ABSTRACT

This work outlines an approach for the treatment of stresses in complex three-dimensional components for the purpose of conducting probabilistic creep-fatigue lifetime assessments. For conventional deterministic assessments, the stress state in a plant component is found using thermal and mechanical (elastic) finite element (FE) models. Key inputs are typically steam temperatures and pressures, with the three principal stress components (PSCs) at the assessment location(s) being the outputs. This paper presents an approach which was developed based on application experience with a tube-plate ligament (TPL) component, for which history data was available. Though both transient as well as steady-state conditions can have large contributions towards the creep-fatigue damage, this work is mainly concerned with the latter. In a probabilistic assessment, the aim of this approach is to replace time intensive FE runs with a predictive model to approximate stresses at various assessment locations. This is achieved by firstly modelling a wide range of typical loading conditions using FE models to obtain the desired stresses. Based on the results from these FE runs a probability map is produced and input(s)-output(s) functions are fitted (either using the Response Surface Method or simple Linear Regression). These are thereafter used to predict stresses as functions of the input parameter(s). This mitigates running an FE models for every probabilistic trial (of which there typically may be more than 10^4), an approach which would be computationally prohibitive.

1 INTRODUCTION

High temperature structural integrity assessments, an example of which is the R5 procedure developed by EDF Energy [1], have built-in conservatism to account for various sources of uncertainty. As a result, conventional deterministic approaches use conservative values for material properties and loading histories (temperature and stress). Probabilistic approaches have been gaining popularity and support for their application in lifetime assessments of components operating within power plants where the main modes of damage are creep and creep-fatigue. This has been driven by a need for identifying and addressing the main sources of uncertainty, which is not formally addressed by conventional deterministic assessments.

A crucial part of a creep-fatigue assessment for a plant component operating at high temperature cyclic conditions is the approximation of the stress state during nominal operation. However, with variations in loading conditions, chief of which is the operating temperature, assuming a conservative stress state for long periods of the component’s history can produce overly pessimistic results. A key part of the R5 V2/3 procedure for creep-fatigue crack initiation [1] is evaluating the stress at the beginning of the creep dwell (σB in Fig 2), for which the three principal stress components (PSCs) during nominal operating conditions are required. The principal stresses are needed because the stress ranges, which will ultimately be needed by the assessment procedure, are calculated from the ranges of individual principle stresses. The PSCs are typically obtained from thermal and mechanical (elastic) FE models which can be computationally costly, and therefore not suitable for direct use in a probabilistic calculation. This issue is explored in [2, 3] and the adoption of the Response Surface Method (RSM), which is effectively a mul-
multivariate regression approach [2,4,5]) in conjunction with Design of Experiments (DoE) was suggested for reducing computational efforts.

This work explores an approach for preparing stresses for use in a probabilistic assessment for a tubeplate ligament (TPL) plant component. An FE model of the TPL is shown in Fig 1. The aim was to develop an approach for predicting the response of an FE model based on some input parameter(s). This arises from a need to model the evolution of the stress state (i.e. the PSCs) for each tubehole as a function of operating conditions (mainly the steam temperature). A procedure was devised which starts by treating the raw temperature data coming from plant measurements, and ends by producing a predictive model which provides the stress state (with an uncertainty) for a given set of operating temperatures.

This work builds on previous research [6] where various probabilistic techniques were examined and proposed as constituents of an under development probabilistic creep methodology. The treatment of assessment stresses and temperatures is part of this methodology which is being developed in conjunction with the new probabilistic appendix (Appendix A15) to the R5 Volume 2/3 procedure for creep-fatigue crack initiation.

2 COMPONENT DESCRIPTION

The TPL has 37 tubeholes and is made of type 316H stainless steel forgings. The failure mechanisms are driven by creep-fatigue, large thermal transients and over-heating due to tube restrictions. Furthermore, the component is thought to be subjected to complex stress states which include large thermal stress gradients due to large steam temperature differences (called tilts) between the tubes attached to the TPL. Fig 2 shows the location of \( \sigma_B \) at an intermediate position in the hysteresis cycle, which is typical for a point near the surface of a tube bore. During a start up (SU) transient the surface of a tubehole is heated up while surrounding metal remains colder, thus the difference in thermal expansion produces a compressive stress near the surface of the tubehole. The reverse effect occurs during a reactor trip (RT) transient where a tensile stress is induced on the surface. Nominal operation can include a period of steady-state (SS) or a period of instability (IN), with the main distinction being the maximum difference between individual tubes’ steam temperatures. Instabilities are characterised by large temperature gradients (> 60°C, see Table 1) which induce large thermal stresses.

3 DATA PROCESSING

3.1 Treatment of raw temperature history data

For periods of the component’s history, the available data had been hourly recorded using rake thermocouple (TC) measurements. These thermocouples are situated so as to sample the steam emanating from individual tubes. The main challenge was arranging the large amounts of hourly data available across decades of operation. This large amount of data cannot be directly used in a deterministic, let alone probabilistic, assessment due to computational issues. In previous assessments the temperature history was discretised by predefining types of events based on the maximum temperature recorded and the maximum tilt (defined by the difference between the maximum and minimum steam temperatures) from the recorded data. Eight types of steady-state (i.e. Normal) and instability history events were predefined, the conditions for which are detailed in Table 1. Therefore, in this work an event is a period of time during which the maximum temperature and the tilt persist within the bounds outlined in Table 1. This provides a convenient approach for cat-
3.2 Treatment of FE data results

The following challenge was related to the choice of point from which the principal stress components (PSCs) would be taken in the FE models. An approach was needed to deal with the issue that each FE run yields itself a large amount of data for each event. For the TPL the total damage was deemed to be creep dominated, and thus the location of maximum positively signed von Mises stress (MPS-VMS) was deemed to be the most damaged. The concept of a signed Mises stress is important in R5 V2/3 and is defined as a VM stress signed according to the sign of the maximum PSC. Therefore for each tubehole, the outputs needed from each of the FE runs were the three PSCs taken from the integration point which had the MPS-VMS. The output of this data processing stage is a list of 1300+ history events each having $37 \times 3$ PSCs. This output data is referred to in this report as the processed FE data.

It is worth noting that this approach does not account for the fact that from one event to the following the MPS-VMS may change location within the surface of a single tubehole. This effectively assumes that the maximum stress always occurs at the same location for a given tubehole for the entire lifetime. This is somewhat conservative, but a practical necessity for dealing with the data coming out of the FE runs.

4 APPROACH FOR APPROXIMATING STRESSES

4.1 Overview

The processed FE data provides some useful insight which can be used to predict the stress states of each tubehole for a given value of tilt. However, there are two distinct issues that need to be addressed:

1. The choice of which tubehole will have the maximum stress and thus will be treated as the main, but not the unique, assessment location.
2. The magnitude of each stress component at each assessment point.

The proposed approach involved treating these two issues separately by:

1. Computing the frequency of each tubehole having the most stressed point, the least stressed point, and all degrees of stressing in between. This is easily done by ordering the tubeholes in terms of their MPS-VMS for each event, and then counting the number of reoccurring tubeholes for each level of stressing. This analysis effectively produces a matrix of probabilities where one dimension is the tube number ($TN = 1, 2, \ldots, 37$), and the other dimension is the relative magnitude of stress (the tube order, $TO = 1, 2, \ldots, 37$). This matrix of probabilities can be visualised by the concentration plot shown in Fig 3. In MCS trials, this probability matrix can be used to produce configurations of stresses across the TPL e.g. for $\sim 50\%$ of the trials, $TN = 2$ would be assigned the most stressed location ($TO = 1$).

2. Statistically characterise the stress components corresponding to each tube order as a function of the most influential input variables. This produces statistical information which can be used to model each stress component for a given level of stressing.

The first step is actually a precursor for the second. Once the data coming from each FE run is condensed to some useful data, the next step is to establish input(s)-output(s) relationships in order to predict the PSCs as functions of some input(s).

4.2 Treatment of processed FE data results

In order to establish what key inputs are to be used to predict the required outputs, a sensitivity study was required in order to assess which input parameters are the most dominant. For the TPL, the two main parameters which were believed to have a strong effect on the PSCs were:

1. The tilt which is the difference between the maximum and minimum steam temperatures across the 37 tubeholes.
2. The distance between the hottest and coldest tubeholes.

A simple approach for examining the strength between the output-input data is to simply plot the outputs against each of the inputs. Fig 4 shows scatter plots of the above inputs against the maximum principal stress during steady-state loading conditions for the most stressed tubehole i.e. $TO = 1$. The conclusion was that the tilt strongly dominated the output stress state, and thus only one input parameter was deemed to be worth considering at this stage. Although strong, the relationship between the PSCs and tilt exhibited a bimodal behaviour. Modelling this behaviour required a separate treatment which is the subject of the remainder of Section 4.2.

4.2.1 Data discrimination

For a set of observations the purpose of discrimination is to segregate between subsets based on some measure. A comprehensive explanation as to how data discrimination is conducted using a Bayesian discriminant rule can be found in [7]. In [8] a number of tools for data analysis are explored, including the rda function in MATLAB®, which
TABLE 1: Conditions used for categorising hourly temperature history into discreet events.

<table>
<thead>
<tr>
<th>Event type</th>
<th>Normal</th>
<th>Instability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normal-0</td>
<td>Normal-1</td>
</tr>
<tr>
<td>Tilt/[°C]</td>
<td>&lt; 60</td>
<td></td>
</tr>
</tbody>
</table>

**FIGURE 3**: Probability map showing the relative frequency of each tube ($TN = 1 – 37$ on the vertical axis) being the highest-lowest ($TO = 1 – 37$ on the horizontal axis) stressed tube. The size of each bubble is a measure of probability.

was used in this work to segregate the processed FE data in order to account for the bimodal behaviour. An example of the outcome of this analysis is shown in Fig 5 where all three stress components exhibited bimodal behaviours. The segregated subsets are plotted along with their respective least-squares regression (LSR) fits. Linear or quadratic fits were used depending on which produced the least scattered residuals. In the same figure, the residuals relative to the regression fits are also presented as histograms. Therefore, for an input tilt, the three PSCs for each of the 37 tubeholes can be predicted as:

\[
(\sigma_i)_{TO} = (\mu_i)_{TO} + (\epsilon_i)_{TO}
\]

where $(\sigma_i)_{TO}$ is the $i^{th}$ stress component $(i = 1 : 3)$ for a required tube order $(TO = 1 : 37)$, $(\mu_i)_{TO}$ is the predicted mean value (obtained from the least-squares regression fits) and $(\epsilon_i)_{TO}$ is the residual (error) component which is sampled from the associated histogram.
It is worth noting that although least-squares regression was used for this work, the Response Surface Method (RSM) [2,4,5] and Artificial Neural Networks (ANN) [9] can be viable alternatives depending on the application.

### 4.2.2 Input range dependent linear regression

Once the stress data was segregated into two distinct subsets, for each stress component and for each tube order two fits were produced based on the data in each group. Thereafter it was observed that the bimodal behaviour was more pronounced for high tilts and high stresses. Therefore, it was deemed important to consider the tilt ranges over which the two subsets of data were observed. According to these ranges, and for a value of input tilt, there would be two possible scenarios for producing a stress prediction:

1. If the tilt lies within a range exclusive to a single subset of data, then the least-squares fit specific to that subset (and associated histogram of residuals) would be used to predict the stress components.
2. If the tilt lies within a cross-over range, where the data suggests that a bimodal behaviour can occur (e.g. in Fig 5a this range is 15 – 185 MPa), then a bimodal prediction can be produced. The probability of each mode \( p_m \) where \( m = 1, 2 \) was dictated by the number of data points that belonged to each mode within the cross-over range. For example, if \( D \) is the total number of data points, \( D_B \) is a portion of \( D \) which lies within the cross-over (bimodal) region, and \( D_m \) is the number of data points belonging to each mode, then the probabilities can be easily defined as:

\[
p_m = \frac{D_m}{D_B}
\]

In a MCS this can be accounted for by separating the number of MCS trials \( N \). If the first scenario applies then all \( N \) trails would be sampled according to the same mode. However, for the other scenario, \( N \) would be divided into two subsets each of \( N_m \) trials according to the probability of each mode:

\[
N_m = N \times p_m
\]

Thereafter, for each subset the PSCs would be predicted according to the relevant mode using Eq 1.

### 4.2.3 Correlations

Correlations were considered in the probabilistic stress predictions as they are believed to have a strong effect on the start-of-dwell stress calculation results. Care must be taken in terms of identifying the correlations of interest and their interpretation. For a probabilistic calculation, what is needed is a correlation that links the sampled stress components for a single input tilt. That should not be confused with the correlation between the stress components coming from the FE results which in fact were for a wide range of tilts. Therefore, the required correlations can be interpreted as correlations between residuals (or errors) relative to the regression fit, rather than correlations between deterministically evaluated stress components. The latter correlation is indeed important, as the maximum and middle principal stresses are strongly correlated for a range of tilts, and thus can have significant effects on stress range calculations. However, that correlation is accounted for by virtue of the regression fits i.e. the three components are linked (or correlated) since they are all functions of tilt. The residuals (which can be rather large) need to be correlated separately, and as it will be discussed, are advised not to be sampled independently.

For each tube order, it was found that the correlations between stress component residuals (e.g. the histograms shown in...
FIGURE 5: Example showing the three sets of fits used to predict the three stress components as functions of tilt for the most stressed tubehole (i.e. TO = 1).
Fig 5) were fairly weak. The most stressed tubehole (i.e. TO = 1), for example, had the following correlation matrix:

\[
\begin{bmatrix}
C_{11} & C_{12} & C_{13} \\
C_{21} & C_{22} & C_{23} \\
C_{31} & C_{32} & C_{33}
\end{bmatrix} = \begin{bmatrix}
1.000 & -0.212 & -0.031 \\
-0.212 & 1.0000 & 0.106 \\
-0.031 & 0.106 & 1.000
\end{bmatrix}
\]  

(4)

where \(C_{ij}\) refers to a Spearman correlation between residuals for the \(i^{th}\) and \(j^{th}\) principal stresses. The Spearman correlation, which is a non-parametric equivalent to its Pearson counterpart, determines the strength and direction of the monotonic relationship between two parameters [10]. It is calculated as the Pearson correlation between the ranks of the two sets of data being correlated. The value of the Spearman correlation ranges from 0 to 1 (0 indicating no correlation and 1 indicating perfect correlation), while its sign indicates whether the parameters are correlated or anti-correlated [10, 11]. For the application presented here, the Spearman correlation was deemed to be more appropriate as it does not assume linearity (an assumption which may not apply for parameters of interest) and is a non-parametric statistic which does not impose any a priori assumptions on the distributions of the input parameters. Indeed, parameters following different types of distributions can still be correlated using the Spearman correlation.

A convenient approach for generating multivariate datasets with arbitrary probability density functions (PDFs) and Spearman correlations is based on using a Gaussian copula. A detailed account of this method can be found in [12], whilst a practical example showing its implementation in MATLAB® can be found in [13].

The correlations above seem weak which may lead to the conclusion that they should not have a strong impact on the probabilistic stress predictions. However, upon including these correlations into the MCSs (which were used to predict \(\sigma_B\) for a wide range of input tilts) it was found that larger stresses were predicted when compared with the case where the error terms were treated independently. Furthermore, during the validation stage of this analysis (see Section 4.3), it was found that including correlations helped achieve predictions which were in better agreement with the \(\sigma_B\) data based on the deterministic FE results. Thus it was concluded that although modest, the correlations between stress component residuals are important and should be accounted for in probabilistic calculations.

### 4.3 Validation of probabilistic stress predictions

The validation procedure (outlined in Fig 6) involved producing mean, upper-lower confidence intervals (CIs) and maximum-minimum limits for the stress at the beginning of the dwell (\(\sigma_D\)) as predicted by the approach above. These are then compared with the deterministic \(\sigma_B\) based on the FE results. The procedure for calculating \(\sigma_B\) can be found in Appendix 7 of [1]. In this work this procedure was implemented with the three stress components at point B in Fig 2 being treated stochastically while the remaining parameters (including the stress components for transients, points A and C in Fig 2) were taken at fixed values and were obtained from [14]. An example of the results for this validation approach is shown in Fig 7b for TO = 1. A judgement on whether the probabilistic stress predictions are appropriate can be made by counting the number of data points that lie outside the CIs and the maximum-minimum limits. Ideally no data should lie outside the latter, whilst a small number can be allowed to lie outside the former. For example if 98% confidence intervals are used, then around 2% of the data should lie outside the upper-lower CIs. If these requirements are not achieved, then a refinement of the predictive approach may be needed. In the example shown in Fig 7a, these conditions were not satisfied which was attributed to the lack of correlations between stress component residuals. By comparing the two plots in Fig 7, it was concluded that in spite of the correlations being small (e.g. see Eq 4), they had a significant impact on the probabilistic stress predictions. Numeric measures which were used to compare the two plots in Fig 7 are summarised in Table 2, which clearly indicate that including correlations produced probabilistic predictions more in agreement with the deterministic data.

A further stage of validation can be added, although it was not implemented for this work, which includes a similar comparison to the one discussed above, but using deterministic data (called the validation data) separate to the data used to fit the least-squared regression models (the training data).

### 5 CONCLUSION

This work provides an example of treating steady-state stresses during nominal plant operation in a probabilistic creep-fatigue assessment. The component which was the subject of this work, the TPL, had 37 possible points of creep-fatigue crack initiation which were treated as independent assessment locations. The proposed approach effectively deals with the issue

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Percentage of data within 98% CI</th>
<th>Percentage of data within Max-Min limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncorrelated</td>
<td>91.4</td>
<td>96.7</td>
</tr>
<tr>
<td>Correlated</td>
<td>98.4</td>
<td>99.9</td>
</tr>
</tbody>
</table>
of choosing the assessment locations and the magnitude of the PSCs at these locations separately. A systematic approach was formalised which can be adopted with plant components of different geometries and comprising of multiple assessment locations. This included:

1. Processing the raw plant data.
2. Running a batch of typical loading conditions (based on history or DoE) in the thermal and mechanical (elastic) FE models.
3. Extract key data from the FE results.
5. Identify the key input parameters through sensitivity analysis.
6. If the data is not well behaved, some post-processing may be required (e.g. data segregation)
7. Fitting input(s)-output(s) relationships to predict the behaviour of the FE models. This can be done using LSR, RSM or ANN.
8. A critical investigation into which correlations should be included.
9. Characterising the uncertainty introduced by using this approach for predicting the outputs in the form of histograms of the relevant residuals.
10. Validating the probabilistic predictions this approach yields against deterministic data.

For a probabilistic creep-fatigue damage assessment, the value of this approach is in providing a means for approximating stresses as simple functions of key input(s). This mitigates the resorting to running time consuming FE models multiple times within one probabilistic MCS trial (of which there typically may be more than $10^4$, if not orders of magnitude larger), which would be computationally prohibitive.

6 FURTHER WORK

Natural progressions for the work presented in this paper include:

- The probabilistic modelling of the metal temperatures at the assessment locations during steady-state operation as functions of input fluid temperature(s).
- The extension of current probabilistic stress modelling to include transient stresses which directly influence the extremities of the hysteresis cycle (e.g. points A and C in Fig 2).

Furthermore, a full probabilistic assessment of the TPL component using R5 V2/3 for creep-fatigue crack initiation is planned. This will implement the findings of this work as well as any further work related to transient stresses. Finally, the main purpose of this collective body of work is to promote a shift from over-conservative deterministic assessments towards a probabilistic framework. The aim of which is to identify, quantify and incorporate real-life uncertainties in high temperature assessment procedures.

ACKNOWLEDGMENT

The authors would like to express their gratitude for the support of EDF Energy towards this project and in particular Marc Chevalier and Joy Tao.

References


Copyright © 2018 by ASME
(a) Results from analysis which did not included correlations.

FIGURE 7: For the most stressed tubehole (i.e. TO = 1), this figure shows probabilistic stress predictions for a range of tilts superimposed onto the deterministic values of $\sigma_B$ for validation purposes.

(b) Results from analysis which included correlations.


