Never Use Intuition to Guess Probability

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The Monty Hall Game

The game show host, Monty Hall, shows the contestant three closed boxes. Inside just one is a prize (and Monty knows which one, of course). The contestant chooses a box. Monty then takes away one of the remaining two boxes saying the prize is not in that box. He then gives the contestant the option of changing his choice to the (now only) alternative box. Is it advantageous for the contestant to change his mind?

The interesting thing about this puzzle is that even strong mathematicians tend to get it wrong – and find it hard to believe the answer even when told. Intuition suggests that both remaining boxes have a 50/50 chance – so there is no advantage in the contestant changing his mind. This is wrong! Actually the contestant doubles his chances of winning by changing to the other box.

The easiest way of seeing this is to increase the number of boxes to 100. Have Monty Hall remove 98 boxes. Obviously it is overwhelmingly more likely that the prize is in the final box that Monty left behind – not in the box the contestant picked.

Analysis of the 3 box case is as follows: Call the boxes A, B and C, and suppose the prize is in box A. Picking the right box (A) initially has a probability of 1/3. If box B were picked, then Monty is obliged to take away box C leaving box A as the alternative. Conversely, if box C were picked, then Monty is obliged to take away box B again leaving box A as the alternative. In both these cases, which each have a probability of 1/3, the prize is obtained if the contestant changes to the box Monty has left. So the probability of getting the prize by changing boxes is 2/3, whilst the probability of getting the prize by sticking is only 1/3.

A Man has Two Children…

For the purposes of these questions, ignore the possibility of twins and assume girls and boys are equally likely.

Version 1: A man tells you that he has two children and one is a boy. What is the probability that the man has two sons?

Version 2: A man tells you that he has two children and the elder is a boy. What is the probability that the man has two sons?

Version 3: A man tells you that he has two children and one is a boy born on a Tuesday. What is the probability that the man has two sons?

Version 4: A man tells you that he has two children and the elder is a boy born on a Tuesday. What is the probability that the man has two sons?

The answers are,

Version 1: 1/3

Version 2: 1/2

Version 3: 13/27

Version 4: 1/2

It is hard to believe that being told that the boy in question is the elder child can change the probability that the other child is a boy. But it does.

For version 1 the possibilities are: BB, BG, GB, GG. Being told that one child is a boy only rules out option GG, leaving BB, BG, GB. Hence the BB possibility is 1 out of 3, i.e., a probability of 1/3. **QED**.

For version 2 the possibilities initially are: BB, BG, GB, GG where we now interpret the first specified as the elder child. Being told that the elder child is a boy now rules out both GB and GG, leaving BB, BG. Hence the BB possibility is 1 out of 2, i.e., a probability of 1/2. **QED**.

Having grasped that it is still hard to believe that Tuesday can have anything to do with it. But it does…

For version 3 we must now recognise that there are not two types of child (B and G) but 14. Each boy may have been born on any of 7 days, as may each girl, making 14 altogether. We are told that one child is a boy born on a Tuesday. Labelling the children arbitrarily as A and B, suppose that A is a boy born on a Tuesday. Then B may be any of the 14 possibilities. Now suppose that B is a boy born on a Tuesday. Then A may be any of the 14 possibilities, but if we choose A to also be a boy born on a Tuesday we must remember that we have already counted that possibility before. So there are 13 new possibilities, making 27 in all. How many of these involve two boys? Well, of the 14 possibilities for B when A is the boy-Tuesday, exactly half, i.e., 7, are boys. But of the 13 possibilities for A when B is the boy-Tuesday only 6 are boys (because the rejected double-accounting case is a boy). Hence 13 of the 27 possibilities involve two boys. **QED**.

For version 4 we can repeat the analysis as for version 3, but we now interpret A as the elder child. Hence it is A who must be the boy-Tuesday. So there are 14 possibilities for B of which half (7) are boys. The probability of two boys is thus 7/14 = 1/2. **QED**.

Whether a piece of information changes the probability depends upon what other information you have. Thus, being told that the boy in question was born on a Tuesday changes the probability when you have no other information (compare versions 1 and 3). But being told that the boy in question was born on a Tuesday does not change the probability when you already know that he is the elder child (compare 2 and 4).