So You Think Magnetic Monopoles Do Not Exist?

Whether or not magnetic monopoles exist is just a matter of convention, not fact. The true position is that all particles appear to have the same ratio of electric charge to magnetic monopole strength. This empirical fact makes it possible to consistently claim that all monopoles are zero. This is established simply as follows.

Maxwell's equations are,

$$-\overline{\nabla}\times\overline{E}=\frac{\partial\overline{B}}{\partial t} \qquad \overline{\nabla}\cdot\overline{D}=\rho_e \qquad (1,2)$$

$$\overline{\nabla}\times\overline{H}=\frac{\partial\overline{D}}{\partial t}+\overline{J}_e \qquad \overline{\nabla}\cdot\overline{B}=0 \qquad (3,4)$$

Look, no monopoles! There are source terms corresponding to the 4-current of electric charge, $j_e^\mu=\left(\rho_e,\overline{J}_e\right)$, but no such source terms for magnetic monopoles. Consequently the equations are rather unsymmetrical. But suppose magnetic monopoles *did* exist. The equations would then take the more symmetrical form,

$$-\overline{\nabla}\times\overline{E}=\frac{\partial\overline{B}}{\partial t}+J_m \qquad \overline{\nabla}\cdot\overline{D}=\rho_e \qquad (5,6)$$

$$\overline{\nabla}\times\overline{H}=\frac{\partial\overline{D}}{\partial t}+\overline{J}_e \qquad \overline{\nabla}\cdot\overline{B}=\rho_m \qquad (7,8)$$

In Equs.(5-8) there are now also source terms corresponding to the 4-current of magnetic 'charge', $j_m^\mu=\left(\rho_m,\overline{J}_m\right)$. But note a curious thing. If we carry out the following so-called *duality transformation*,

$$\overline{E}=\overline{E}'\cos\xi+\overline{H}'\sin\xi \qquad \overline{D}=\overline{D}'\cos\xi+\overline{B}'\sin\xi \qquad (9,10)$$

$$\overline{H}=-\overline{E}'\sin\xi+\overline{H}'\cos\xi \qquad \overline{B}=-\overline{D}'\sin\xi+\overline{B}'\cos\xi \qquad (11,12)$$

$$\rho_e=\rho_e'\cos\xi+\rho_m'\sin\xi \qquad \overline{J}_e=\overline{J}_e'\cos\xi+\overline{J}_m'\sin\xi \qquad (13,14)$$

$$\rho_m=-\rho_e'\sin\xi+\rho_m'\cos\xi \qquad \overline{J}_m=-\overline{J}_e'\sin\xi+\overline{J}_m'\cos\xi \qquad (15,16)$$

we find that the Maxwell equations remain true if all fields and all source terms are replaced by the dashed quantities. The duality transformation is a symmetry of the equations.

Moreover, nothing which is physically measurable is affected. For example, the energy-momentum tensor of the electromagnetic field, $T^{\mu\nu}$, is invariant under the transformations (9-12), as is readily seen by confirming that both $E_iD_j+H_iB_j$ and $\overline{E}\times\overline{H}$ are invariant.

What about the effect of the electromagnetic field on the motion of a particle? We can no longer assume the Lorentz force $\overline{F}=\left(\rho_e\overline{E}+\overline{J}_e\times\overline{B}\right)$ because there is now a magnetic 'charge' to create an extra force. Symmetry suggests that the force should become $\overline{F}=\left(\rho_e\overline{E}+\overline{J}_e\times\overline{B}\right)+\left(\rho_m\overline{H}-\overline{J}_m\times\overline{D}\right)$. This is readily confirmed as invariant under the transformations (9-16) also.

Consequently, because nothing physical depends upon the value we choose for the symmetry parameter ξ we can set it to any value we wish. Suppose, then, that we choose to set it such that $\tan\xi = \frac{q'_m}{q'_e}$ where q'_e, q'_m are the electric and magnetic charges for a given type of particle. It follows that for any number of these particles in an arbitrary state of motion $j'^m_\mu = j'^e_\mu \tan\xi$ and hence, from (15,16), that $j^m_\mu = 0$. So with this convention, in the un-dashed system there is no magnetic source corresponding to this particle type. The magnetic monopole for this particle has been transformed away.

But does this mean there are no monopoles at all? What about other particle types? Well, if there was another particle type which had a *different* ratio $\frac{q'_m}{q'_e}$ we would not be able to work simultaneously with two different values of $\tan\xi$. In this case, one or other of the particle types would retain a magnetic monopole 'charge' whatever convention we adopted for ξ. We would only be able to conclude that all monopoles could be transformed away if all particles have the same ratio $\frac{q'_m}{q'_e}$ for an arbitrary ξ.

So we conclude that…

(A) The true ξ-invariant, and thus absolute, claim is that all particles have the same magnetic monopole to electric charge ratio. This is a contingent fact, i.e., it requires experiment or observation to confirm or deny it.

(B) Claiming that there are no magnetic monopoles is not a ξ-invariant concept. It is only a convention achieved by choosing ξ appropriately and upon the assumption that (A) is true.