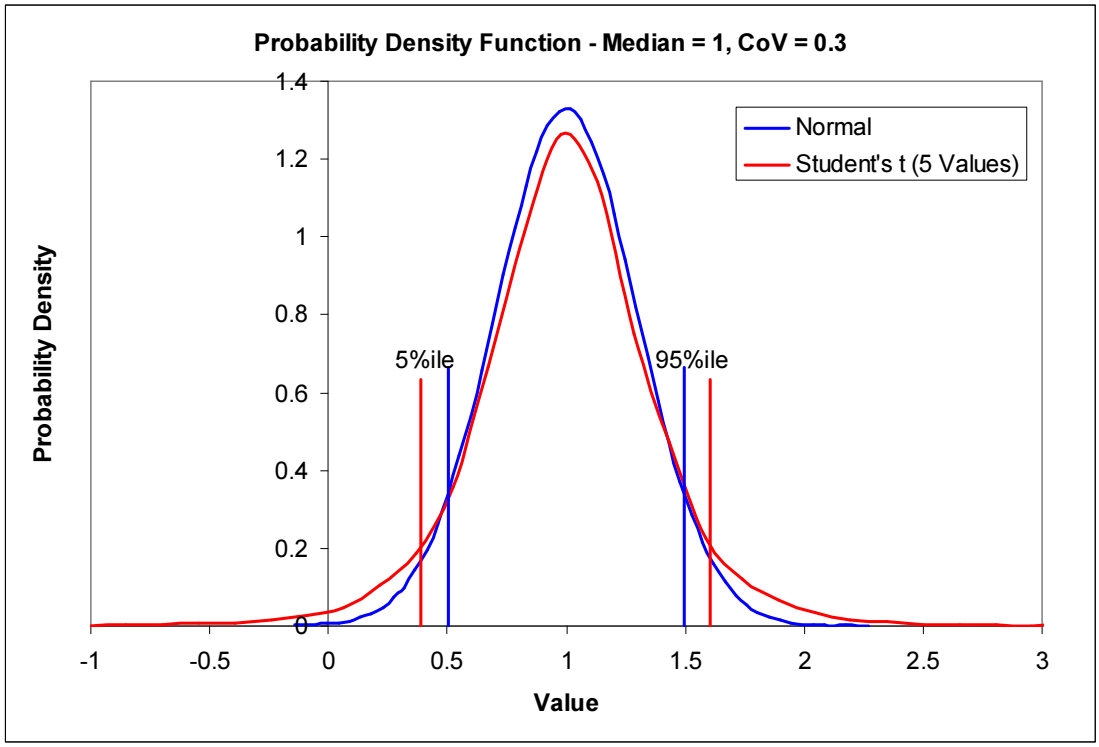
Comparison of Normal and Log-Normal for High CoV

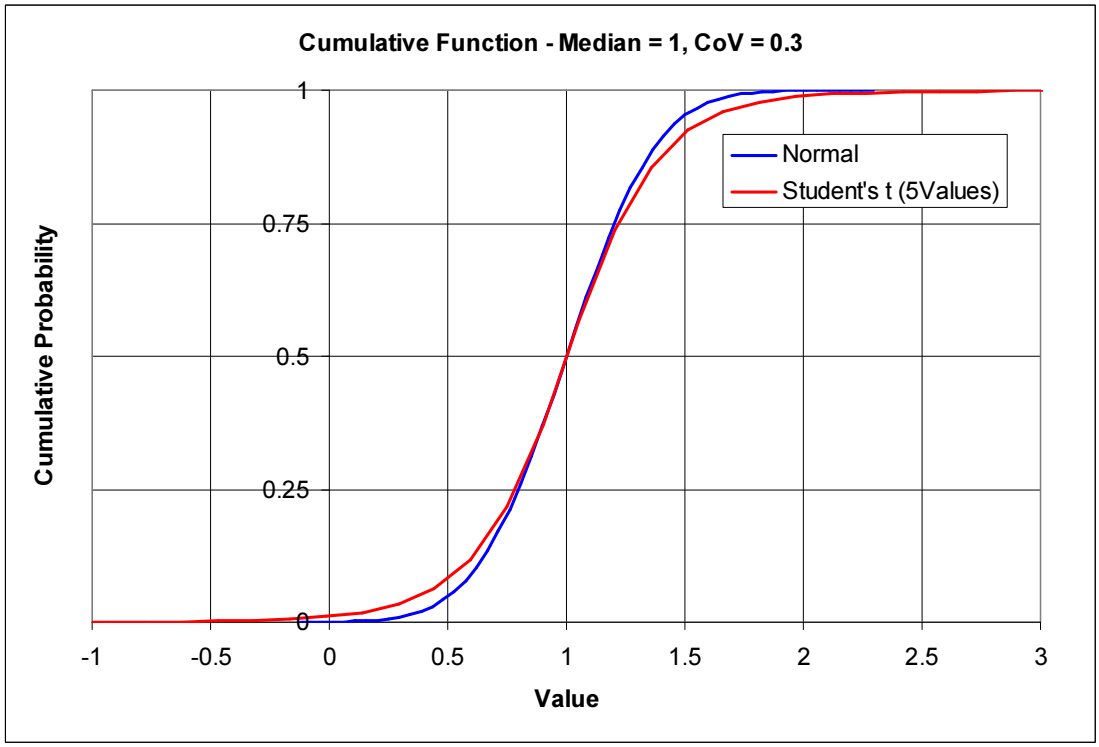




Student's t Distribution

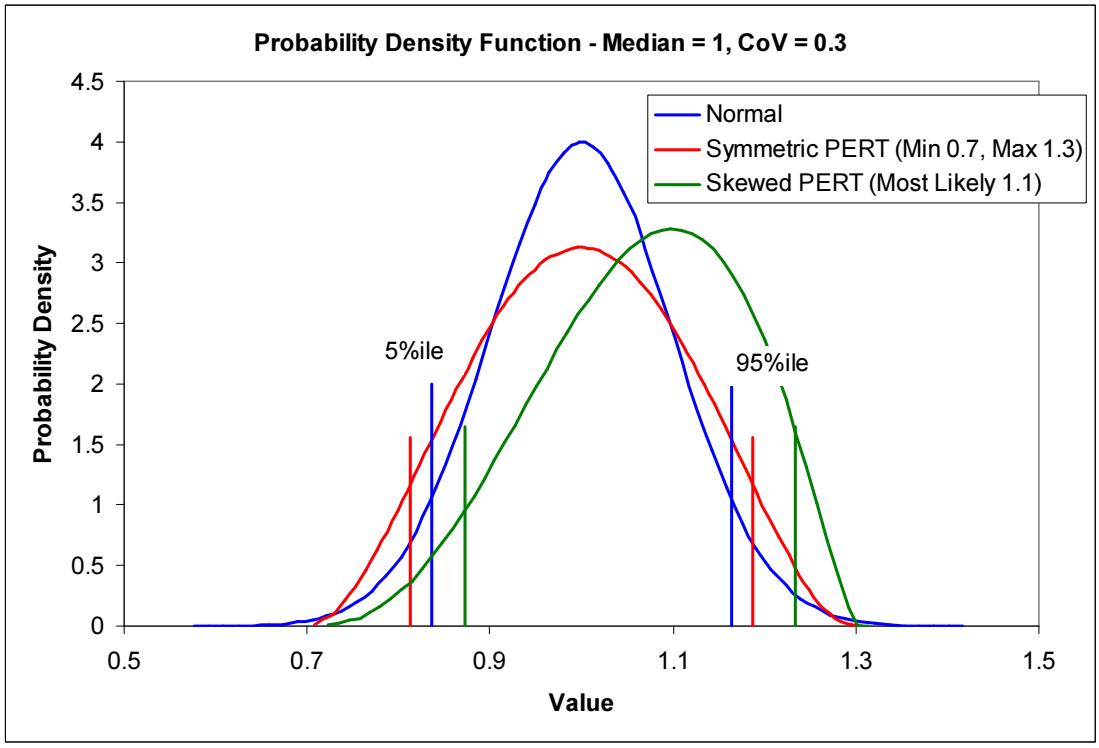
- Used for a distribution based on few data values.

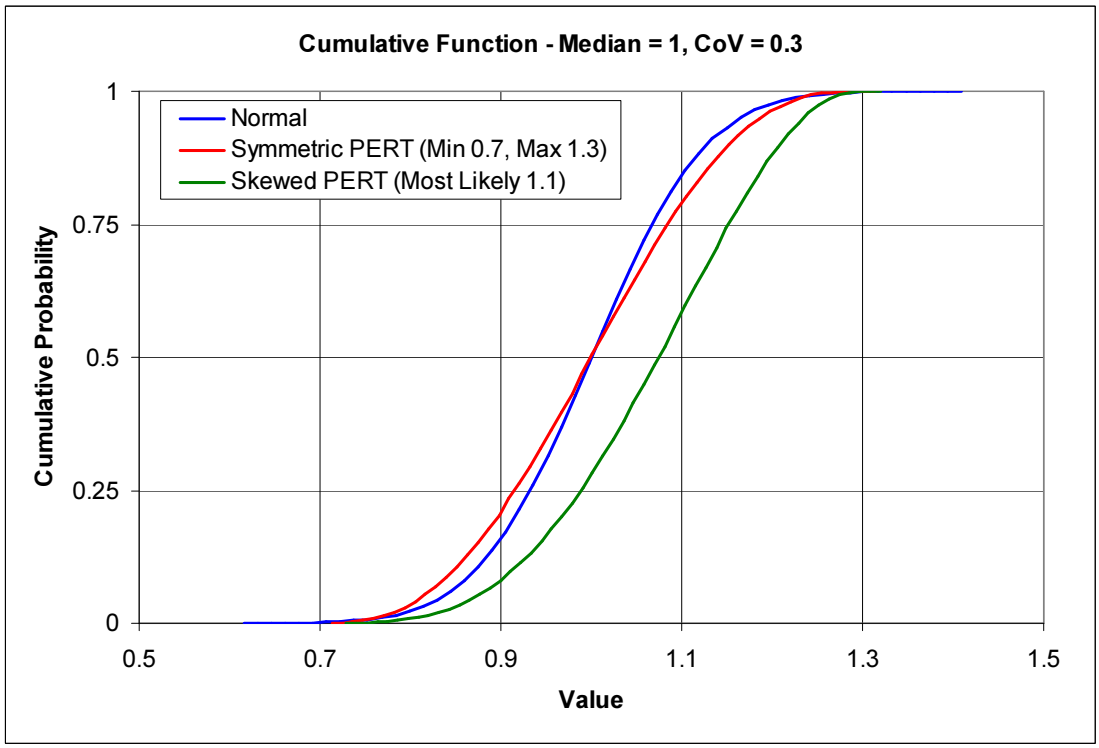




PERT Distribution

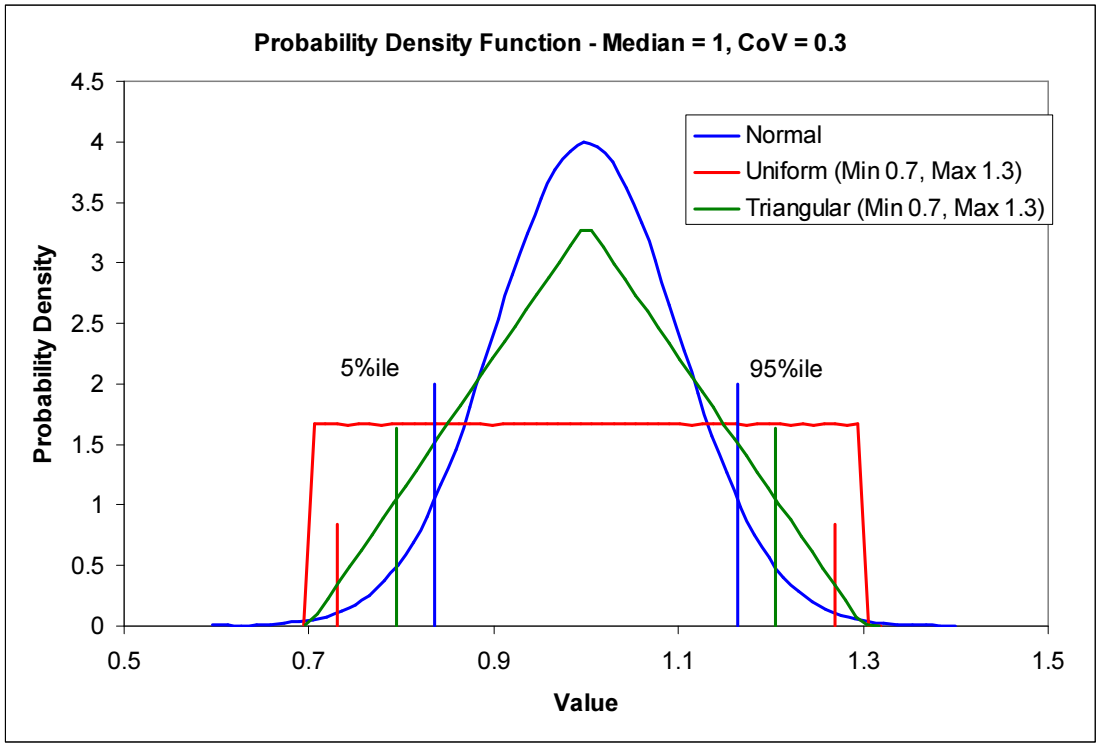
- Used for an 'expert' distribution.
- Minimum, maximum and most likely values are specified

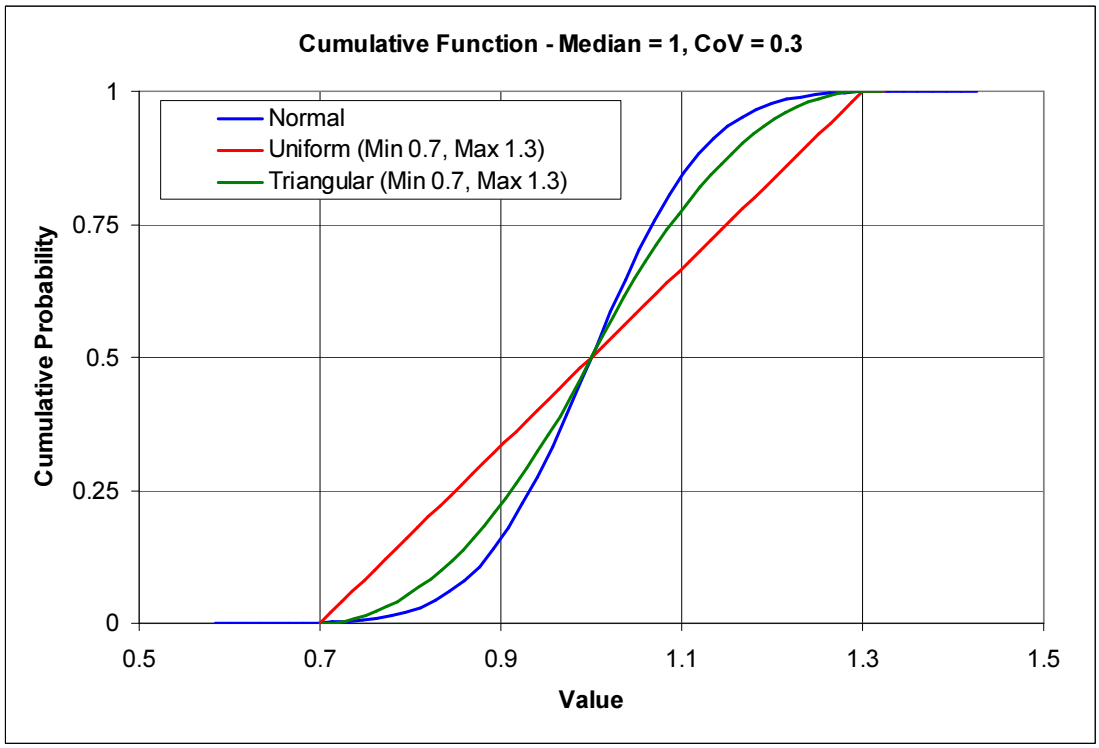




Uniform and Triangular Distributions

- Also used for distributions with known minimums and maximums.
- Triangular distribution can also be skewed if required.





Composite Distributions

- Can combine standard distributions using logical statements.
- For example, a Normal Distribution over a subset x% of the population.
- Generate a random sample from a Normal Distribution.
- Generate a random sample from a uniform distribution between 0 and 100.
- If the second sample is < x, select the first sample, otherwise select zero.

Correlated Distributions

- It is not always valid to assume distributions are entirely independent.
- Monte Carlo programs/addins will usually allow the specification of a correlation coefficient between two random variables, though this may be only for a few distribution types (e.g. Normal and Log-Normal).
- In more complex cases, multivariate cross-correlation may be allowed.