Why is Interference Destroyed by "Which Path" Information?

Last Update: 24/9/11

Consider a familiar two-slit arrangement. An interference pattern is found on the screen. In terms of the Hilbert space states this can be understood as follows. The state of the wave/particle prior to hitting the screen is a coherent superposition of the two states representing passage through each slit, thus,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|U\rangle + |L\rangle\right) \tag{1}$$

where *U* and *L* stand for the upper and lower slits. Each of the states $|U\rangle$ and $|L\rangle$ is normalised to unity, representing a single particle, and hence so is $|\psi\rangle$. Consider a given position, *x*, on the screen. Coherence means that we can consider the upper wave differing from the lower only by some phase difference which depends upon *x*, $\theta(x)$, so that $|U\rangle = e^{i\theta(x)}|L\rangle$. Consequently (1) becomes,

$$|\psi\rangle = \frac{|L\rangle}{\sqrt{2}}\left(e^{i\theta(x)} + 1\right) \tag{2}$$

The intensity of the pattern at position x on the screen is proportional to $|\psi|^2 = \langle\psi|\psi\rangle$ which (2) gives to be,

$$\langle\psi|\psi\rangle = 1 + \cos(\theta(x)) \tag{3}$$

Equ.(3) produces the familiar interference pattern, with alternating dark bands corresponding to θ taking values equal to half-integral multiples of π (zero intensity), and bright bands where θ equals an integral multiple of π (intensity 2 units). The average intensity is unity, as it should be.

Now let us introduce a two-state device, *M*, in the beam path of the lower slit. This device is set up in state $|M:U\rangle$ but it will change to state $|M:L\rangle$ if a particle passes through it. In other words *M* is a device which records which path the particle takes.

Readers will recall that such a measurement is expected to destroy the interference pattern. But why should this always be the case – for any sort of measurement, *M*, however we choose to contrive it? The impression that is sometimes given is that the wave/particle is physically disturbed by the measurement, and that it is this physical disturbance which destroys the interference. This is seriously misleading and misrepresents the nature of quantum mechanics. The vanishing of the interference arises from the very possibility of distinguishing paths and is a result of the state algebra alone (or, if you will, the quantum logic). The means by which the "which path" information is obtained, however subtle, is irrelevant.

Suppose that *M* is a perfect measuring device, and suppose that we are certain that exactly one particle has passed through our apparatus. This means that *M* will infallibly be left in state $|M:L\rangle$ if the particle takes the lower path and infallibly be left in state $|M:U\rangle$ if the particle takes the upper path. However, to be a prefect measurement we must be able to distinguish with certainty between the two states $|M:L\rangle$ and $|M:U\rangle$. In other words, these two states must be orthogonal,

Perfect Measurement: $$\langle M:L|M:U\rangle = 0 \tag{4}$$

Prior to any interaction between our particle and the measuring device their combined state is just the direct product state $|\Psi\rangle = |\psi\rangle|M:U\rangle = \frac{1}{\sqrt{2}}(|U\rangle + |L\rangle)|M:U\rangle$ because M is set up in state $|M:U\rangle$. However, after the wave/particle has had opportunity to interact with M the state evolves into,

$$|\Psi\rangle \to |\Psi'\rangle = \frac{1}{\sqrt{2}}(|U\rangle|M:U\rangle + |L\rangle|M:L\rangle) \tag{5}$$

The intensity of the signal received on the screen is proportional to $\langle\Psi'|\Psi'\rangle$ which, from (5), is now simply,

$$\langle\Psi'|\Psi'\rangle = 1 \tag{6}$$

i.e., uniform illumination with no interference fringes. Why? Because the orthogonality of $|M:L\rangle$ and $|M:U\rangle$ causes the two terms on the RHS of (5) to be orthogonal. The cross-product between them is now zero, whereas previously it was this cross-product which gave rise to the $\cos\theta$ terms in (3) and hence the interference.

Hence it is now clear that the nature of the measurement, *M*, is irrelevant. The interference disappears by virtue of the perfect nature of the measurement as expressed by the orthogonality of the measurement state, (4).

The destruction of the interference pattern need not b an all-or-nothing affair. Suppose that *M* is an imperfect measuring device which has $\langle M:L|M:U\rangle \neq 0$.

$$\langle\psi|\psi\rangle = 1 + \Re\left(e^{i\theta(x)}\langle M:L|M:U\rangle\right) \tag{7}$$

Consequently the interference pattern is merely dimmed if $0 < |\langle M:L|M:U\rangle| < 1$.

In the early days of quantum mechanics, great emphasis was placed on measurements requiring physical interaction with the measured system and the (supposed) impossibility of making measurements without disturbing what was attempting to be measured. Such physical disturbance was held to be responsible for the destruction of the interference in cases like that considered above. However, this demonstrably incorrect. A graphic illustration of this is provided by the Elitzur-Vaidman bomb test. This involves a bomb whose sensitivity is so extreme that it is guaranteed to exploded if exposed any physical interaction whatsoever. If this bomb is used as a measuring device it can successfully destroy an interference pattern without exploding.

Another way of appreciating this point is to note that the evolution of the state into the form of Equ.(5) does not really constitute a measurement. It is merely the first part of a measurement process. The second part is the 'collapse of the wavepacket' in which only one or other of the two possibilities, either $|U\rangle|M:U\rangle$ or $|L\rangle|M:L\rangle$, actually realised. But if the combined particle-plus-*M* system is unperturbed by interaction with any third party, including our own selves, then their combined state remains as given in (5). But this state can be obtained by a purely unitary evolution. In other words, $|\Psi\rangle \to |\Psi'\rangle$ is calculable from the Schrodinger equation, so that $|\Psi(t)\rangle = \exp\left\{-\frac{i}{\hbar}\hat{H}t\right\}|\Psi(0)\rangle$ where $\hat{H}$ is the appropriate Hamiltonian operator.

Consequently it is possible, by another unitary evolution, for (5) to be converted back again, clearly so because $|\Psi(0)\rangle = \exp\left\{\frac{i}{\hbar}\hat{H}t\right\}|\Psi(t)\rangle$. This is known as "quantum erasure" and will be considered further in Chapter ? The quantum erasure process re-establishes an interference pattern which would otherwise have been destroyed by the interaction with *M*. This would not be possible for a genuine measurement since the collapse of the wavepacket is irreversible.

Three Slits and Partial Information

What happens if we use three slits? In obvious notation, and with no measuring device, the state is,

$$|\psi\rangle = \frac{1}{\sqrt{3}}\left(|1\rangle + |2\rangle + |3\rangle\right) \tag{8}$$

Suppose the relative phase between the waves from slits *j* and *k* is $\theta_{jk}(x)$ at position *x* on the screen, then there is an interference pattern on the screen given by,

$$\langle\psi|\psi\rangle = 1 + \frac{2}{3}\left[\cos\theta_{12}(x) + \cos\theta_{23}(x) + \cos\theta_{13}(x)\right] \tag{9}$$

For example, if $\theta_{12} = \theta_{23} = \theta$ and $\theta_{13} = 2\theta$ then,

$$\langle\psi|\psi\rangle = 1 + \frac{2}{3}\left[2\cos\theta(x) + \cos 2\theta(x)\right] \tag{10}$$

The average is unity, as it must be, and the intensity varies from 0 to 3.

Now, suppose we introduce our two-state device *M* into the path of slit 3 (only). This does not provide a measurement which discriminates between slits 1 and 2. So does a modified interference pattern remain, namely that due to slits 1 and 2 alone? The answer is "yes" as may be seen by considering the state algebra as follows. The combined state evolves thus,

$$|\Psi\rangle \to |\Psi'\rangle = \frac{1}{\sqrt{3}}\left(|1\rangle|M:U\rangle + |2\rangle|M:U\rangle + |3\rangle|M:L\rangle\right) \tag{11}$$

Here $|M:U\rangle$ is the state in which *M* is initially setup, and hence it remains in this state if the particle takes paths 1 or 2. Only path 3 results in the modified *M* state, $|M:L\rangle$. Consequently when we form the absolute square of the combined state, which provides the intensity seen at the screen, the third term in (11) is orthogonal to the first two, but the first two terms continue to provide an interference cross-product,

$$\langle\Psi'|\Psi'\rangle = 1 + \frac{2}{3}\cos\theta(x) \tag{12}$$

Hence (12) continues to show an interference pattern, though diminished in contrast from previously. The minima are now 1/3 and the maxima at 5/3, compared with 0 and 3 when all three slits contribute. The average is again unity, as it must always be.

This is a further illustration of how partial information will degrade the interference pattern without removing it completely.