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The Inevitability of Fine Tuning in a Complex Universe

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ABSTRACT

Why should the universe need to be fine tuned? It is pointed out that neither the God hypothesis nor the Multiverse hypothesis address this question, since they do not explain why complexity requires parameter sensitivity. The thesis is presented that parameter sensitivity arises as a natural consequence of the mathematics of dynamical systems with complex outcomes. Hence, fine tuning is a mathematical correlate of complexity and should not elicit surprise. It is noted that each example of parameter sensitivity takes the form of a constraint between two or more constants. Hence, alternative complex universes might exist (in the mathematical sense) with values of the universal constants well outside the fine-tuned bounds that are usually claimed.

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1 The Two Distinct Problems of Fine Tuning

It is over 40 years since Carter [1967] observed that the universal constants of physics appear to be peculiarly fine tuned. Relatively small variations in the universal constants, it is claimed, would produce radical changes in the universe. If true, this is an occurrence which requires explanation.

A great deal has been written about the merits, or otherwise, of God or the Multiverse as the explanation of this apparently improbable state of affairs. It seems to have gone largely unnoticed that an explanation is required of two quite separate phenomena. The first is the parameter sensitivity of the universe. This is the (claimed) property of the universe that small changes in the parameters of physics produce catastrophic changes in the evolved universe. In particular its complexity, and hence its ability to support life, would be undermined by relatively small changes in the universal constants (it is said). Thus, ‘parameter sensitivity’ is the claim that the target in parameter space which is compatible with a complex universe is small. The smallness of the target, if true, is one of the features which requires explanation.

The second, and quite distinct, problem is how nature manages to hit this small target. To do so requires that the actual constants in our universe be fine tuned to coincide with the requirements for a complex outcome. In other words, given that only special values for the parameters will do (i.e., given parameter sensitivity), how does nature contrive to adopt these particular values for the parameters (i.e., how does fine tuning arise)?

It is the second question which appears to have received all the attention. The contrivance by which the small target is successfully hit is purported to be God or the Multiverse, according to taste. But neither of these postulates even attempts to explain parameter sensitivity, i.e., why the target is small in the first place. They aim only to explain how a small target may be hit, i.e., how the universe may be fine tuned. Why, though, is the universe parameter sensitive? It appears that no explanation has previously been offered.

There is a danger of misunderstanding this point. Physicists might argue that every instance of a phenomenon which requires a universal constant to be fine tuned constitutes a demonstration, via physical calculation, that the target is small. They might opine that the question "why parameter sensitivity?" is answered by the totality of such calculations. But our calculations are merely observations that parameter sensitivity appears to prevail in our universe. They do not provide an explanation of why this should be so.

The point can be illustrated in the following way. Before one looks into the physics of these things, it is not obvious that there could not be complex universes corresponding to the bulk of the volume of parameter space (or the 'string landscape' as it tends to be called these days, Susskind [2003], Hogan [2006]). Take life as an exemplar of complexity and consider the universes which might result if changes were made to the universal constants. We can imagine, without any nonsense being apparent, that as we follow some path through parameter space, the carbon based life forms we know give way to a continuum of different life forms. A sequence of small changes would give rise eventually to completely different forms of life – not unlike biological evolution, but far more radical since the physics of the environment is also changing. This description of the consequences of changing the universal constants is precisely what 'parameter sensitivity' claims is *not* true. But why is this?

Neither God nor the Multiverse explains why there should be no complexity in these alternative universes. They do not explain parameter sensitivity. Parameter sensitivity must be addressed as a consequence of physics, and is, we shall argue, a result of the mathematical properties of dynamical systems. We shall argue that parameter sensitivity is inevitable in any complex universe, and hence, as a consequence, so is fine tuning.

2 How Fine is Fine Tuning?

Some people regard any consideration of variations in the universal constants as intrinsically nonsensical. If the universal constants are, in fact, fully prescribed by pure mathematics, as has long been the dream of physicists, then considering them to be varied is as foolish as considering a variation in the result of 1 plus 1. However, there has been precious little encouragement over the last 100 years that the ambition of deriving the universal constants from pure mathematics may be fulfilled. The standard model of particle physics did not lead to a reduction in the number of universal constants, nor did the grand unified theories. The hope that string theory might do so has been dashed by the colossal number of different string theories (Kachru et al [2003], Susskind [2003], Dine [2004]), notwithstanding the likelihood that any one string theory would constrain the constants (Kane et al [2002]). Pragmatically, we simply admit that the subject matter of this paper is contingent upon the consideration of variations in the universal constants not being intrinsically nonsensical.

We are interested in the degree of fineness of the tuning because the finer the tuning the more remarkable the coincidence. So one is tempted to think. If a parameter can tolerate only a very small percentage variation before the universe is rendered abiophilic it is natural to regard this universe as an improbable state of affairs. "Natural" it may be, but not necessarily defensible. A number of authors have pointed out that the smallness of the numerical window within which a parameter must lie to result in a complex universe says nothing at all about its probability (e.g., Manson [2000], McGrew et al [2001]). The implicit appeal to small windows implying small probabilities is false. To make the small window into a small probability we must know something about the possible range of values, and their probability distribution. This is difficult to dispute, though Koperski [2005] and Monton [2006] have attempted to do so. Koperski opines that, "*whilst both design and multiverse proponents might be wrong, their common demand for an explanation of fine tuning is justified (at present)*".

Klee [2002] reminds us that many of the claims of fine tuning are not at all fine. Many are very coarse, as discussed in the next Section. Klee observes that a claimed 'order of magnitude agreement' is often stretched to cover cases which actually differ by two or three or even four orders of magnitude. One source of such elasticity in order-of-magnitude estimates lies in the use of dimensional analysis. The trouble with dimensional analysis is that it does not provide the magnitude of the dimensionless multiplier. So this is merely assumed to be of order unity. Barrow and Tipler [1986] refer to the tendency of this assumption to be about right as 'the unreasonable effectiveness of dimensional analysis' and proceed to attempt to explain it on the basis of the low dimensionality of space. However, it just is not true. There are formulae in physics which display very large numerical factors. For example, dimensional analysis suggests that the lifetime of the muon should be in the order of $G_F^{-2} m_\mu^{-5}$. A field theory calculation confirms that the lifetime is proportional to this quantity, but evaluates the numerical multiplier to be $3 \times 2^6 \pi^3 = 5{,}953$. So dimensional analysis is not 'unreasonably effective' in this case, on the contrary it fails by nearly 4 orders of magnitude.

However, the intention of this paper is to demonstrate that parameter sensitivity is actually a natural state of affairs, i.e., not improbable at all, given that the universe is complex. The probability that the universe be complex (i.e., the probability that the actual parameters be suitably fine tuned) is another matter, but this is beyond the scope of the present work.

3 The Physical Evidence for Parameter Sensitivity

Before expending a great deal of effort in seeking an explanation for parameter sensitivity, how confident should we be that parameter sensitivity is truly a property of our universe? There are two aspects to this. The first is the quality of the physical arguments and calculations which underpin the claim; the second is their interpretation. Philosophical enquiry has tended to focus on these interpretational issues, whilst taking on trust that the physics has been dealt with properly by the physicists. This is perhaps unfortunate. Whilst there have been many compilations of 'cosmic coincidences', as cited below, some have tended to be rather polemical. Even putting to one side the thorny issue of interpretation, a balanced critique of the physical status of the coincidences is overdue. Unfortunately, this is far too large an undertaking for the present paper, so just a few illustrative examples must suffice.

Our contention will be that many instances of tuning are not required to be terribly 'fine' (illustrated by subsections 3.1 to 3.3). Nevertheless, not-so-fine tuning is widely evident and is no less in need of explanation for being imprecise. Moreover, there are some cases in which the required tuning does appear to be quite impressively fine (illustrated by subsections 3.4 and 3.5). These issues have been widely discussed previously. Individual citations are omitted in 31.-3.5, but example sources are Agrawal et al [1998a,b], Barrow and Tipler [1986], Barrow et al [2008], Carr and Rees [1979], Carr [2007], Carter [1967,1974], Davies [1972,1982,2004,2006], Dyson [1971], Gribbin and Rees [1989], Hogan [2000,2006] and Rees [1999,2003], and there are many more. In the discussions of 3.1-3.5 we consider varying a single universal constant alone, in line with common practice. The shortcomings of this approach are discussed in Section 4.

3.1 Nucleon and Electron Masses for Atomic Stability

The first example concerns the constraints on particle masses which arise from the requirement that atomic matter be stable. For atomic matter to be stable, the mass of the neutron must exceed the sum of the masses of the proton and the electron. Otherwise the nucleus would capture the atomic electrons via the reaction $e + p \to n$, and all matter would reduce to neutrons. There would then be no chemistry. But the neutron mass is 939.565MeV and the sum of the proton and electron masses is 938.783MeV, so it is a close run thing, disaster being averted by a mass deficit of just 0.08% of the neutron mass. Moreover, the fact that we have required $\Delta = M_n - (M_p + m_e) > 0$ leads to a free neutron being unstable. This means that atomic nuclei may be unstable via beta decay, i.e. $n \to p + e + \bar{\nu}$. The lighter nuclei escape this instability only because the daughter nucleus often has a smaller binding energy than the original nucleus, by an amount, ΔB, which exceeds the mass deficit, Δ. This is not mere theory: nuclei with $\Delta B < \Delta$ are invariably unstable. Stable nuclei exist, therefore, only because they have $M_n < M_p + m_e + \Delta B$. The magnitude of ΔB varies, often being negative for the heavier, unstable nuclei. For the commoner isotopes of the lighter nuclei ΔB is usually a few MeV, say 10 MeV for illustration. Hence $M_p + m_e + \Delta B \sim 949$ MeV, in comparison with the neutron mass of 939.565MeV. This narrowly misses a universe with no stable atoms by just 1% of the neutron mass. Hence if the neutron were lighter by more than 0.08%, or if it were heavier by more than ~1%, then there would be no stable atomic matter in the universe.

This seems impressive, but becomes much less so when it is recalled that the neutron and the proton share a common structure. About 99% of a nucleon's mass is due to the virtual gluons and virtual quarks which comprise the strong nuclear force. This feature is shared by the neutron and the proton, which differ only as regards the 'valence' quarks which provide the nucleons with their net quantum numbers. Since the u and d quarks in question have masses of just a few MeV, it is no longer particularly surprising that the neutron-proton mass difference is also of this order. In fact this is to be expected. The moral is that there are mechanistic reasons why the neutron and proton masses should be very close. This is not to say that there is no 'tuning' at all, just that it is not so terribly 'fine' as it first appears. It is more indicative to compare $M_n - M_p$ with the mass of the electron or the mass of the u or d quarks. On this scale the tuning is at the level of tens or hundreds of percent, rather than less than 1%. Nevertheless, there is *some* tuning. For example, the d quark must be heavier than the electron for atomic stability[1].

3.2 The Weak Force (Fermi) Constant

Our second example concerns the survival of hydrogen in the first seconds after the Big Bang and the constraint this places on the strength of the weak nuclear force. Within the first second, leptonic reactions interconvert neutrons and protons such that their relative abundance is determined by dynamic equilibrium, and hence by the temperature. By the time nucleosynthesis starts, after the first minute or two, the ratio of neutrons to protons has been set by the temperature prevailing when the leptonic reactions are frozen-out by cosmic expansion. The fact that the universe contains comparable quantities of hydrogen and helium is a consequence of the 'coincidence' that $M_n - M_p$ is of the same order as kT at the time of freeze-out of the leptonic reactions. This requires that the weak nuclear force, which drives the leptonic reactions, be of a particular strength close to its actual strength. Had the weak force been sufficiently weaker then freeze-out would occur earlier when the temperature was higher, resulting in the abundance of neutrons and protons being closely matched. Big Bang nucleosynthesis (BBN) would then result in a universe consisting of virtually all helium and very little hydrogen. A universe with no hydrogen would contain no water, no hydrocarbons such as amino acids, and no hydrogen bond chemistry. Whilst we cannot be confident what such a universe would be like in detail, it would certainly not support life as we know it.

Because the neutron:proton ratio is the exponential of $-(M_n - M_p)/kT$, it is often claimed that it is highly sensitivity to changes in the freeze-out time, and hence to the strength of the weak force. Actually, a closer examination shows that if the Fermi constant, G_F (the parameter which controls the rate of the leptonic reactions) were reduced by an order of magnitude, the universe would still be 18% hydrogen (by mass, and nearly 50% by number of atoms). This would still support hydrogen burning stars with lives in the order of billions of years, long enough for biological evolution. Reducing G_F by a factor of 100 would still leave the universe with ~14% hydrogen by number of atoms. Admittedly if the hydrogen abundance were reduced too much this would ultimately prejudice the formation of the first stars, which is believed to rely on a cooling mechanism via molecular hydrogen. However, quantitative knowledge of these

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[1] That is, if we make the rather sweeping assumption that $M_n - M_p \approx M_d - M_u$. Far more carefully argued constraints on the u, d and s quark masses which produce a congenial universe have been discussed recently by Jaffe, Jenkins and Kimchi [2009] and by Damour and Donoghue [2008].

mechanisms is poor. Nevertheless, there is no obvious reason to regard as catastrophic a reduction in G_F by somewhat more than a factor of ten.

If G_F were increased, then there would be less helium in the universe. For example, a factor of 4 increase in G_F results in only ~0.2% helium by mass. But this would seem unimportant. Helium appears to play no essential role in the formation of large scale structure or stellar physics[2]. Although no upper bound on G_F results from these considerations, there are suggestions that Type II supernovae require G_F to lie close to its actual value. This is because crucial aspects of the mechanism of Type II supernovae involve neutrino interactions, i.e., weak-force interactions. The neutrinos seem to be required to interact just weakly enough to escape the core of the collapsed star, but strongly enough to transfer almost all their energy to the mantle to cause the explosion. Unfortunately the quantitative understanding of Type II supernovae is too poor to deduce just how fine tuned G_F must be. In any case, it is not clear that Type II supernovae are crucial for life.

The claim is that Type II supernovae are essential in order to release the stars' precious product of chemical elements into the interstellar medium (ISM), ultimately contributing to some biosphere. In truth, only the elements beyond iron require Type II supernovae. There are other mechanisms which ensure release of the lighter elements into the ISM. It is by no means clear that the heavier elements are required for life. The essential biochemistry is provided by the lighter elements. On the other hand, a case can certainly be made for the importance of planetary geothermal and tectonic activity in supporting the emergence of life, and these depend upon heating by the radioactive heavy elements formed in Type II supernovae.

In summary, there is a case for considering G_F to be a genuine instance of tuning, but it is not necessarily terribly 'fine'. To produce a universe with sufficient hydrogen, there is a relatively weak requirement, i.e., that G_F be greater than a few percent of its actual value.

3.3 The Low Energy Strength of the Strong Nuclear Force

Our third example of fine tuning relates to the strength of the strong nuclear force and the constraint upon it to ensure that nuclei are indeed bound states of their nucleons. Both BBN and hydrogen burning in stars proceed via deuterium ($^2_1 H$) as an intermediate product. Consequently the production of any elements would be prevented if deuterium were not a stable nucleus. It requires a reduction in g_s, the low energy effective coupling of the strong force, of only 15% to unbind the deuteron and hence prevent the formation of the chemical elements. This does appear to be a genuine instance of reasonably 'fine' tuning, requiring that g_s exceed ~85% of its actual value.

Claims are often made that there is also an upper bound on g_s to avoid diproton stability. If g_s were ~10% larger, then the diproton ($^2_2 He$) would be a bound state[3]. It has frequently been claimed that this would lead to an all-helium universe. The argument is

[2] The ppII and ppIII reaction sequences would be slowed by the absence of initial helium, but the ppI sequence is unaffected.

[3] The diproton is not bound in this universe. This is because the spin-singlet nuclear force is weaker than the spin-triplet nuclear force which binds the deuteron. It is *not*, as some authors have claimed, due to electrostatic Coulomb repulsion.

that all the nucleons would end up as helium during BBN, either via the conventional sequence starting with $n+p \to {}_1^2 H$, or via the diproton $p+p \to {}_2^2 He \to {}_1^2 H + e^+ + \nu$. The inverse beta decay which converts the diproton to a deuteron is possible because the binding energy of the deuteron (2.224 MeV) exceeds $M_n - M_p + m_e = 1.804$ MeV. However, this argument is just wrong. The reason is that, even if the diproton were stable, the rate of its formation via $p+p \to {}_2^2 He$ is too slow for any significant number of diprotons to be formed during BBN (Bradford [2009]). It is true that the nuclear physics of stars would subsequently be very different, but there is no obvious reason why biophilic stars would not be stable (Bradford [2009]).

Hence, once again we conclude that there is a case for some tuning of g_s (a lower bound) but that the case tends to have been overstated in the past (no obvious upper bound).

3.4 The Hoyle Coincidence

For our final example we consider the famous Hoyle [1954] coincidence concerning the production of carbon and oxygen in stars. This bucks the trend of the previous examples in that our view is that this ‘coincidence’ is genuinely impressive in terms of numerical precision. The instability of beryllium-8 (${}_4^8 Be$) means that carbon (${}_6^{12} C$) can be produced only by virtue of the subsequent alpha capture reaction ${}_4^8 Be + {}_2^4 He \to {}_6^{12} C$ being extremely fast due to the existence of a resonance of the carbon nucleus at just the right energy level. Moreover, the subsequent burning of all the carbon into oxygen is avoided by the fortuitous placing of the energy levels of the oxygen nucleus so that resonance is just avoided. The Hoyle coincidence often gets a bad press even amongst those who might be anthropically sympathetic (see for example, Weinberg [2005]).

To reproduce the observed state of affairs we require the production of Be^8 and C^{12} to be resonant, but the production of O^{16} to be non-resonant. Thus, the first two must have small positive energies and the third a small negative energy with respect to the alpha capture thresholds. Moreover, the numerical sizes of these energies (i.e. +91.9, +287.7 and -45.0 keV respectively) are crucial in producing a universe with a balance of carbon and oxygen. The coincidence becomes more impressive when it is realized just how sensitive the energy levels of these nuclear states are to the strength of the strong nuclear force.

A mere 0.4% change in the strength of the nuclear force can produce a change in the C^{12} 0_2^+ resonance energy of up to 38% (Oberhummer [1999,2000], Csoto [2000,2001], Schlattl [2004]). But these same sources have also shown via detailed stellar models that a reduction in the C^{12} 0_2^+ resonance energy of this size will result in a reduction in carbon production of around two orders of magnitude. Alternatively, an increase in the C^{12} 0_2^+ resonance energy of the same amount will result in a reduction in oxygen production of around two orders of magnitude.

Consequently it seems that the Hoyle coincidence remains one of the more impressively fine tuned, requiring changes in the strong force of only ±0.4% to challenge the likelihood of conventional biochemistry by serious depletion of either carbon or oxygen.

3.5 The Cosmological Constant

The existence of a non-zero cosmological constant, Λ, is one possible explanation for the 'dark energy' which is apparently driving the acceleration of the universal expansion. If the cosmological constant has its origin in the zero-point energy of quantum fields, dimensional analysis would suggest that the energy density should be of the order of the Planck density. The trouble is that this density is ridiculously huge, exceeding the critical density by 123 orders of magnitude. One perspective on this is that Λ is the product of extreme fine tuning, at an accuracy of the 123rd decimal place.

An alternative point of view is that the very extremity of the required tuning tends to speak against this 'explanation', suggesting instead that a mechanistic explanation will be forthcoming in the future (and assuming that the apparent acceleration of the cosmic expansion stands the test of time). Another cosmological parameter which previously appeared to require an extreme degree of fine tuning was the density parameter, Ω. But the consensus view at present is that a mechanistic explanation for extreme flatness ($\Omega = 1$) is provided by inflation theory. A theoretical explanation for the anomalously small magnitude of Λ may also be found in the future[4].

4 The Congenial Parameter Surface and Alternative Biophilic Universes

It might be imagined that parameter sensitivity necessarily implies that each affected constant must lie within a certain range of values. Indeed most discussions give this impression (e.g., the sources cited in Section 3), and we have kept to this traditional manner of exposition in Sections 3.1-3.5. But this is quite wrong. This conclusion cannot be drawn from analyses which vary only one parameter at a time.

Suppose we have N universal constants, $\{c_i\}$. Parameter sensitivity might be thought to imply $c_1 \in [c_1^L, c_1^U]$, and $c_2 \in [c_2^L, c_2^U]$, and $c_3 \in [c_3^L, c_3^U]$, etc., so that the 'allowed' region of parameter space is an N-dimensional rectangle. This is not the case. Let us suppose that varying the constant c_1 alone does indeed imply that it must lie in the range $c_1 \in [c_1^L, c_1^U]$ to produce a complex universe. Let us suppose also that varying the constant c_2 alone implies that it must lie in the range $c_2 \in [c_2^L, c_2^U]$. A corresponding statement is found to hold for each constant varied individually. But it is an elementary logical error to suppose that this implies that the permissible points in parameter space lie with the rectangular block specified by $c_1 \in [c_1^L, c_1^U]$, $c_2 \in [c_2^L, c_2^U]$, $c_3 \in [c_3^L, c_3^U]$, etc., all holding true simultaneously. The fallacy is exposed most simply by the following illustration.

Suppose that, within the N-dimensional C-space, a complex universe will result if and only if the constants, $\{c_i\}$, lie on a particular N-1 dimensional hyper-surface. Suppose that our universe lies on a typical point of this surface, and hence that the surface is not parallel to any of the c_1, c_2,… axes at this point. It follows that varying any single c_j, i.e. moving parallel to axis j, will take us off the 'magic surface' and into the region of non-

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[4] This is without prejudice to the anthropic arguments of Martel et al [1998] and Weinberg [2005]. It is always valid to use our knowledge of the universe to provide constraints on the parameters of physics, but this does not preclude the possibility that specific mechanistic explanations might be found.

complex universes. Hence, the observation that each c_j is fine tuned when varied alone does not imply that we are restricted to a hyper-cube in C-space. If this were all the information we had, we could only conclude that we are restricted to an N-1 dimensional sub-space (hypersurface) of C-space. This crucial point is often overlooked. It is illustrated by Figure 1.

Figure 1 is typical of parameter sensitivity, showing a lower bound and an upper bound curve in the 2- parameter subspace illustrated. Rather than an N-1 dimensional biophilic surface, the coarseness of not-so-fine tuning leads to a 'thick surface' defined by the region between the curves. By varying just one parameter at a time, tuning is observed to be required in both the parameters c_1 and c_2, the extent of which is indicated by the arrowed lines. However, it is incorrect to conclude that the constants are therefore restricted to the box indicated by the red dashed lines. By assumption they are actually restricted to the more extensive region defined by the lower and upper bound curves (or a higher dimensional analogue if three or more parameters are involved). This means that there may be parameter values far distant from the red dashed box which nevertheless give rise to a complex universe.

Is Figure 1 representative of the parameter sensitivity that is observed? We claim that it is. The characteristic of all examples of parameter sensitivity is a relationship involving two or more parameters. The point is illustrated by examples as follows:-

- Consider the bound on the neutron mass as discussed in Section 3.1, i.e., $M_p + m_e < M_n < M_p + m_e + \Delta B$. The left-hand inequality can be undermined by an increase in the electron mass. However, by increasing the quark masses and the strong nuclear coupling constant appropriately we can contrive to ensure that $M_p, M_n, \Delta B$ all increase roughly in proportion to the change in electron mass, thus preserving the inequalities. To put it even more simply, the inequalities involve more than just one constant.
- Another example is the preservation of hydrogen during the Big Bang. We have seen in Section 3.2 that reducing the weak coupling constant, G_F, by a factor of ~100 could challenge this. However, the excess of protons over neutrons at the time of the freeze-out of the leptonic reactions depends upon the product $G_F^{2/3}(M_n - M_p)$, so that a reduction in G_F can be compensated by an increase in the nucleon mass difference. (This is likely to involve a reduction in the neutron lifetime, which also influences the final proportion of hydrogen surviving the Big Bang, but the photon:nucleon ratio can be re-tuned to negate that effect if necessary).
- The lower bound on the strength of the nuclear force, g_s, to bind the deuteron was given in Section 3.3 as $g_s > 0.85 g_{s,actual}$. But closer inspection reveals that the range of the nuclear force, and the nucleon mass, are also part of this calculation. The combination of parameters which is bounded below is actually $g_s^2 M_n / M_\pi$, where M_π is the pion mass. So the numerical bound on g_s can be changed by varying the nucleon:pion mass ratio. Again, the situation is as depicted in Figure 1 since the inequality involves more than just one constant.

- The stability of larger nuclei requires that the quantum of charge is not too great or else the Coulomb repulsion between the protons will blow the nucleus apart. But a larger quantum of charge can be compensated by also increasing the strength of the nuclear force. This produces an inequality involving both g_s and also α, the electromagnetic fine structure constant.
- One of the original fine-tunings of Carter [1967] was the requirement that small stars be convection dominated whilst large stars be radiation dominated. This leads to the coincidence $\alpha^{12}\left(\frac{m_e}{M_N}\right)^4 \sim GM_N^2$. Again this is a statement about the relative strengths of the gravitational and electromagnetic forces (though a non-linear one), and involves several different constants.

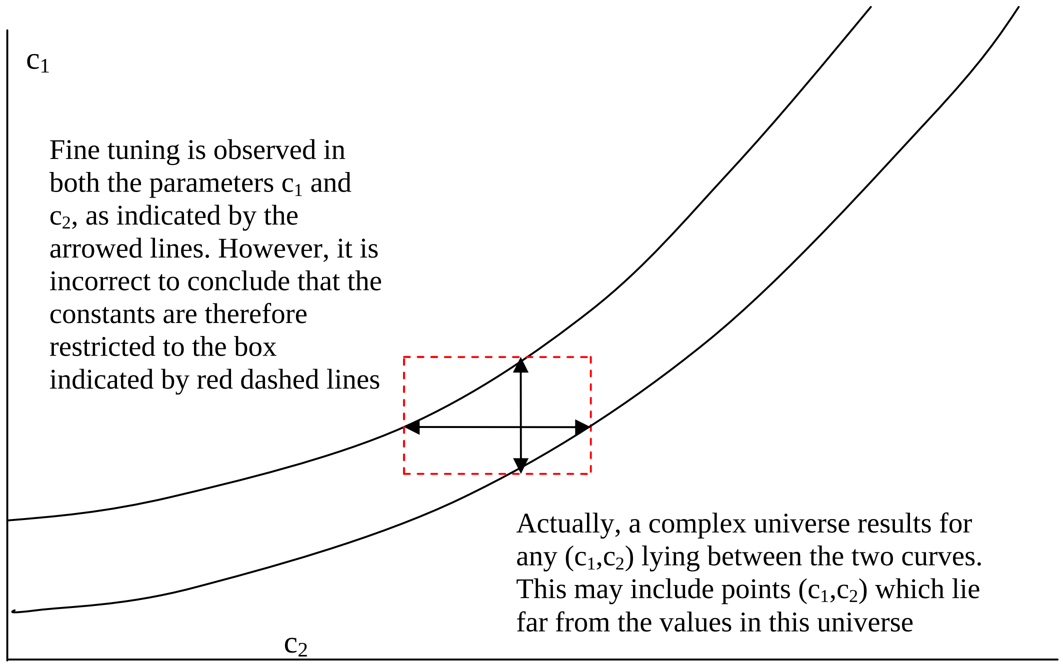

Figure 1: *Illustrating a logical fallacy: the observation of fine-tuning in parameters c_1 and c_2 does not imply that they are confined to the red dashed box.*

It is clear that Figure 1 must be a faithful depiction of each case of fine tuning. No universal constant has been calculated from pure mathematics, and neither has any *bound* been so calculated. All examples of parameter sensitivity are relations between two or more constants. Note, however, that there will be many constraints like Figure 1 which must be satisfied simultaneously, potentially up to one for each case of fine tuning.

Whether it is possible to make the changes to multiple parameters as suggested above and still preserve other effects, such as the balance of the production of carbon and oxygen in stars, is very difficult to determine. However, it may not matter. What matters is attaining complexity, rather than preserving the particular strategies employed in this universe for achieving specific outcomes. Hence, preserving the Hoyle effect is not necessary if the change in the parameters opens up a different pathway for carbon and oxygen production (e.g., Section 4.1). Once we move significantly away from the red box in Figure 1 we must accept that the universe will probably be qualitatively very different indeed, despite still being complex provided we stay within the bounding curves.

The universe does not need to be fine tuned provided we vary the parameters in the 'congenial' direction, i.e., parallel to the bounding curves in Figure 1. Radically different, but still complex, universes may exist in these directions. And they will all exhibit parameter sensitivity.

Support for this contention is provided by a number of radically different universes which have been constructed by Aguirre [2001], by Harnik, Kribs and Perez [2006] and by Adams [2008].

4.1 Aguirre's Cold Big Bang Universe

If the photon:baryon ratio were less than $\sim 10^6$ then structure formation would be prevented because the universe would be permanently opaque and hence small density fluctuations would be supported against gravitational collapse by radiation pressure. Furthermore, Tegmark and Rees [1998] argue that the magnitude of the primordial density fluctuations, Q, is fine tuned to be within an order of magnitude of its value in this universe. Despite these parameter sensitivities, Aguirre [2001] has presented a case for a universe capable of supporting life in which the photon:baryon ratio is of order unity, and Q is smaller than its value in this universe by a factor of between a thousand and a million. Aguirre argues that such a cosmology can produce stars and galaxies comparable in size and longevity to our own. As a bonus, a rich chemistry, including carbon, oxygen and nitrogen, can arise within seconds of the Big Bang.

The moral of Aguirre's work is that by varying more than one universal constant at once, and by being bold enough to vary them by many orders of magnitude, it is possible to discover distant regions of parameter space which appear to support a complex, biophilic universe. The key is varying more than one parameter at once, the change in one parameter effectively offsetting the change in the other. In addition, by making very large changes, the nature of the physics involved changes qualitatively.

4.2 The Weakless Universe of Harnik, Kribs and Perez

Harnik, Kribs and Perez [2006] (HKP) consider a universe which has no weak nuclear force. In Section 3.2 we discussed how reducing the value of the Fermi constant sufficiently would lead to a universe with insufficient hydrogen to support familiar chemistry. The reason is that the smaller G_F, the earlier the freeze-out of the leptonic reactions, and hence the higher the temperature, and hence the closer to equality is the density of neutrons and protons. However, we have taken for granted that the neutrons and protons achieve their thermal equilibrium densities. This will only be the case if the weak interaction exists, since this provides the mechanism for the inter-conversion of neutrons and protons. Thus, the situation is entirely different if the weak interaction does not exist at all. In this case, the relative abundance of protons and neutrons (in the primordial universe prior to BBN) would be determined by whatever CP symmetry violating mechanism gives rise to baryogenesis. In other words, we can presumably fix the relative neutron and proton abundance by *fiat*. This was the line taken by HKP. So there is no reason to assume equal numbers of protons and neutrons, and hence an all-helium universe does not result.

Moreover, the same argument applies to the baryon:photon ratio, which HKP also adjusted at will. They found that they could contrive a universe with a similar hydrogen:helium ratio as ours, but with about 25% of the hydrogen being deuterium rather than protons. To do so they chose a baryon:photon ratio of 4×10^{-12}, i.e., about a thousand times smaller than in our universe. HKP argue that galaxies could still form despite the much reduced visible baryon density, but that the number density of stars in the galaxies would be appropriately reduced. They can claim that stars would form, because they have taken the precaution of making the chemical composition of their universe sufficiently similar to ours, thus ensuring that there would be a cooling mechanism to permit gravitational collapse.

The main difference for stars in the HKP universe would be that the initial fusion reaction would be the formation of helium-3 from a proton and a deuteron. Note that HKP have cunningly contrived to have substantial quantities of deuterium formed during BBN, so there is no need for the usual weak-force-mediated deuteron formation reaction from two protons. Since the first stellar reaction in HKP stars is very fast compared with the usual weak-mediated deuteron formation reaction, the core temperature of such stars would be lower. It has to be lower to keep the reaction rate down to a level at which the thermal power does not outstrip the available mechanisms of heat transport away from the core.

The moral once again is that by varying more than one universal constant at once, and by being bold enough to vary them by many orders of magnitude (and making one vanish entirely), it is possible to discover distant regions of parameter space which appear to support a complex, biophilic universe. The key is varying more than one parameter at once, consistent with Figure 1. By making very large changes, the strategies adopted by the universe to achieve its complexity change qualitatively.

4.3 Adams' Parametric Survey of Stellar Stability

Adams [2008] has considered how common the formation of stars might be in universes with different values for the universal constants. The most important quantities which determine stellar properties are the gravitational constant G, the fine structure constant α, and a composite parameter that determines nuclear reaction rates. Adams uses a simple analytical model to determine the region within this 3-dimensional parameter space which permits stellar stability. The result is about one-quarter of it. So the requirement that stars be stable is hardly a strong constraint on the universal constants, a dramatically different conclusion from Smolin's [1997]. Yet again, so long as more than one parameter is varied, complexity is obtained even for parameter values very different from our own.

5 The Dynamical Explanation of Parameter Sensitivity

In this final section we outline our proposal regarding the mathematical cause of parameter sensitivity. The contention is that this occurs as a mathematical consequence of any dynamics which evolves into complexity.

What do we mean by a complex universe? This is a difficult question, so it is fortunate that we do not require a complete answer for our purposes. We shall see that conditions which are clearly necessary for the emergence of complexity turn out to be sufficient to imply parameter sensitivity.

In considering the meaning of "a complex universe" we generally think of the universe as it is now. The living organisms and the ecosystem of planet Earth are the epitome of complexity. However, all this did not emerge fully formed in a single step from the fireball of the Big Bang. Rather it is the current state of (one part of) a universe which has been evolving for 13.7 billion years. The history of the universe is one of increasing complexity. Thus, the formation of helium nuclei after the first few minutes represents an increase in complexity compared with what preceded it. The same is true of the formation of the first neutral atoms at ~360,000 years, and the first stars at some hundreds of millions of years. The gravitational congealing of matter provided the opportunity for complex, orderly structures to arise. Despite their gaseous form, stars have a considerable complexity of structure and evolution. The structure of galaxies is vastly more complex still, acting as they do as stellar nurseries. And the solid astronomical bodies: planets, comets and asteroids, provide the opportunity for great complexity on smaller size scales.

From the point of view of the second law of thermodynamics it is curious that the initial Big Bang fireball, which was in thermal equilibrium, spontaneously produced the orderly structures of the universe. The reason is that the orderly, and complex, structures occur in regions of gravitational collapse. Such regions have shrugged off their unwelcome excess entropy, using the vast tracts of almost empty universe as a dumping ground. This is the salient fact: inhomogeneity of the entropy distribution is a necessary condition for the emergence of complexity.

This world was not always complex. It became complex. The complexity of the world is a product of dynamics; especially, but not exclusively, the dynamics of gravitational collapse. Since parameter sensitivity is defined via complexity, it follows that parameter sensitivity should be understood as a property of dynamics - or evolution, if you will.

Phase space is the natural arena for further discussion. Phase space comprises the totality of degrees of freedom of the system in question, both generalised coordinates, q_i, and generalised momenta, p_i. A point in phase space, defined by phase coordinates $\{x_k\} \equiv \{q_i, p_i\}$, specifies a unique microstate of the system. Studies of bulk behaviour are generally conducted thermodynamically. This considers, not individual microstates, but macrostates corresponding to large numbers of possible microstates, and hence to large volumes of phase space. The volume of phase space within which a system might lie is a measure of the number of possible microstates, and hence is related to its entropy and hence to its potential for complexity.

The evolution of complexity is thus related to the variation of phase space volumes over time.

Because a given phase space point, $\{x_k\}$, is a complete specification of a microstate, it follows for any deterministic physics that the state of the system is defined uniquely at all later times. The phase space trajectory $\{x_k(t)\}$ is uniquely defined by $\{x_k(0)\}$. This statement can be written in differential form as,

$$\frac{dx_i}{dt} = f_i\left(\{x_k\}, \{c_j\}\right) \tag{1}$$

The functions f_i specify the physics which determines the system's evolution, and depend both upon the current state, $\{x_k(t)\}$, and upon the universal constants, $\{c_j\}$. Consider a small volume of phase space at time *t* and location $\{x_k\}$ defined by $dx_1, dx_2, \ldots$, with volume $dV = \prod_i dx_i$. A time δt later, the evolved system effectively defines a new set of coordinates $x_i' = x_i(t + \delta t) = x_i + \delta t \cdot f_i$. The evolved volume $dV' = \prod_i dx_i'$ is related to the initial volume by the Jacobian determinant: $dV' = \left\| \left\{ \frac{\partial x_i'}{\partial x_j} \right\} \right\| dV$. Since $\frac{\partial x_i'}{\partial x_j} = \delta_{ij} + \delta t \cdot \frac{\partial f_i}{\partial x_j}$ we find that $\left\| \left\{ \frac{\partial x_i'}{\partial x_j} \right\} \right\| = 1 + \delta t \cdot \sum_i \frac{\partial f_i}{\partial x_i}$ to first order in δt. Re-arranging and integrating to form a finite volume yields the evolution of an arbitrary phase space volume to be,

$$\frac{dV}{dt} = \int_V \left(\sum_i \frac{\partial f_i}{\partial x_i} \right) dV \tag{2}$$

In the case of a conservative system defined by Hamiltonian mechanics, the RHS of (2) is identically zero by virtue of Hamilton's equations[5]. Hence, the phase space volume does not change for Hamiltonian systems (Liouville's theorem).

We are interested, not in the whole universe, but in those parts of the universe which are forming structure, order and complexity. Suppose this consists of some subspace of phase space, $\{x_i \in \aleph\}$. This subspace within which complexity could potentially arise is not conservative. It is a dissipative subsystem, necessarily ejecting energy and entropy into its surroundings. By definition, $\aleph$ is a region whose entropy is reducing, so that its phase space volume must be reducing, i.e., $\frac{dV}{dt} < 0$.

This is the familiar behaviour of dissipative dynamic systems, whose phase space volume tends to shrink asymptotically onto some attractor, typically of lower dimension than the phase space (albeit probably fractal). Thus, the phase space volume might shrink to zero. However, an equation like (2) would continue to hold with the volume reinterpreted as being of reduced dimension.

[5] Hamilton's equations can be interpreted as stating that the divergence of the phase velocity field is zero. Alternatively expressed, the phase space of a Hamiltonian system is a symplectic manifold.

Since the volume cannot be negative, the condition $\frac{dV}{dt} < 0$ inevitably leads eventually to a minimum volume, which may or may not be zero[6]. In either case, this implies that $\frac{dV}{dt} \to 0$ after a sufficiently long time. But this innocent observation implies, by virtue of (2), that,

$$\int_{\aleph} \left(\sum_i \frac{\partial f_i(\{x_k\}, \{c_j\})}{\partial x_i} \right) dV \to 0 \qquad (3)$$

where the integral is over the structure-forming region, $\aleph$. Unlike the case of a conservative Hamiltonian system, (3) is not an algebraic identity. It is a dynamic constraint on any subsystem whose entropy is continually reducing. But note that all the dynamic variables, $\{x_i\}$, vanish from the LHS of (3) by virtue of the volume integration. Hence, (3) is actually a constraint on the universal constants, $\{c_j\}$, which must be fulfilled if the region $\aleph$ is to have continually reducing entropy.

We claim that the existence of a region of continually reducing entropy is a necessary (but not sufficient) requirement for complexity to arise. Consequently, the emergence of complexity requires that the universal constants obey the constraint,

$$F(\{c_j\}) \approx 0 \qquad (4)$$

where F is given by the LHS of (3). Equation (4) is the algebraic expression of Figure 1.

In general, different fine tunings will correspond to different phase space regions $\aleph$, each with a corresponding constraint like (4).

As developed, equation (4) would suggest an extreme degree of parameter sensitivity, i.e., an infinitely thin rather than a 'thick' N-1 dimensional subsurface as in Figure 1. There are a number of reasons why the equality in (4) may only be approximate. One is that the RHS, actually dV/dt, may not be quite zero. Another reason is that the distinction between the structure forming region, $\aleph$, and the rest of phase space is likely to be both imprecise and also impermanent. Stars do not last forever. The mantles of large stars may shortly find themselves subjected to a supernova, thence becoming part of the non-$\aleph$. The reverse also occurs due to the formation of new stars.

Whilst equations (3,4) seem rather abstract, they are the precise expression of a form of behaviour that becomes familiar after conducting a few numerical experiments on simple invented systems. Complex behaviour seems to be found only for a limited range of the tuneable parameters.

There are a number of potential challenges to the generality of the arguments expressed by equations (1-4). Firstly, it might be argued that not all physical processes are described by dynamic equations like (1). For example, BBN is not normally formulated in this manner. However, it could be. Although gravitational collapse is an obvious example of entropy reduction, the formation of larger nuclei from smaller, or the formation of

[6] Note that zero volume does not imply zero entropy because the entropy would then be determined by the volume on the attractor of reduced dimensionality.

chemical compounds, are also examples of phase space contraction. They result from the nuclear force and the electromagnetic force respectively. The formation of a bound state of two particles involves a 12 dimensional phase space reducing to 6 dimensions, and an associated entropy reduction (balanced by the emission of some secondary particles or quanta into the non-$\aleph$). The reduction of phase space volume does not require the reduction of the physical volume of the system, nor is it restricted as regards the nature of the physical force involved.

Our scheme might appear to depend upon a cosmic time coordinate. This is probably not an essential limitation since we could interpret the system in question as being any sufficiently large portion of the universe. In any case, standard cosmology is based on a cosmic time coordinate (the existence of which follows from large scale homogeneity).

Finally, equations (1) are deterministic and do not address quantum mechanical behaviour. We do not explore here whether an analogue of equations (3,4) would also be found via a quantum mechanical approach. Of course, all physical processes are ultimately quantum mechanical. And quantum calculations are essential in providing much of the input to astrophysical calculations (e.g., reaction rates). However, as far as bulk behaviour is concerned, and providing we do not attempt to address what happens at t = 0, it is likely that the classical formulation is sufficient.

6 Conclusion

The problem of elucidating why the universe is parameter sensitive has not previously been addressed, despite much effort expended on the issue of fine tuning.

The thesis has been presented that parameter sensitivity arises as a natural consequence of the mathematics of dynamical systems with complex outcomes. The argument is as follows: the emergence of complexity requires regions of entropy reduction, which can be interpreted as a reducing phase space volume. This leads, via a very general formulation of system evolution, to a constraint on the set of universal constants, $F(\{c_j\}) \approx 0$, for each contracting phase space region, $\aleph$. Each fine tuning is identified with an instance of $\aleph$.

Hence, parameter sensitivity is inevitable given that the universe is complex, and therefore fine tuning will always be required to produce a complex world.

However, the present considerations do not address how or why the universe is fine tuned, only that fine tuning is an inevitable requirement for a complex outcome. Consequently the motivation for postulating a Creator or a Multiverse is unchanged by the present work, except in one respect.

Fine tuning has been interpreted by some as strengthening the argument from design compared to its nineteenth century form. This is not so. The observation that the world is complex is sufficient to imply parameter sensitivity, and hence that fine tuning must occur. So, the physicists' discovery of fine tuning adds nothing to the argument from design based directly on the world's complexity, because the former is inevitable given the latter.

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