Entanglement, Interference and Delayed Erasure: Various Conundrums Illustrating Quantum Weirdness, But <u>Not</u> Faster-Than-Light Communication

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1. A False Conundrum

We have seen previously that interference type phenomena are destroyed if we make "which state" measurements, http://rickbradford.co.uk/QM13Counterfactuals.pdf.

What will happen if we attempt to perform an interference experiment with one half of an entangled pair? Let's say a pair of particles are produced in an entangled state, and the particles then fly apart. When the particles are safely at a space-like separation we use one of them, say particle A, in an interference experiment. This might be a Mach-Zehnder interferometer or a double slit diffraction grating. The conundrum comes about if we consider making a measurement on the other particle, B. By this means we determine the state of particle A as well, due to their entanglement. Does this destroy the interference pattern made by particle A?

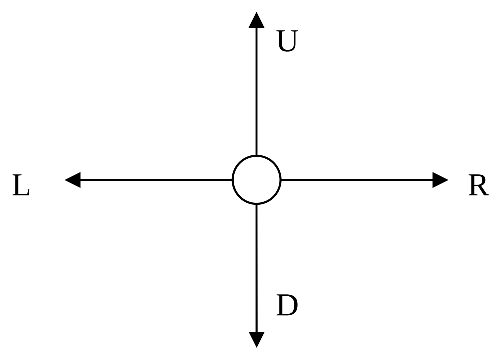
If the measurement of particle B does *not* destroy the interference pattern made by particle A, then we seem to have violated the rule that determining the state of A *will* destroy interference phenomena. On the other hand, if the measurement of particle B *does* destroy the interference pattern made by particle A, then we have created a means of trans-luminal communication. The disappearance of the interference at particle A can be induced, instantaneously, by a measurement carried out at particle B. This should not be possible since A and B are space-like separated. So both options are unacceptable.

Where did we go wrong? There are crucial ambiguities in the above description as regards both what is being measured and also what we mean by interference. These points are illustrated by the examples which follow.

2. Entangled Photons Input to Mach-Zehnder Interferometers

Suppose we have a means of generating pairs of entangled photons. Further suppose that these photons can be emitted only along either the x-axis or the y-axis, and these occur with equal probability. This can be represented diagrammatically by,

Figure 1



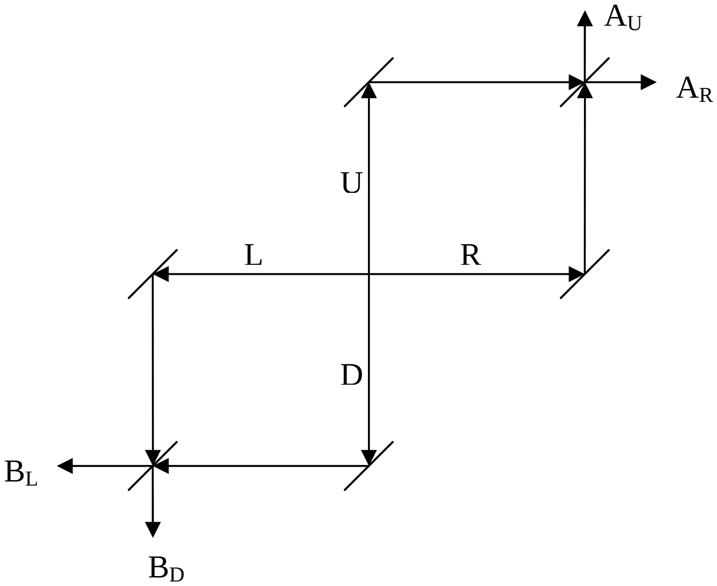
The corresponding quantum state is,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|R\rangle|L\rangle + |U\rangle|D\rangle\right) \tag{1}$$

This means that either the two photons emerge in the left and right directions, or in the up and down directions. I don't know whether such an arrangement would be easy to achieve experimentally, but this does not matter to the principle being illustrated. The two photons are entangled since (1) is not a product state.

Now suppose we erect two Mach-Zehnder interferometers around these photons, thus,

Figure 2



The first mirror that each photon encounters is a full mirror. These induce phase changes by a factor -1. The second mirror is half-silvered (i.e., a beam splitter). We refer to the top right interferometer as the A device (or the A photon), and the bottom left as B. The labels A_U, A_R, B_L, B_D are four photon detectors.

The two beam splitters both have their silvered surface (their "front") on the lower surface as shown in the diagram. The phase changes caused by the beam splitters are,

- Reflection off the front (silvered) face causes a factor of -1;
- Transmission causes a factor of $e^{i\Delta}$ where Δ depends upon the thickness and refractive index of the glass substrate;
- Reflection off the back of the mirror causes a factor $e^{2i\Delta}$ due to the passage twice through the glass substrate (the reflection itself causing no phase change).

These rules are derived in http://rickbradford.co.uk/QM13Counterfactuals.pdf. Also, the state after a beam splitter picks up a factor of $1/\sqrt{2}$.

Using these rules we can work out what the quantum state will be for photons entering any of the four detectors. Before we do this, though, pause to consider what we might expect based on our knowledge of Mach-Zehnder interferometers, and in the context of the conundrum of §1.

Consider the A device. Since the initial state is a superposition of U and R, we would expect that all the photons would emerge into just one of the detectors A_U and A_R, and none in the other (providing that we have tuned the set-up appropriately). For an explanation see http://rickbradford.co.uk/QM13Counterfactuals.pdf. This behaviour is the manifestation of interference for this device. Let's say all the photons would be expected in detector A_R and none in A_U. Now suppose we make a "which path" measurement in device B. We can make the arms of the B interferometer very long so as to ensure that this measurement is at a space-like separation from the A-detectors. We now know which path contains the photon in the B device, and hence we also know which path contains the photon in the A device. But this must destroy the interference in the A device (as well as in the B device) and this will be apparent because the A_U detector will start registering photons. So we have achieved faster than light communication!

Of course we have not really achieved FTL communication. It turns out that where we went wrong in this analysis is in assuming that the entangled photons in this set-up behave in the same way as single, un-entangled photons in a Mach-Zehnder interferometer. They do not, as we will now show.

Consider firstly photons entering detector A_U. We call their state $|A_U\rangle$. This state can be arrived at via either of paths R or U. Following the phase factors at the mirrors and beam splitter we find that the input photon state becomes,

$$|\psi\rangle \to |\psi : A_U\rangle = \frac{1}{2}\left(-e^{i\Delta}|A_U\rangle|L\rangle - e^{2i\Delta}|A_U\rangle|D\rangle\right) = -\frac{e^{i\Delta}}{2}|A_U\rangle\left(|L\rangle + e^{i\Delta}|D\rangle\right) \qquad (2)$$

Similarly the state entering detector A_R is,

$$|\psi\rangle \to |\psi : A_R\rangle = \frac{1}{2}\left(-1\times -1|A_R\rangle|L\rangle - e^{i\Delta}|A_R\rangle|D\rangle\right) = \frac{1}{2}|A_R\rangle\left(|L\rangle - e^{i\Delta}|D\rangle\right) \qquad (3)$$

Assuming orthogonality, $\langle L|D\rangle = 0$, we see that $|\psi : A_U\rangle$ and $|\psi : A_R\rangle$ are also orthogonal. The square moduli of both these states is ½. Hence half the photons enter each detector. There is no interference (which would have been characterised by all photons entering just one detector).

This exposes the error in the initial analysis: the entangled photons do not exhibit interference in this set-up. So there is no FTL communication.

But there is another sense in which interference *does* occur in this experimental arrangement – but without any possibility of FTL communication. This involves correlated behaviour between the photons in the A and B interferometers. To see this, consider how the state in (2) is modified by the other photon's passage through the B interferometer. The state for entry into both A_U and B_D is,

$$|\psi : A_U, B_D\rangle = -\frac{e^{i\Delta}}{2\sqrt{2}}|A_U\rangle\left(-e^{i\Delta}|B_D\rangle + e^{i\Delta}\times(-1)^2|B_D\rangle\right) = 0 \qquad (4)$$

The state for entry into both A_U and B_L is,

$$|\psi : A_U, B_L\rangle = -\frac{e^{i\Delta}}{2\sqrt{2}}|A_U\rangle\left(-e^{2i\Delta}|B_D\rangle + e^{i\Delta}\times\left(-e^{i\Delta}\right)|B_D\rangle\right) = \frac{e^{3i\Delta}}{\sqrt{2}}|A_U\rangle|B_D\rangle \qquad (5)$$

We conclude that if a photon is detected in detector A_U then there will be no coincident photon in detector B_D, but rather there will always be a coincident photon in detector B_L. For photons entering A_R we find,

$$\left|\psi : A_R, B_D\right\rangle = \frac{1}{2\sqrt{2}}\left|A_R\right\rangle\left(-e^{i\Delta}\left|B_D\right\rangle - e^{i\Delta}\times(-1)^2\left|B_D\right\rangle\right) = -\frac{e^{i\Delta}}{\sqrt{2}}\left|A_R\right\rangle\left|B_D\right\rangle \qquad (6)$$

$$\left|\psi : A_R, B_L\right\rangle = \frac{1}{2\sqrt{2}}\left|A_R\right\rangle\left(-e^{2i\Delta}\left|B_D\right\rangle - e^{i\Delta}\times\left(-e^{i\Delta}\right)\left|B_D\right\rangle\right) = 0 \qquad (7)$$

So similarly we conclude that if a photon is detected in detector A_R then there will be no coincident photon in detector B_L, but rather there will always be a coincident photon in detector B_D. Note that the four states in (4-8) maintain normalisation to unity.

So we see that interference *does* occur in a modified sense that depends upon coincident observations of photons in the A and B devices. Recall that the signature of interference in a Mach-Zehnder interferometer is that all the photons appear in one detector and none in the other. This is indeed found for the B device, provided that we filter out all the instances in which A_U records a photon and keep only cases when A_R detects a photon (or *vice-versa*). But this type of coincident, or correlated, interference does not provide any means of FTL communication. If the A detectors are at a space-like separation from the B detectors, we can only discover the 'interference' some time later, when a classical signal has communicated the results of the B detectors to the operator at A.

Caution is needed when reading the literature since this type of coincident, or correlated, interference is sometimes simply called "interference" without qualification. This is horribly confusing since true 'local' interference in such situations *would* violate causality.

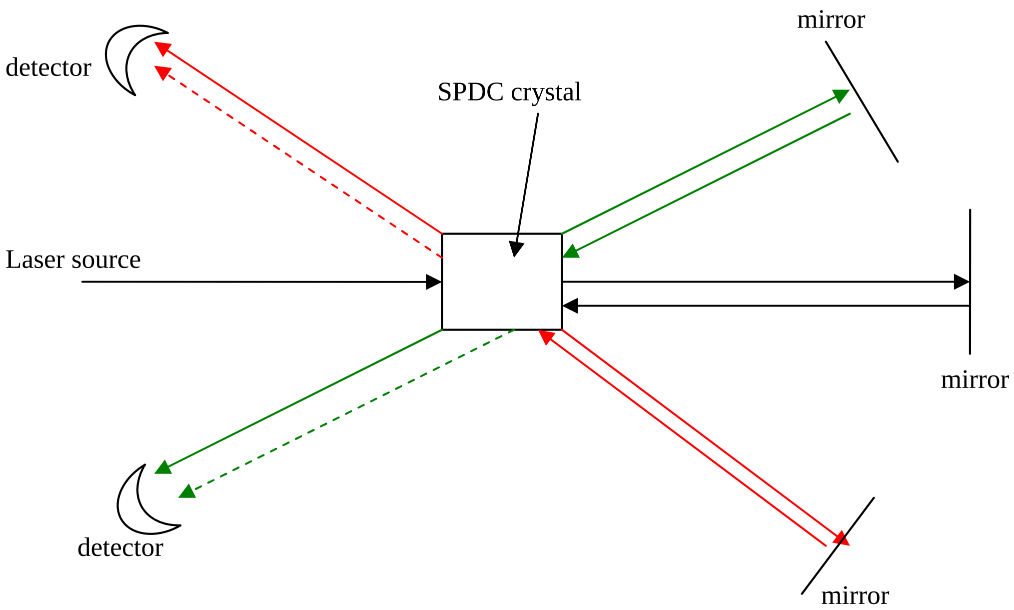
3. Entangled Interference with Crossed SPDC Beams

Here we illustrate exactly the same phenomena described in §2 but for what initially appears to be a very different experimental arrangement. The algebraic properties, though, turn out to be essentially the same. This apparatus uses spontaneous parametric down-conversion (SPDC) which splits an input photon into two identical photons of half the energy. This can be done using, for example, a lithium iodate crystal. The arrangement is shown in Figure 3.

The incoming beam from the laser is split by the SPDC crystal into two beams (shown green and red) which emerge at some characteristic angle. These beams are reflected off mirrors back through the crystal and into detectors (continuous green and red lines). These beams are not significantly affected by the crystal on their second passage (due to their reduced energy). However, the crystal acts as a beam splitter also in another sense. Not all the incoming beam is initially down-converted. What is not is transmitted through the crystal and gets reflected back to the crystal by a mirror. On this second passage there is therefore a second down-conversion which creates the green and red *dashed* beams. These are also directed into the same detectors.

Hence, a 'green' photon can reach the detector by either of two paths: by being down-converted at the first pass and following the continuous green beam, or by being down-converted at the second pass and following the dashed green beam. These two beam paths can (potentially) cause interference at the detectors. But do they? We are now alert to the possibility that they may not, by virtue of being one half of an entangled pair.

Figure 3



How is the state of the photon expressed algebraically? If the 'green photon' follows the continuous line, then so does the red photon. Conversely, if the green photon follows the dashed line, then so does the red photon. So the state of the entangled pair at the detectors is,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|G\rangle|R\rangle + |Gd\rangle|Rd\rangle\right) \tag{8}$$

Here $|G\rangle, |R\rangle$ represent the continuous green and red beams, and $|Gd\rangle, |Rd\rangle$ represent the dashed beams. We can express the Hilbert states in position representation, using the variable *x* for the green beams and *y* for the red beams. So (8) becomes,

$$\psi(x,y) = \frac{1}{\sqrt{2}}\left(\psi_G(x)\psi_R(y) + \psi_{Gd}(x)\psi_{Rd}(y)\right) \tag{9}$$

But the dashed beams differ from the continuous beams where they enter the detectors only by their phase (because they are coherent). So we can write,

$$\psi_{Gd}(x) = e^{i\theta_G}\psi_G(x) \quad \text{and} \quad \psi_{Rd}(y) = e^{i\theta_R}\psi_R(y) \tag{10}$$

Here the phase angles will be functions of position, and so properly written $\theta_G(x)$ and $\theta_R(y)$. Hence (9) becomes,

$$\psi(x,y) = \frac{1}{\sqrt{2}}\psi_G(x)\psi_R(y)\left(1 + e^{i(\theta_G + \theta_R)}\right) \tag{11}$$

Now for plane waves we can take $|\psi_G| = |\psi_R| = 1$ so that the square modulus of (11) is,

$$|\psi(x,y)|^2 = 1 + \cos(\theta_G + \theta_R) \tag{12}$$

If we ignore whereabouts on the y-screen the red photon is detected, do we see an interference pattern on the x-screen which is detecting the green photon? The answer is “no” since the intensity on the x-screen is then the sum of (12) over all y values, i.e.,

$$\langle|\psi(x,y)|\rangle_y = \frac{1}{2\pi}\int_0^{2\pi} d\theta_R\left(1+\cos(\theta_G+\theta_R)\right) = 1 \qquad (13)$$

This retains no θ_G (which is equivalent to *x*) dependence, and hence is a uniform illumination without an interference pattern.

However, just as with the Mach-Zehnder interferometer example, we can resurrect an interference pattern by considering correlated measurements. Thus, we choose any position on the y-screen (i.e., any θ_R) and scan our detector slowly over the x-screen recording counts only if there is a coincident count at the fixed y-position detector. The pattern which emerges from the x-screen is just (12), noting that $\theta_G \propto x$, for a constant value of θ_R. This is an interference pattern, varying from 0 to 2 as the cosine varies from -1 to +1.

The absence of a “local” interference pattern, i.e., one which does not require knowledge of the results from the other detector, saves us from a causality disaster. However, interference is occurring but can only be revealed when the results of both detectors are brought together – by some classical (sub-luminal) communication.

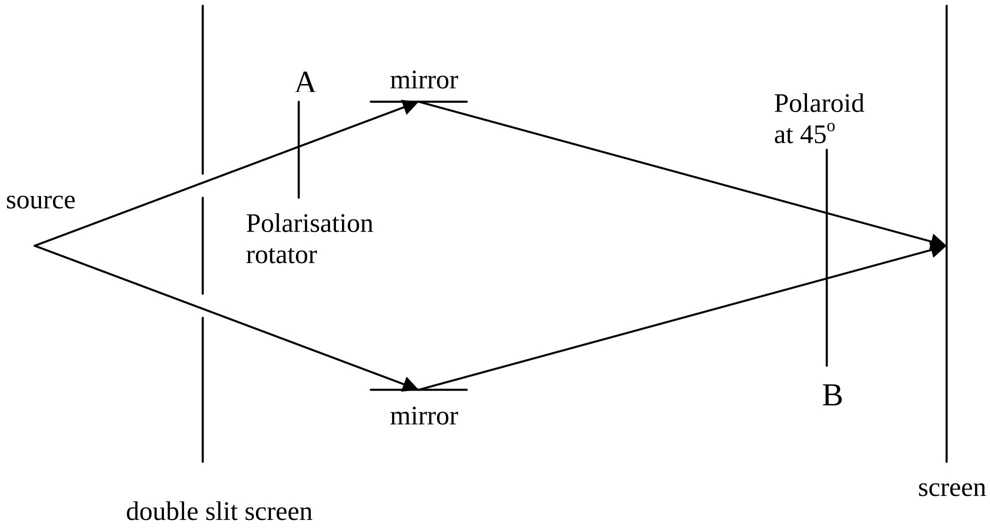
So this SPDC crossed-beam arrangement is essentially the same as the Mach-Zehnder example. The reason is that the algebraic structure of the quantum states, given by (8) and (1) respectively, are the same.

4. Quantum Erasure – A Simple Example

Recall that if we measure which path a photon travels by in an interferometer then we destroy the interference pattern. Remarkably such a “measurement” can be undone – or erased – and the interference pattern regained. There is a caveat, though, which most sources are guilty of failing to emphasise. The caveat hinges upon exactly what is meant by “measurement”. We shall return to this at the end of this section. Firstly a brief reminder of how “which path” information destroys interference.

Consider a double slit interference experiment as shown in Figure 4.

Figure 4



The source is assumed to supply vertically polarised light. Initially the polarisation rotator (A) and the 45° Polaroid (B) are not present. There is then an interference pattern on the screen. Algebraically this occurs as follows. The state arriving at the screen can be written,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|U\rangle + |L\rangle\right) \tag{14}$$

Here $|U\rangle, |L\rangle$ represent the upper and lower beam paths. At the screen they will differ by a phase $e^{i\theta}$, so that, in the x-basis, where x is the position on the screen, we have,

$$\psi(x) = \frac{1}{\sqrt{2}}\left(1 + e^{i\theta}\right)\psi_U(x) \tag{15}$$

Hence,

$$|\psi(x)|^2 = 1 + \cos\theta(x) \tag{16}$$

(assuming $\psi_U(x)$ is a plane wave with $|\psi_U| = 1$) which displays the usual interference pattern.

We now insert the polarisation rotator (A), which we arrange to rotate the polarisation of the upper beam to the horizontal. (The 45° Polaroid is still not present). Since the upper and lower beams are now distinguishable by their distinct polarisations we expect the interference pattern to disappear. This is indeed the case, and the reason may be seen algebraically as follows. The polarisation part of the state will be written $|v\rangle$ or $|h\rangle$, for vertical and horizontal polarisation respectively. These are perfectly distinguishable states, so that $\langle v|h\rangle = 0$. The state before encountering the polarisation rotator is $\frac{1}{\sqrt{2}}\left(|U\rangle|v\rangle + |L\rangle|v\rangle\right)$ but afterwards it is,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|U\rangle|h\rangle + |L\rangle|v\rangle\right) \tag{17}$$

In the x-basis this becomes,

$$\psi(x) = \frac{1}{\sqrt{2}}\left(1 \cdot |h\rangle + e^{i\theta} \cdot |v\rangle\right)\psi_U(x) \tag{18}$$

Hence,
$$|\psi(x)|^2 = 1 \tag{19}$$

as a consequence of $\langle v|h\rangle = 0$. So the interference pattern has disappeared. From this it is algebraically transparent how the distinguishability of the two paths eradicates the interference: it makes the two terms in (17) orthogonal and hence the cross-product is zero.

Now, what about erasure? Can we undo the effects of the polarisation rotator later? We insert the 45^o Polaroid. Let us write the state emerging from this Polaroid as $|45\rangle$, and the perpendicular polarisation as $|-45\rangle$. We can express the vertical and horizontal polarisations in terms of these as,

$$|v\rangle = \frac{1}{\sqrt{2}}\left(|45\rangle + |-45\rangle\right) \tag{20}$$

$$|h\rangle = \frac{1}{\sqrt{2}}\left(|45\rangle - |-45\rangle\right) \tag{21}$$

Substituting (20,21) into (17) gives,

$$|\psi\rangle = \frac{1}{2}\left(|U\rangle\left(|45\rangle - |-45\rangle\right) + |L\rangle\left(|45\rangle + |-45\rangle\right)\right) \tag{22}$$

The 45^o Polaroid traps the component $|-45\rangle$ and lets through the component $|45\rangle$. So the state reaching the screen is,

$$|\psi\rangle = \frac{1}{2}\left(|U\rangle|45\rangle + |L\rangle|45\rangle\right) = \frac{1}{2}\left(|U\rangle + |L\rangle\right)|45\rangle \tag{23}$$

Because the unique polarisation state factors, we now have once again,

$$|\psi(x)|^2 = \frac{1}{2}\left(1 + \cos\theta(x)\right) \tag{24}$$

So the interference fringes are regained! The "which path" information transiently provided by the polarisation rotator has been successfully erased by the Polaroid. (Note that the reason why (24) has a mean square modulus of only ½ is because the Polaroid has absorbed half the photons).

Are we entitled, though, to refer to what the polarisation rotator does as being "a measurement"? I suggest not. A measurement actually consists of two things,

- the establishment of entanglement between the system being measured and the apparatus, and,
- the "collapse of the wavepacket", i.e., the actual selection of just one of the possible outcomes.

If the second of these steps had occurred in respect of our so-called measurement due to the polarisation rotator, then, before reaching the Polaroid, the state would have been either $|\psi\rangle = |U\rangle|h\rangle$ or $|\psi\rangle = |L\rangle|v\rangle$ but not $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|U\rangle|h\rangle + |L\rangle|v\rangle\right)$. It would then have been quite impossible to reverse such a *true* measurement to yield an interference pattern. Instead, the polarisation rotator carries out only the first step of a measurement. The state remains coherent, with no increase in entropy, and hence is reversible.

The morale is that the term "measurement" is used rather loosely in the literature. Measurement in the restricted sense of merely bringing the system into entanglement with a set of pointer states ***is*** sufficient to eliminate interference, by virtue of the above algebra (i.e., that the state is not a product state). And this is reversible, unlike true measurement.

5. Quantum Erasure in the Crossed SPDC Beam Setup

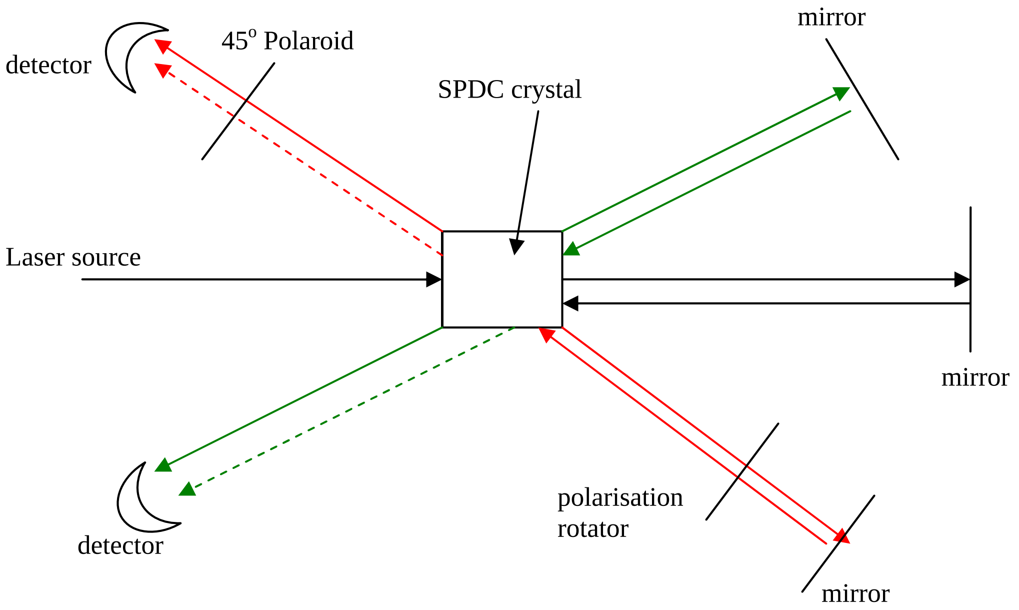
What happens in the crossed SPDC beam setup if we introduce a polarisation rotator, and perhaps subsequently erase its effect using a 45^o Polaroid? We are assuming now that the two beams to emerge from the lithium iodate SPDC crystal are both vertically polarised. Including the polarisation state, the state entering the detectors in the absence of any polarisers/Polaroids would be,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|G\rangle|v\rangle|R\rangle|v\rangle + |Gd\rangle|v\rangle|Rd\rangle|v\rangle\right) \qquad (25)$$

where the polarisation ket written first/second is understood to refer to the 'green' and 'red' photon respectively. We have already seen that this experiment would not produce a *local* interference pattern, but *would* produce an interference pattern provided that green and red detector correlations were taken into account – by virtue of (12).

Suppose we now introduce a polarisation rotator into the red beam at the position shown in Figure 5 so that the affected beam emerges from it with horizontal polarisation. (The 45^o Polaroid shown in the Figure is not yet present):-

Figure 5



The state entering the detectors is now,

$$\left|\psi\right\rangle = \frac{1}{\sqrt{2}}\left(\left|G\right\rangle\left|v\right\rangle\left|R\right\rangle\left|h\right\rangle + \left|Gd\right\rangle\left|v\right\rangle\left|Rd\right\rangle\left|v\right\rangle\right) \tag{26}$$

Note that it is only the state $\left|R\right\rangle$ which is affected by the polarisation rotator. The dashed red beam does not pass through the rotator. This change is sufficient to make the two terms in (26) orthogonal and hence to eliminate any interference – even the non-local interference which might result from green/red correlations. To spell this out, the equivalent of (11) is,

$$\left|\psi(x,y)\right\rangle = \frac{1}{\sqrt{2}}\psi_G(x)\psi_R(y)\left(\left|v\right\rangle\left|h\right\rangle + e^{i(\theta_G+\theta_R)}\left|v\right\rangle\left|v\right\rangle\right) \tag{27}$$

So that,
$$\left\langle\psi(x,y)\middle|\psi(x,y)\right\rangle = 1 \tag{28}$$

because $\left\langle v\middle|h\right\rangle = 0$. Consequently even the correlated interference which previously resulted from (12) does not occur.

We know, of course, that local interference cannot occur in this entangled setup because this would provide the opportunity for FTL communication. But can the correlated interference be resurrected by erasure of the effects of the polarisation rotator? Consider placing a 45^o Polaroid into the red beam path as shown in Figure 5. Expressing the state (27) *before* encountering this Polaroid in terms of the 45^o polarisation states (for the red beam only) gives, using (20,21),

$$\left|\psi(x,y)\right\rangle = \frac{1}{2}\psi_G(x)\psi_R(y)\left(\left|v\right\rangle\left(\left|45\right\rangle - \left|-45\right\rangle\right) + e^{i(\theta_G+\theta_R)}\left|v\right\rangle\left(\left|45\right\rangle + \left|-45\right\rangle\right)\right) \tag{29}$$

Since the effect of the Polaroid is to filter the perpendicular state, $\left|-45\right\rangle$, the state entering the detectors is, (30)

$$\left|\psi(x,y)\right\rangle = \frac{1}{2}\psi_G(x)\psi_R(y)\left(\left|v\right\rangle\left|45\right\rangle + e^{i(\theta_G+\theta_R)}\left|v\right\rangle\left|45\right\rangle\right) = \frac{1}{2}\psi_G(x)\psi_R(y)\left(1 + e^{i(\theta_G+\theta_R)}\right)\left|v\right\rangle\left|45\right\rangle$$

Hence,
$$\left\langle\psi(x,y)\middle|\psi(x,y)\right\rangle = \frac{1}{2}\left(1+\cos(\theta_G+\theta_R)\right) \tag{31}$$

Thus, non-local (correlated) interference is re-introduced by the Polaroid. This comes about because its effect is to make the two terms in the wave-vector proportional, rather than orthogonal.

So, just as true local interference is destroyed by "which path" data, but can be restored by erasure of this effect, so too can non-local, correlated interference be destroyed by "which path" data obtained on one of the entangled particles alone – and this effect can also be erased by appropriate interaction with this same single particle beam alone.

6. Double-Slit Interference & Erasure: The Arrangement of Walborn et al

The preceding experimental arrangements may have given the impression that local interference cannot be observed using one of an entangled pair of particles. However this would be too loose a statement. Careful examination of these examples shows that the reason why local interference does not occur is due to the entangled degrees of freedom being the same as the degrees of freedom involved in the potential interference. However, it may be possible to obtain interference from one particle of an entangled pair provided that the interference involves different degrees of freedom from those which are entangled.

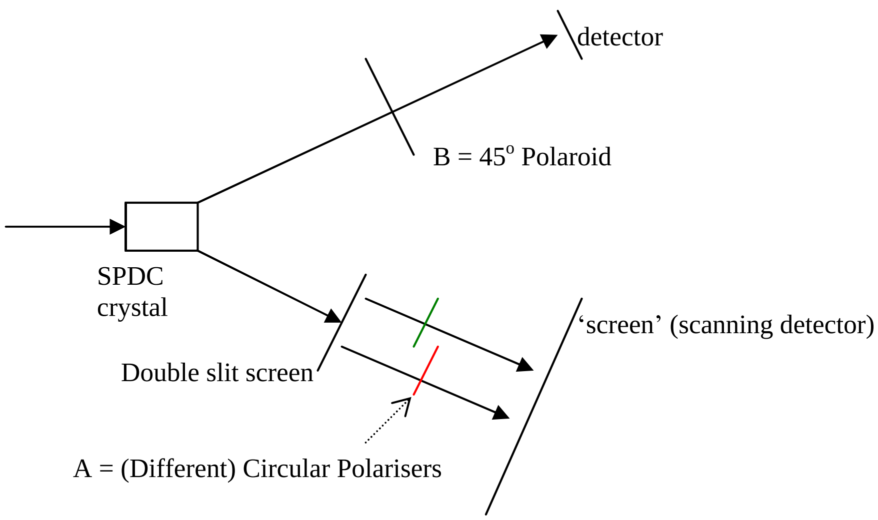
Consider the arrangement shown in Figure 6. Here just one of the beams output from an SPDC crystal (e.g., beta barium borate) is incident on a double slit screen. The other photon path plays no part initially, other than vetoing noise measurements via a coincidence counter. Do we see (purely local) interference from the lower beam? The answer is "yes" as can be seen algebraically as follows. Call the beams emerging from the SPDC crystal U and L, for upper and lower beams. The state prior to encountering the double slit screen is thus $|\psi\rangle = |U\rangle|L\rangle$. After the double slit screen the lower beam gets further split into beams which we label as 1 and 2. Thus, if we have no polarisation rotators nor the Polaroid in place, the state entering the detectors is,

$$|\psi\rangle = \frac{1}{\sqrt{2}}|U\rangle(|L1\rangle + |L2\rangle) \quad (32)$$

In the x-basis, where x is the position on the lower screen (detector), this is,

$$|\psi(x)\rangle = \frac{1}{\sqrt{2}}|U\rangle\psi_{L1}(x)(1 + e^{i\theta(x)}) \quad (33)$$

Figure 6



So we do indeed get local interference since (33) gives an intensity on the screen of,

$$|\psi(x)|^2 = 1 + \cos\theta(x) \quad (34)$$

So we see that it would be quite wrong to simply claim that "entangled particles cannot produce local interference". In this example the entanglement with the upper beam is simply irrelevant. The reason is that the splitting of the lower beam into two parts which subsequently interfere takes place quite independently of the entanglement with the upper beam. So the upper beam state simply factors in (32,33).

One may object to this on the grounds that no entanglement has been displayed explicitly in the initial state $|\psi\rangle = |U\rangle|L\rangle$. Let us do so now. Suppose that either particle can emerge from the SPDC crystal in a vertical or horizontal polarisation state. We shall write this as *v* or *h* following the path designation. The initial state is now written,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|Uv\rangle|Lh\rangle + |Uh\rangle|Lv\rangle\right) \tag{35}$$

The two photons are thus explicitly entangled in the description (35). When the lower beam is split by the double slit this becomes,

$$|\psi\rangle = \frac{1}{2}\left(|Uv\rangle\left[|L1h\rangle + |L2h\rangle\right] + |Uh\rangle\left[|L1v\rangle + |L2v\rangle\right]\right) \tag{36}$$

In the x-basis this is,

$$\begin{aligned} |\psi\rangle &= \frac{1}{2}\left(|Uv\rangle\psi_{L1}(x)\left[|h\rangle + e^{i\theta}|h\rangle\right] + |Uh\rangle\psi_{L1}(x)\left[|v\rangle + e^{i\theta}|v\rangle\right]\right) \\ &= \frac{1}{2}\left(|Uv\rangle|h\rangle + |Uh\rangle|v\rangle\right)\psi_{L1}(x)\left(1 + e^{i\theta}\right) \end{aligned} \tag{37}$$

noting, as before, that $\langle x|\psi_{L2}\rangle = e^{i\theta(x)}\langle x|\psi_{L1}\rangle = e^{i\theta(x)}\psi_{L1}(x)$. Now taking the absolute square we get the intensity on the x-screen to be,

$$|\psi(x)|^2 = 1 + \cos\theta(x) \tag{38}$$

exactly as (34). The two ket terms on the second line of (37) are, of course, orthogonal – so there is no cross-term to cause interference arising from them. However, the interference does not arise from that term in (37) – it arises from the last term, the factor of $\left(1 + e^{i\theta}\right)$. And this arises from the purely lower beam based cross-terms in (36), i.e., the cross-term arising from $\left[|L1h\rangle + |L2h\rangle\right]$ and equally that with the opposite polarisation. So the entanglement of the polarisation degrees of freedom does not prejudice local interference arising from the spatial degrees of freedom.

Now let's see what happens when we insert the two polarisation rotators adjacent to the two slits in the lower beam (but the 45^o Polaroid is not yet present in the upper beam path). These are such that an input polarisation state $|v\rangle$ becomes a clockwise polarised beam, $|C\rangle$, after passing through the 'green' polarisation rotator (aligned with the higher slit), whereas the same input state would become an anti-clockwise polarised beam, $|AC\rangle$, after passing through the 'red' rotator (aligned with the other slit). If the input polarisation state were $|h\rangle$, on the other hand, then the green rotator would produce $|AC\rangle$ and the red rotator would produce $|C\rangle$. Consequently, from (36), the state after passing through the polarisation rotators is,

$$|\psi\rangle = \frac{1}{2}\left(|Uv\rangle\left[|L1\rangle|AC\rangle + |L2\rangle|C\rangle\right] + |Uh\rangle\left[|L1\rangle|C\rangle + |L2\rangle|AC\rangle\right]\right) \tag{39}$$

It is clear that there will no interference now because all terms in (39) are orthogonal, so all cross-terms are zero.

Why is this? Well we could carry out a measurement of the vertical/horizontal polarisation of the upper photon, and this would then tell us which slit the lower photon had travelled through via its circular polarisation. This "which path" information provided by the two polarisation rotators destroys the interference, as we would expect. Note that it is not necessary to actually obtain this information, i.e., to collapse the wavepacket to determine which path was taken. To destroy the interference it is sufficient that the first phase of a true measurement, the entanglement phase, has been done.

The interesting question which now arises is, “can we erase this ‘measurement’ and resurrect the interference by interacting only with the upper beam?” In popular accounts the answer is given as an unqualified “yes”. However, we shall see below that the correct answer is “yes, with a crucial proviso”. The proviso is that local interference cannot be resurrected. What can be regained is only non-local, correlated interference of the type discussed previously. Thus it is not quite accurate to refer to ‘erasure’ in this case. It is only partial erasure. Whereas initially we had local interference, what we regain is the weaker phenomenon of correlated interference.

To see how correlated interference is regained let us now introduce the 45^o Polaroid into the upper beam path. This simply filters out the polarisation component $|-45\rangle$ and lets $|45\rangle$ through unimpeded. So we must first re-write (35) in terms of the states $|\pm 45\rangle$ by substituting from (20,21). This gives,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|U,45\rangle|L,45\rangle - |U,-45\rangle|L,-45\rangle\right) \tag{40}$$

Consequently, passing the upper beam through the 45^o Polaroid causes the state to become,

$$|\psi\rangle = \frac{1}{\sqrt{2}}|U,45\rangle|L,45\rangle \tag{41}$$

Using (20,21) this can also be written,

$$|\psi\rangle = \frac{1}{2}|U,45\rangle\left(|L\rangle|v\rangle + |L\rangle|h\rangle\right) \tag{42}$$

When the lower beam passes through the double slits the state therefore becomes,

$$\begin{aligned}|\psi\rangle &= \frac{1}{2\sqrt{2}}|U,45\rangle\left(|L1\rangle|C\rangle + |L2\rangle|AC\rangle + |L1\rangle|AC\rangle + |L2\rangle|C\rangle\right)\\ &= \frac{1}{2\sqrt{2}}|U,45\rangle\left(|L1\rangle + |L2\rangle\right)\left(|C\rangle + |AC\rangle\right)\end{aligned} \tag{43}$$

We thus (apparently) get interference separately for the clockwise and anti-clockwise polarised waves, with the result that,

$$|\psi(x)|^2 = \frac{1}{2}\left(1 + \cos\theta(x)\right) \tag{44}$$

The factor of ½ is again because the 45^o Polaroid stops half the photons getting through (or does it?). So we have successfully erased the effect of the polarisation rotators and restored the interference on the x-screen. From (44) this would appear to be local interference observable using the x-detector data alone – but is it?

How does the erasure happen? It is because the measurement of the polarisation at 45^o forces the lower beam into a 45^o state, and this is a superposition of states with vertical and horizontal polarisation. It is only these vertical and horizontal polarisation states that the polarisation rotator turns deterministically into circularly polarised states. A 45^o polarisation state produces a superposition of clockwise and anti-clockwise polarisation states after passing through either rotator. Consequently knowing whether the photon at the x-screen has clockwise or anti-clockwise polarisation no longer tells us which slit it passed through. Either slit could result in either clockwise or anti-clockwise polarised photons, since the slits are illuminated by a superposition of vertical and horizontal polarised photons.

Fair enough - but hold on one moment. Can we really have resurrected *local* interference? Wouldn't this mean that we have a FTL communication channel? The position of the 45^o Polaroid can be arbitrarily distant from the x-screen. And by inserting or removing the Polaroid we create or destroy the interference pattern at the x-screen. Bingo – FTL communication! Where is the snag this time?

The snag is that we have failed to notice that half the time the Polaroid will not produce a $|\psi\rangle = \frac{1}{\sqrt{2}}|U,45\rangle|L,45\rangle$ state but will absorb the upper beam photon. But in these cases the lower beam photon still gets detected at the x-screen. In this case the lower beam photon is left in the state $\frac{1}{\sqrt{2}}|L,-45\rangle = \frac{1}{2}\left(|L\rangle|v\rangle - |L\rangle|h\rangle\right)$. After passage through the double slits this becomes,

$$\begin{aligned}|\psi\rangle &= \frac{1}{2\sqrt{2}}\left(|L1\rangle|C\rangle + |L2\rangle|AC\rangle - |L1\rangle|AC\rangle - |L2\rangle|C\rangle\right) \\ &= \frac{1}{2\sqrt{2}}\left(|L1\rangle - |L2\rangle\right)\left(|C\rangle - |AC\rangle\right)\end{aligned} \tag{45}$$

Taking the absolute square gives, for the intensity at the x-screen due to those cases when the upper beam photon is absorbed by the Polaroid,

$$|\psi(x)|^2 = \frac{1}{2}\left(1 - \cos\theta(x)\right) \tag{46}$$

The only difference from (44) is the sign of the cosine term, resulting from the signs in (45). Now what is *actually* observed locally at the x-detector is the *sum* of (44) and (46). So the cosine term cancels and we get a total signal of $|\psi(x)|^2 = 1$ at the x-screen, i.e., no interference pattern!

The claim that was made above regarding observing a *local* interference pattern was incorrect. Actually the interference corresponding to Equ.(44) will be observed only if coincidence counts are used. That is, only photons arriving at the x-screen coincident with photons reaching the upper detector will result in an interference pattern. The interference is not local but correlated. There is, of course, no FTL communication.

The experiment described here has actually been carried out, with results just as anticipated by the theory above, see Ref.[1].

7. Delayed Erasure

The term "delayed erasure" is given to experiments which involve erasure that is carried out *after* detection of the photons which produce the interference pattern. This is easily accomplished in the experimental setup of Walborn et al, §6, simply by making the upper beam path long enough prior to the 45^o Polaroid. Note that in §6 the analysis implicitly assumed that the Polaroid was encountered first and the double slits with their polarisation rotators encountered second. This order is easily reversed in the algebra. After the double slits and polarisation rotators, but before the 45^o Polaroid, the state is given by (39), i.e.,

$$|\psi\rangle = \frac{1}{2}\left(|Uv\rangle\left[|L1\rangle|AC\rangle + |L2\rangle|C\rangle\right] + |Uh\rangle\left[|L1\rangle|C\rangle + |L2\rangle|AC\rangle\right]\right) \tag{39}$$

Using (20,21) to re-express the upper beam polarisation in terms of the $|\pm 45\rangle$ states this becomes, (47)

$$|\psi\rangle = \frac{1}{2\sqrt{2}}\left(|U\rangle\left(|45\rangle + |-45\rangle\right)\left[|L1\rangle|AC\rangle + |L2\rangle|C\rangle\right] + |U\rangle\left(|45\rangle - |-45\rangle\right)\left[|L1\rangle|C\rangle + |L2\rangle|AC\rangle\right]\right)$$

The effect of the 45^o Polaroid is therefore to reduce (47) to,

$$\begin{aligned} |\psi\rangle &= \frac{1}{2\sqrt{2}}|U,45\rangle\left(\left[|L1\rangle|AC\rangle + |L2\rangle|C\rangle\right] + \left[|L1\rangle|C\rangle + |L2\rangle|AC\rangle\right]\right) \\ &= \frac{1}{2\sqrt{2}}|U,45\rangle\left(|L1\rangle + |L2\rangle\right)\left(|AC\rangle + |C\rangle\right) \end{aligned} \tag{48}$$

Which is exactly as (43), and hence also produces the intensity at the x-screen given by (44), i.e., an apparent interference pattern (but which can actually be seen only by coincidence counting with arrivals at the upper detector, as before).

Consequently the delay of the erasure makes no difference to the results, which are the same as in §6.

A great play is made in popular accounts of the weirdness of this outcome. The lower beam photon appears to ‘know’ that a 45^o Polaroid will be inserted into the upper beam *some time after* the lower photon has already been detected. The lower photon has to ‘know’ this in order to know whether or not to produce an interference pattern on the x-screen.

But these protestations of weirdness tend to forget the crucial issue – that an interference pattern is found only when the x-detector counts are vetoed according to whether there is a corresponding count in the upper detector. The interference is not local, but only a correlated interference.

The apparent weirdness is largely the result of a conceptual error. One confuses the situation with an instantaneous effect at the x-screen due to the insertion of the Polaroid in the remote upper beam. This would be FTL communication and would indeed be very weird – in fact, impossible. But there is no such effect. Ask yourself, when the Polaroid is inserted what exactly is the instantaneous effect on the signal in the x detector? Say that you have placed your x detector carefully at a minimum of the potential interference pattern – and that this pattern is perfect so that no photons at all would reach this point if interference did occur. Will you detect a photon or not? You cannot tell because it depends upon whether the Polaroid in the upper beam happens to pass a 45^o state photon or to absorb a -45^o state photon – and you don’t know which has happened. So the insertion of the remote Polaroid predicts…nothing. A prediction can be made only if one is also given the result of the upper detector.

Suppose the Polaroid is so far away that we can count large numbers of photons and develop the whole x-screen whilst remaining causally disconnected. What do we see? We see no interference. Whether or not the Polaroid has been inserted we see a uniformly illuminated x-screen. Look – nothing weird at all. Only when the signal received at the upper detector is used as a mask to retain or veto individual x-screen counts does the interference pattern emerge. What does this mean? It means that there is a correlation between two sets of measurements which were carried out in a causally disconnected manner. How weird is this? It is not at all weird.

If two sub-systems have a common source, their properties will often be correlated. This is an objective fact and does not require subsequent causal connection between the sub-systems to manifest it. For example, if a mass breaks up into two smaller masses it will surprise no one to find that the mass of one part is correlated with the mass of the other – even if the measurements of these masses is carried out when the parts are light years apart.

However this goes rather too far in dismissing quantum weirdness. There *is* a degree of weirdness, namely non-locality in spite of the absence of hidden variables. In the classical example the matter is clear because each part can be considered as possessing a definite mass before any measurement is made. This is not the case for the polarisation of the photons – they are in a superposed polarisation state initially. So the weirdness lies in the fact that the upper and lower detectors produce correlated results despite this.

References

[1] Walborn, Terra Cunha, Padua, and Monken (2002), Physical Review A, (**65**, 033818, 2002).