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The Inevitability of Fine Tuning in a Complex Universe

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ABSTRACT

Why should the universe be fine tuned? It is pointed out that neither the God hypothesis nor the multiverse hypothesis address this question. The thesis is presented that the tuning of the parameters of physics arises as a natural consequence of the mathematics of dynamical systems. A complex outcome from a dynamical system will tend to occur only for finite ranges of its tuneable parameters. Hence tuning is a mathematical correlate of complexity. This contention is illustrated by numerical examples. We also note that in an N-dimensional parameter space, the observation of tuning in all N parameters need only reduce the number of degrees of freedom by one. Alternative complex universes might exist (in the mathematical sense) with values of the universal constants well outside the fine-tuned bounds that are usually claimed.

1 The Two Distinct Problems of Fine Tuning

It is over 40 years since Carter [1967] observed that the universal constants appear to be peculiarly fine tuned. Relatively small variations in the universal constants, it is claimed, produce radical changes in the universe. If true, this is an occurrence which requires explanation.

A great deal has been written about the merits, or otherwise, of God or the multiverse as the explanation of this apparently improbable state of affairs. It seems to have gone largely unnoticed that an explanation is required of two quite separate phenomena. The first is fine tuning itself. This is the (claimed) property of the universe that small changes in the parameters of physics produce catastrophic changes in the evolved universe. In particular its complexity, and hence its ability to support life, would be undermined (it is said). Thus, 'fine tuning' is the claim that the target in parameter space which is compatible with a complex universe is small. The smallness of the target, if true, is one of the features which requires explanation. The second, and quite distinct, part of the required explanation is how nature manages to hit this small target. In other words, given that only special values for the parameters will do (i.e., given that fine tuning is true), how does nature contrive to adopt these particular values for the parameters?

It is the second question which appears to have received all the attention. The contrivance by which the small target is successfully hit is purported to be God or the multiverse, according to taste. But neither of these postulates even attempts to explain why the target is small in the first place. They aim only to explain how a small target may be hit. Why, then, is the universe fine tuned? It appears that no explanation has previously been offered.

The point can be illustrated in the following way. Before one looks into the physics of these things, it is not obvious that there could not be complex universes corresponding to the bulk of the volume of parameter space (or the 'string landscape' as it tends to be called these days, Susskind [2003], Hogan [2006]). Take life as an exemplar of complexity and consider the universes which might result if changes were made to the universal constants. We can imagine, without any nonsense being apparent, that as we follow some path through parameter space, the carbon based life forms we know give way to a continuum of different life forms. A sequence of small changes would give rise eventually to completely different forms of life – not unlike evolution, but far more radical since the physics of the environment is also changing. This description of the consequences of changing the universal constants is precisely what 'fine tuning' claims is *not* true. But why is this?

Neither God nor the multiverse explain why there should be no complexity in these alternative universes. They do not explain fine tuning. Fine tuning must be addressed as a consequence of physics, and is, we shall argue, a result of the mathematical properties of dynamical systems. We shall argue that fine tuning is inevitable in any complex universe.

2 The Physical Evidence for Fine Tuning

Before expending a great deal of effort in seeking an explanation for fine tuning, how confident should we be that fine tuning is truly a property of our universe? There are two aspects to this. The first is the quality of the physical arguments and calculations which underpin the claim. The second is their interpretation. Just how 'fine' is the fine tuning? What is meant by a *small* change in a universal constant? Philosophical enquiry has tended to focus on these interpretation issues, whilst taking on trust that the physics has been dealt with properly by the physicists. This is perhaps unfortunate. Whilst there have been many compilations of 'cosmic coincidences'[1], some have tended to be rather polemical. Even putting to one side the thorny issue of interpretation, a balanced critique of the physical status of the coincidences is overdue. Unfortunately, this is far too large an undertaking for the present paper, so just a few examples must suffice.

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[1] A few examples are Agrawal et al [1998a,b], Barrow and Tipler [1986], Barrow et al [2008],Carr [2007], Carter ([1967,1974]), Davies [1972,1982,2004,2006], Dyson [1971], Gribbin and Rees [1989], Hogan [2000,2006] and Rees [1999,2003], but there are many more.

Lest the reader get lost in the detail, our contention will be that most instances of tuning are not terribly ‘fine’. Nevertheless, not-so-fine tuning is widely evident and is no less in need of explanation for being imprecise.

The first example concerns the constraints on particle masses which arise from the requirement that atomic matter be stable. For atomic matter to be stable, the mass of the neutron must exceed the sum of the masses of the proton and the electron. Otherwise the nucleus would capture the atomic electrons via the reaction $e + p \to n$, and all matter would reduce to neutrons. There would then be no chemistry. But the neutron mass is 939.565MeV and the sum of the proton and electron masses is 938.783MeV, so it is a close run thing, disaster being averted by a mass deficit of just 0.08% of the neutron mass. Moreover, the fact that we have required $\Delta = M_n - (M_p + m_e) > 0$ leads to a free neutron being unstable. This means that atomic nuclei may be unstable via beta decay, i.e. $n \to p + e + \bar{\nu}$. The lighter nuclei escape this instability only because the daughter nucleus often has a smaller binding energy than the original nucleus, by an amount, ΔB, which exceeds the mass deficit, Δ. This is not mere theory: nuclei with $\Delta B < \Delta$ are invariably unstable. Stable nuclei exist, therefore, only because they have $M_n < M_p + m_e + \Delta B$. The magnitude of ΔB varies, often being negative for the heavier, unstable nuclei. For the commoner isotopes of the lighter nuclei ΔB is usually a few MeV, say 10 MeV for illustration. Hence $M_p + m_e + \Delta B \sim 949$ MeV, in comparison with the neutron mass of 939.565MeV. This narrowly misses a universe with no stable atoms by just 1% of the neutron mass. Hence if the neutron were lighter by more than 0.08%, or if it were heavier by more than ~1%, then there would be no stable atomic matter in the universe.

This seems impressive, but becomes much less so when it is recalled that the neutron and the proton share a common structure. About 99% of a nucleon’s mass is due to the virtual gluons and virtual quarks which comprise the strong nuclear force. This feature is shared by the neutron and the proton, which differ only as regards the ‘valence’ quarks which provide the nucleons with their net quantum numbers. Since the u and d quarks in question have masses of just a few MeV, it is no longer particularly surprising that the neutron-proton mass difference is also of this order. In fact this is to be expected. The moral is that there are mechanistic reasons why the neutron and proton masses should be very close. This is not to say that there is no ‘tuning’ at all, just that it is not so terribly ‘fine’ as it first appears. It is more indicative to compare $M_n - M_p$ with the mass of the electron or the mass of the u or d quarks. On this scale the tuning is at the level of tens or hundreds of percent, rather than less than 1%. Nevertheless, there is *some* tuning. For example, the d quark must be heavier than the electron for atomic stability[2].

Our second example concerns the survival of hydrogen in the first seconds after the Big Bang and the constraint this places on the strength of the weak nuclear force. Within the first second, leptonic reactions interconvert neutrons and protons such that their relative abundance is determined by dynamic equilibrium, and hence by the temperature. By the

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[2] That is, if we make the rather sweeping assumption that $M_n - M_p \approx M_d - M_u$. Far more carefully argued constraints on the u, d and s quark masses which produce a congenial universe have been discussed recently by Jaffe, Jenkins and Kimchi [2009] and by Damour and Donoghue [2008].

time nucleosynthesis starts, after the first minute or two, the ratio of neutrons to protons has been set by the temperature prevailing when the leptonic reactions are frozen-out by cosmic expansion. The fact that the universe contains comparable quantities of hydrogen and helium is a consequence of the 'coincidence' that $M_n - M_p$ is of the same order as kT at the time of freeze-out of the leptonic reactions. This requires that the weak nuclear force, which drives the leptonic reactions, be of a particular strength close to its actual strength. Had the weak force been sufficiently weaker then freeze-out would occur earlier when the temperature was higher, resulting in the abundance of neutrons and protons being closely matched. Big Bang nucleosynthesis (BBN) would then result in a universe consisting of virtually all helium and very little hydrogen. A universe with no hydrogen would contain no water, no hydrocarbons such as amino acids, and no hydrogen bond chemistry. Whilst we cannot be confident what such a universe would be like in detail, it would certainly not support life as we know it.

Because the neutron:proton ratio is the exponential of $-(M_n - M_p)/kT$, it is often claimed that it is highly sensitivity to changes in the freeze-out time, and hence to the strength of the weak force. Actually, a closer examination shows that if the Fermi constant, G_F (the parameter which controls the rate of the leptonic reactions) were reduced by an order of magnitude, the universe would still be 18% hydrogen (by mass, and nearly 50% by number of atoms). This would still support hydrogen burning stars with lives in the order of billions of years, long enough for biological evolution. Reducing G_F by a factor of 100 would still leave the universe with ~14% hydrogen by number of atoms. Admittedly if the hydrogen abundance were reduced too much this would ultimately prejudice the formation of the first stars, which is believed to rely on a cooling mechanism via molecular hydrogen. However, quantitative knowledge of these mechanisms is poor. Nevertheless, there is no obvious reason to regard as catastrophic a reduction in G_F by somewhat more than a factor of ten.

If G_F were increased, then there would be less helium in the universe. For example, a factor of 4 increase in G_F results in only ~0.2% helium by mass. But this would seem unimportant. Helium appears to play no essential role in the formation of large scale structure or stellar physics[3]. Although no upper bound on G_F results from these considerations, there are suggestions that Type II supernovae require G_F to lie close to its actual value. This is because crucial aspects of the mechanism of Type II supernovae involve neutrino interactions, i.e., weak-force interactions. The neutrinos seem to be required to interact just weakly enough to escape the core of the collapsed star, but strongly enough to transfer almost all their energy to the mantle to cause the explosion. Unfortunately the quantitative understanding of Type II supernovae is too poor to deduce just how fine tuned G_F must be. In any case, it is not clear that Type II supernovae are crucial for life.

The claim is that Type II supernovae are essential in order to release the stars' precious product of chemical elements into the interstellar medium (ISM), ultimately contributing to some biosphere. In truth, only the elements beyond iron require Type II supernovae. There are other mechanisms which ensure release of the lighter elements into the ISM. It

[3] The ppII and ppIII reaction sequences would be slowed by the absence of initial helium, but the ppI sequence is unaffected.

is by no means clear that the heavier elements are required for life. The essential biochemistry is provided by the lighter elements. On the other hand, a case can certainly be made for the importance of planetary geothermal and tectonic activity in supporting the emergence of life, and these depend upon heating by the radioactive heavy elements formed in Type II supernovae.

In summary, there is a case for considering G_F to be a genuine instance of tuning, but it is not necessarily terribly 'fine'. To produce a universe with sufficient hydrogen, there is a relatively weak requirement, i.e., that G_F be greater than a few percent of its actual value.

Our third example of fine tuning relates to the strength of the strong nuclear force and the constraint upon it to ensure that nuclei are indeed bound states of their nucleons. Both Big Bang nucleosynthesis and hydrogen burning in stars proceed via deuterium (^2_1H) as an intermediate product. Consequently the production of any elements would be prevented if deuterium were not a stable nucleus. It requires a reduction in g_s, the low energy effective coupling of the strong force, of only 15% to unbind the deuteron and hence prevent the formation of the chemical elements. This does appear to be a genuine instance of reasonably 'fine' tuning, requiring that g_s exceed ~85% of its actual value.

Claims are often made that there is also an upper bound on g_s to avoid diproton stability. If g_s were ~10% larger, then the diproton (^2_2He) would be a bound state[4]. It has frequently been claimed that this would lead to an all helium universe. The argument is that all the nucleons would end up as helium during BBN, either via the conventional sequence starting with $n + p \rightarrow ^2_1H$, or via the diproton $p + p \rightarrow ^2_2He \rightarrow ^2_1H + e^+ + \nu$. The inverse beta decay which converts the diproton to a deuteron is possible because the binding energy of the deuteron (2.224 MeV) exceeds $M_n - M_p + m_e = 1.804$ MeV. However, this argument is just wrong. The reason is that, even if the diproton were stable, the rate of its formation via $p + p \rightarrow ^2_2He$ is too slow for any to be formed during BBN (Bradford [2009]). It is true that the nuclear physics of stars would subsequently be very different, but there is no obvious reason why biophilic stars would not be stable (Bradford [2009]). Hence, once again we conclude that there is a case for some tuning of g_s but that the case tends to have been overstated in the past.

For our final example we consider the famous Hoyle coincidence concerning the production of carbon and oxygen in stars. This bucks the trend of the previous examples in that our view is that this 'coincidence' is genuinely impressive in terms of numerical precision. The instability of beryllium-8 (^8_4Be) means that carbon ($^{12}_6C$) can be produced only by virtue of the subsequent alpha capture reaction $^8_4Be + ^4_2He \rightarrow ^{12}_6C$ being extremely fast due to the existence of a resonance of the carbon nucleus at just the right energy level. Moreover, the subsequent burning of all the carbon into oxygen is avoided by the fortuitous placing of the energy levels of the oxygen nucleus so that resonance is avoided. The Hoyle coincidence often gets a bad press even amongst those who might be anthropically sympathetic. For example, Weinberg [2005] says, "*I don't set much store*

[4] The diproton is not bound in this universe. This is because the spin-singlet nuclear force is weaker than the spin-triplet nuclear force which binds the deuteron. It is *not*, as some authors have claimed, due to electrostatic Coulomb repulsion.

by the famous 'coincidence' emphasised by Hoyle, that there is an excited state of C^{12} with just the right energy to allow carbon production via $\alpha - Be^8$ reactions in stars. We know that even-even nuclei have states that are well described as composites of α-particles. One such state is the ground state of Be^8, which is unstable against fission into two alpha particles. The same α-α potential that produces that sort of unstable state in Be^8 could naturally be expected to produce an unstable state in C^{12} that is essentially a composite of three alpha particles, and that therefore appears as a low-energy resonance in $\alpha - Be^8$ reactions. So the existence of this state doesn't seem to me to provide any evidence of fine tuning."

I disagree. The above argument does explain the existence of Be^8 and C^{12} states which are close to the two-alpha and three-alpha thresholds respectively. And the same argument can be applied to O^{16}. Thus, we could expect states of low energy with respect to the $N\alpha$ threshold for each of Be^8, C^{12} and O^{16}. However, to reproduce the observed state of affairs we require the first two to be resonant, and the third to be non-resonant. Thus, the first two must have small positive energies and the third a small negative energy with respect to the alpha capture thresholds. Perhaps the fact that this is realised is not a particularly remarkable coincidence, i.e. a 1 in 8 chance. However, the numerical size of the deviations of these energies from zero (i.e. +91.9, +287.7 and -45.0 keV with respect to the reactant threshold), as well as their signs, are crucial in producing a universe with a balance of carbon and oxygen. Weinberg's argument would appear to suggest that merely being close to zero is enough. It is not. The signs must be right and the magnitudes must be close to their actual values to an accuracy which surpasses the accuracy of current nuclear physics calculations. Consequently, Weinberg's argument is simply not precise enough to account for the degree of fine tuning which is realised in practice.

The Hoyle [1954] coincidence becomes more impressive when it is realized just how sensitive the energy levels of these nuclear states are to the strength of the strong nuclear force. Suppose we write the energy of the C^{12} 0_2^+ resonance with respect to the three-alpha threshold as $\Delta E = E_C - 3E_\alpha$. Then the fractional change in this energy due to a change in the strength of the nuclear force by a fraction ε is easily shown by an elementary argument to be,

$$\frac{\delta \Delta E}{\Delta E} = 223 \varepsilon \eta_C \left[1.00448 \frac{\eta_\alpha}{\eta_C} - 1 \right]$$

Here $\eta_C \equiv V / E$ is the ratio of the potential energy to the total energy, $E = V + K$, of the nuclear ground state of carbon, where K is the total kinetic energy. Similarly, η_α is the same ratio for helium. These η-factors are not accurately determined by nuclear physics models, current values typically lying in the range 2 to 5 for both carbon and helium. Consequently the ratio η_α / η_C is very uncertain (between 0.4 and 2.5). If η_α / η_C were precisely unity, then the fractional change in ΔE would be $\varepsilon \eta_C$, i.e. about 2 to 5 times greater than the postulated fractional change in the strong force. However, if η_α / η_C were 0.91, then the fractional change in ΔE would be $-19 \varepsilon \eta_C$, and hence a factor of 38 to 95 times greater than the fractional change in the strong force. This phenomenon (derived in a different manner) has been referred to by Csoto et al [2000] as ‘non-linear amplification’. A mere 0.4% change in the strength of the nuclear force can thus produce a change in ΔE of up to 38%. Since ΔE is 379 keV, the change in ΔE is thus 144 keV. But it has been shown via detailed stellar models that a reduction in the C^{12} 0_2^+ resonance energy of this size will result in a reduction in carbon production of around two orders of magnitude (Oberhummer [1999,2000], Csoto [2001], Schlattl [2004]). Alternatively, an increase in the C^{12} 0_2^+ resonance energy of ~144 keV will result in a reduction in oxygen production of around two orders of magnitude. Consequently it seems that the Hoyle coincidence remains one of the more impressively fine tuned, requiring changes in the strong force of only ±0.4% to challenge the likelihood of conventional biochemistry. Moreover, if η_α / η_C is actually less than 0.91, or greater than 1.08, which does not seem unlikely given the current possible range (0.4 to 2.5) then an even smaller % change in the strong force would be abiophilic.

3 How Fine is Fine Tuning?

Some people regard any consideration of variations in the universal constants as intrinsically nonsensical. They may have a point. If the universal constants are, in fact, fully prescribed by pure mathematics, as has long been the dream of physicists, then considering them to be varied is as foolish as considering a variation in the result of 1 plus 1. We have no rigorous answer to this. However, there has been precious little encouragement over the last 100 years that this ambition may be fulfilled. The standard model of particle physics did not lead to a reduction in the number of universal constants, nor did the grand unified theories. The hope that string theory might do so has been dashed by the colossal number of different string theories (Kachru et al [2003], Susskind [2003], Dine [2004]), notwithstanding the likelihood that any one string theory would constrain the constants (Kane et al [2002]). More pragmatically we simply admit that the subject matter of this paper is contingent upon the consideration of variations in the universal constants not being intrinsically nonsensical.

We are concerned with the degree of fineness of the tuning because the finer the tuning the more remarkable the coincidence. So one is tempted to think. If a parameter can tolerate only a very small percentage variation before the universe is rendered abiophilic it is natural to regard this universe as an improbable state of affairs. “Natural” it may be, but not necessarily defensible. A number of authors have pointed out that the smallness of the numerical window within which a parameter must lie to result in a complex universe says nothing at all about its probability (e.g., Manson [2000], McGrew et al [2001]). The implicit appeal to small windows implying small probabilities is false. To make the small window into a small probability we must know something about the possible range of values, and their probability distribution. This is difficult to dispute, though Koperski [2005] and Monton [2006] have attempted to do so. Koperski opines that, “*whilst both design and multiverse proponents might be wrong, their common demand for an explanation of fine tuning is justified (at present)*”. We can defuse the conflict by noting that the intention of the present paper is, in any case, to demonstrate that the tuning of the physical parameters is actually a natural state of affairs, i.e., not improbable at all. However this argument will be made by analogy with example dynamical systems, and is most credible if the tuning is not excessively fine.

In fact Klee [2002] reminds us that many of the claims of fine tuning are not at all fine. Many are very coarse, as we have already noted in Section 2. Klee observes that a claimed 'order of magnitude agreement' is often stretched to cover cases which actually differ by two or three or even four orders of magnitude. One source of such elasticity in order-of-magnitude estimates lies in the use of dimensional analysis. The trouble with dimensional analysis is that it does not provide the magnitude of the dimensionless multiplier. So this is merely assumed to be of order unity. Barrow and Tipler [1986] refer to the tendency of this assumption to be about right as 'the unreasonable effectiveness of dimensional analysis' and proceed to attempt to explain it on the basis of the low dimensionality of space. However, it just is not true. There are formulae in physics which display very large numerical factors. For example, dimensional analysis suggests that the lifetime of the muon should be in the order of $G_F^{-2} m_\mu^{-5}$. A field theory calculation confirms that the lifetime is proportional to this quantity, but evaluates the numerical multiplier to be $3 \times 2^6 \pi^3 = 5{,}953$. So dimensional analysis is not 'unreasonably effective' in this case, on the contrary it fails by nearly 4 orders of magnitude.

At the other extreme we have the flatness problem and the cosmological constant. The density parameter, Ω, is the ratio of the density of the universe to the critical density which is just sufficient to halt the universal expansion after a divergently long time. The flatness problem is the observation that cosmic expansion causes the density parameter to diverge rapidly away from unity. Thus, to obtain a value for Ω consistent with current observations (i.e., within about 1% of unity), its value at 1 second would have to be equal to 1 to an accuracy of 16 decimal places. This could be interpreted as an extreme degree of fine tuning. However, we take the view that it is inappropriate to regard Ω as fine tuned. Rather its value is probably constrained by a mechanism. At the present time the favourite mechanistic explanation is inflation. But even if inflation became discredited, it remains likely that such extreme precision is constrained by theory rather than tuning.

The other parameter which falls into this class is the cosmological constant, Λ. The existence of a non-zero cosmological constant is one possible explanation for the 'dark energy' which is apparently driving the acceleration of the universal expansion. If the cosmological constant has its origin in the zero-point energy of quantum fields, dimensional analysis would suggest that the energy density should be of the order of the Planck density. The trouble is that this density is ridiculously huge, exceeding the critical density by 123 orders of magnitude. One perspective on this is that Λ is the product of extreme fine tuning, at an accuracy of the 123rd decimal place. Of course, it is always valid to use our knowledge of the universe to provide constraints on the parameters of physics, as was done for the cosmological constant by Martel et al [1998] and Weinberg [2005]. But this does not preclude the possibility that specific explanations for its magnitude might be found. We take the view that there is a physical reason for the magnitude of Λ, always assuming the apparent acceleration of the cosmic expansion stands the test of time. For example, it may be that the driving force is some field with contingent parameters unrelated to the Planck scale. No surprise is expressed that atomic or nuclear densities differ markedly from the Planck density, and it may be no more appropriate to be surprised that Λ differs from the Planck density. Alternatively, it may be that the cosmological constant is not determined by local effects but is globally constrained in some manner. In which case it would not be determined by $\hbar$, c and G, i.e., the Planck scale.

So we discount the extreme cases of flatness and the cosmological constant, expecting mechanistic explanations. As for the remaining universal constants, we contend that, in keeping with the examples of Section 2, whilst tuning of the physical parameters is evident, it is generally not-so-fine.

4 The Dynamical Explanation

Having pulled the teeth of fine tuning, we now offer an explanation for what residual tuning remains. Recall that 'tuning' means that the universal constants of physics can only be varied within a relatively limited range without destroying the complexity of the universe. What do we mean by a complex universe?

In considering this question we generally think of the universe as it is now. The living organisms and the ecosystem of planet Earth are the epitome of complexity. However, all this did not emerge fully formed in a single step from the fireball of the Big Bang. Rather it is the current state of (one part of) a universe which has been evolving for 13.7 billion years. The history of the universe is one of increasing complexity. Thus, the formation of helium nuclei after the first few minutes represents an increase in complexity compared with what preceded it. The same is true of the formation of the first neutral atoms at ~360,000 years, and the first stars at some hundreds of millions of years. The gravitational congealing of matter provided the opportunity for complex, orderly structures to arise. Despite their gaseous form, stars have a considerable complexity of structure and evolution. The structure of galaxies is vastly more complex still, acting as they do as stellar nurseries. And the solid astronomical bodies: planets, comets and asteroids, provide the opportunity for great complexity on smaller size scales.

From the point of view of the second law of thermodynamics it is curious that the initial Big Bang fireball, which was in thermal equilibrium, spontaneously produces the orderly structures of the universe. The reason is that the orderly, and complex, structures occur only in regions of gravitational collapse. Such regions have shrugged off their unwelcome excess entropy, using the vast tracts of almost empty universe as a dumping ground. Indeed, it may be that the inhomogeneity of the entropy distribution provides a sufficient condition for the emergence of complexity. It is certainly a necessary condition.

The key message is that this world was not always complex. It became complex. The complexity of the world is a product of dynamics; especially, but not exclusively, the dynamics of gravitational collapse. Since tuning relates to the acquisition of complexity, it follows that tuning should be understood as a property of dynamics - or evolution, if you will.

Suppose that we can describe the state of a hypothetical universe at a given time by a set of parameters $Z = \{z_i\}$. This could involve great detail, such as the positions and velocities of all material particles. However, it might also include more global parameters such as the abundance of each chemical element, the diversity of chemical compounds, the degree of departure from thermodynamic equilibrium, and measures of the departure from homogeneity, such as the number and distribution of galaxies, stars, planets, and so on. It might seem that if many of these numbers were zero then we could say immediately that the universe was not complex. However, this will not do. It might be simply that we have not given the universe time enough to evolve. Our own universe was very simple for the first millions of years. So, to judge its complexity, we must consider the whole biography of the hypothetical universe, from birth to death (if there is one).

The key point is to recognise that the signature of a complex universe is that it does not remain at a fixed point in Z-space, but rather it evolves. It moves around in Z-space. A universe which merely shuffles between a few points in Z-space is unlikely to be complex. Rather, complexity is likely to involve the exploration of large portions of Z-space. For example, our universe started with uniform density and temperature and with few distinct ingredients, but is currently highly inhomogeneous with a great diversity of content. Moreover, evolution still continues, both astronomically and in the biosphere. Being at a complex Z-point guarantees that motion in Z-space will continue. Stasis is the antithesis of life. This is rather convenient since, instead of having to define what property of Z constitutes complexity, we can recognise complexity by how Z changes.

Suppose the set of universal constants which control the physical evolution of a universe is C = $\{c_j\}$. The evolution of a universe with these values for the universal constants, which is currently in state Z, can be written symbolically as,

$$Z \rightarrow Z' = \hat{F}_C Z \qquad (1)$$

Thus, the mapping or operator, $\hat{F}_C$, defines how the universe evolves in some chosen time interval if the universal constants are C = $\{c_j\}$. The biography of the universe is the whole history of points in Z-space, i.e.,

$$\text{Biography of the universe} = \left\{\hat{F}_C^n Z_0, n = 0,1,2....\right\} \qquad (2)$$

assuming we can start the clock going from some known primordial state Z_0 (like a Big Bang fireball at 1 millisecond, say). The integer 'n' is effectively a discrete time. (A continuum time can be adopted instead by changing the notation to time derivatives). From our preceding discussion, we claim that the signature of a complex universe is having a large biography, i.e., it consists of a large number of different Z-points.

Quite generally, an iterated mapping from a given starting point, Z_0, can have one of the following behaviours,

1) $\text{LIM}_{n\to\infty} \hat{F}_C^n Z_0 \rightarrow \text{a limit point } Z_1$

2) $\text{LIM}_{n\to\infty} \hat{F}_C^n Z_0 \rightarrow \text{diverges to infinity}$

3) $\text{LIM}_{n\to\infty} \hat{F}_C^n Z_0 \rightarrow \text{a limit cycle} \left(Z_1, Z_2, Z_3, ...Z_N, Z_1, Z_2, ...\right)$

4) None of the above.

In case (1), the biography of the universe is a sequence of Z-points which converge on a limit point, and hence is dull. Case (2) is really a special case of (1) for which the limit point in question is the point at infinity. In case (3), there is a finite length limit cycle after the initial ‘transient’. This is also dull if N is small but may qualify as interesting if N is sufficiently large. However, it is clear that case (4) is the winner as regards complexity. It involves a never-repeating, and hence infinite, sequence of Z-points.

In the complex dynamics of iterated maps, the union of cases 3 and 4 is the Julia set of the mapping. Thus, for the quadratic map $z \to z^2 + c$, the sets of points in complex z-space which do not converge to a limit point or diverge to infinity are the familiar Julia sets associated with the Mandelbrot set. The Mandelbrot set itself is the set of complex c-points such that the mapping does not diverge, from an assumed starting value of z = 0. The value of c is the analogue of a universal constant, whereas the value of z is the analogue of an instantaneous ‘state of the universe’. The interesting feature of this analogy is how the Julia set varies according to what value of c is chosen.

For values of c well inside the Mandelbrot set, away from its fractal boundary, the Julia set is a simple, closed loop. For values of c well outside the Mandelbrot set, again at some distance from its fractal boundary, the Julia set breaks up into sparse disconnected dust. But for values of c near the fractal boundary of the Mandelbrot set, the Julia set is complex and full of structure. This is illustrated by Figure 1 which shows the Julia sets on a trajectory across the Mandelbrot plane, starting near the centre and ending well outside the Mandelbrot set. The third and fourth Julia sets correspond to values of c just inside and just outside the Mandelbrot set respectively. (The Julia sets were generated using the program of Joyce [1994]).

This provides the first illustration of our thesis. It is mathematically natural for complexity in the universe (the Julia set) to occur only for a relatively narrow range of values of the universal constants (c, namely near the Mandelbrot fractal boundary)[5].

[5] This is, of course, only an analogy. However, the reader may be disturbed by the fact that the Julia set is dynamically unstable. A small perturbation will result in divergence or convergence to the origin. However this can be fixed simply by reversing time, that is, by replacing the quadratic mapping by its inverse, whereupon the Julia set is reinterpreted as the attractor of the dynamical system.

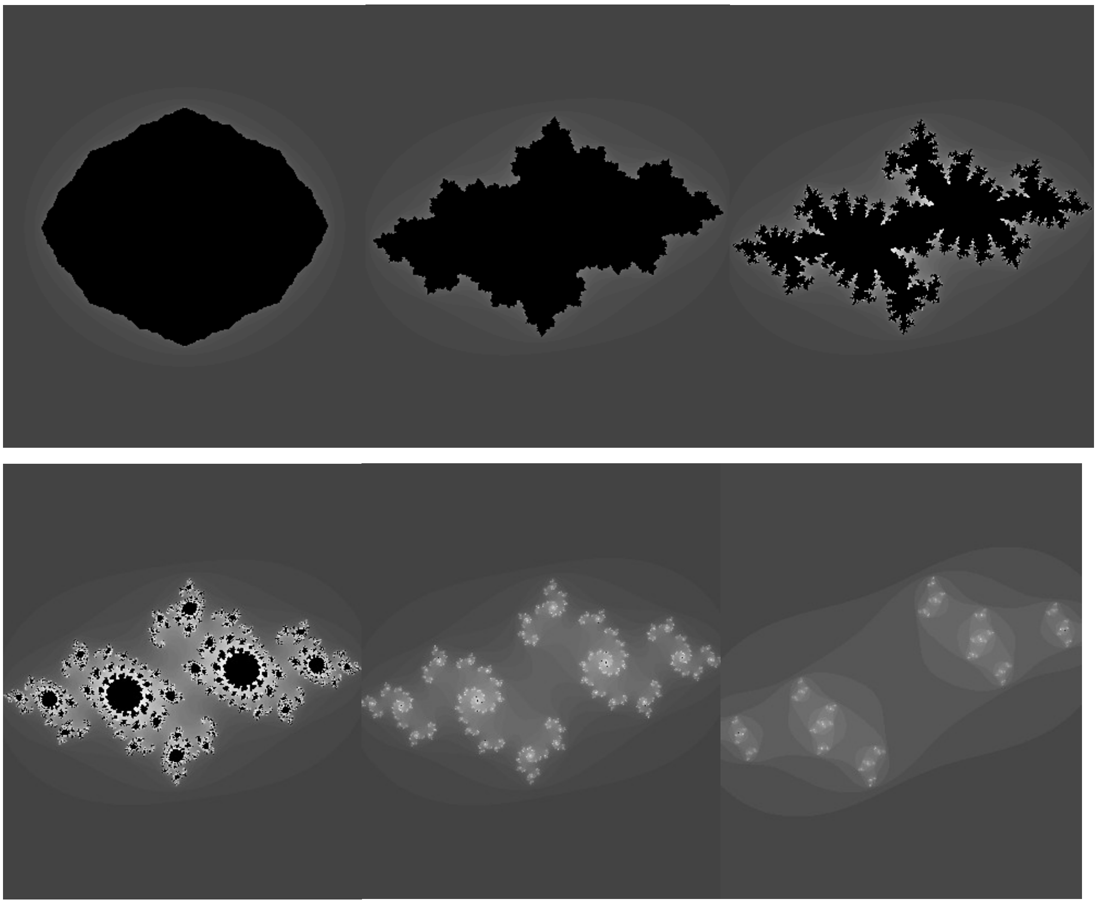

Figure 1: *A sequence of Julia sets on a trajectory across the Mandelbrot plane. The upper row lie inside the Mandelbrot set, and the lower row outside. The third and forth Julia sets are very close to the fractal boundary of the Mandelbrot set. Thus, a complex 'universe' (Julia set) results for only a limited range of 'universal constants', c, lying near the Mandelbrot fractal boundary.*

The Mandelbrot-Julia set example is only an analogy, though an instructive one. But what about true dynamics, formulated in real continuum time? In non-linear dynamics, the long term behaviour of the trajectory in phase space tends to a limit point, including the point at infinity, a finite limit cycle or a strange attractor. A strange attractor thus provides a signature of complexity. Consider the equations which produce Rossler's Band, (Rossler [1976]),

$$\dot{x} = -y - z \qquad \dot{y} = x + ay \qquad \dot{z} = b + z(x - c) \tag{3}$$

where a dot denotes the time derivative. For appropriate choices of the parameters a, b and c, and an appropriate choice for the starting point, these equations produce a phase portrait in (x,y,z)-space which converges onto a Rossler band strange attractor. Rossler's band behaviour is the signature of complexity for this system. It is found to occur only for certain ranges of the parameters a, b and c. For example, setting b = 2, the region of the (a, c) plane which can produce this behaviour is shown in Figure 2 (assuming integration from the (x,y,z) origin). Within this finite region of the (a, c) plane, both Rossler's band behaviour and limit cycle behaviour occurs. This finite region is surrounded by two distinct regions in which the phase space trajectory either diverges or converges to a limit point. The interesting behaviour is thus confined to a finite region of (a, c) parameter space. (Note that Figure 2 is only approximate, the fine detail of the boundaries between the regions has been omitted).

The parameters, a, b and c are, of course, the analogues of the "universal constants" in this illustration. The Rossler equations thus provide a very simple illustration of how complex behaviour results from dynamics only for finite ranges of the tuneable parameters of the system.

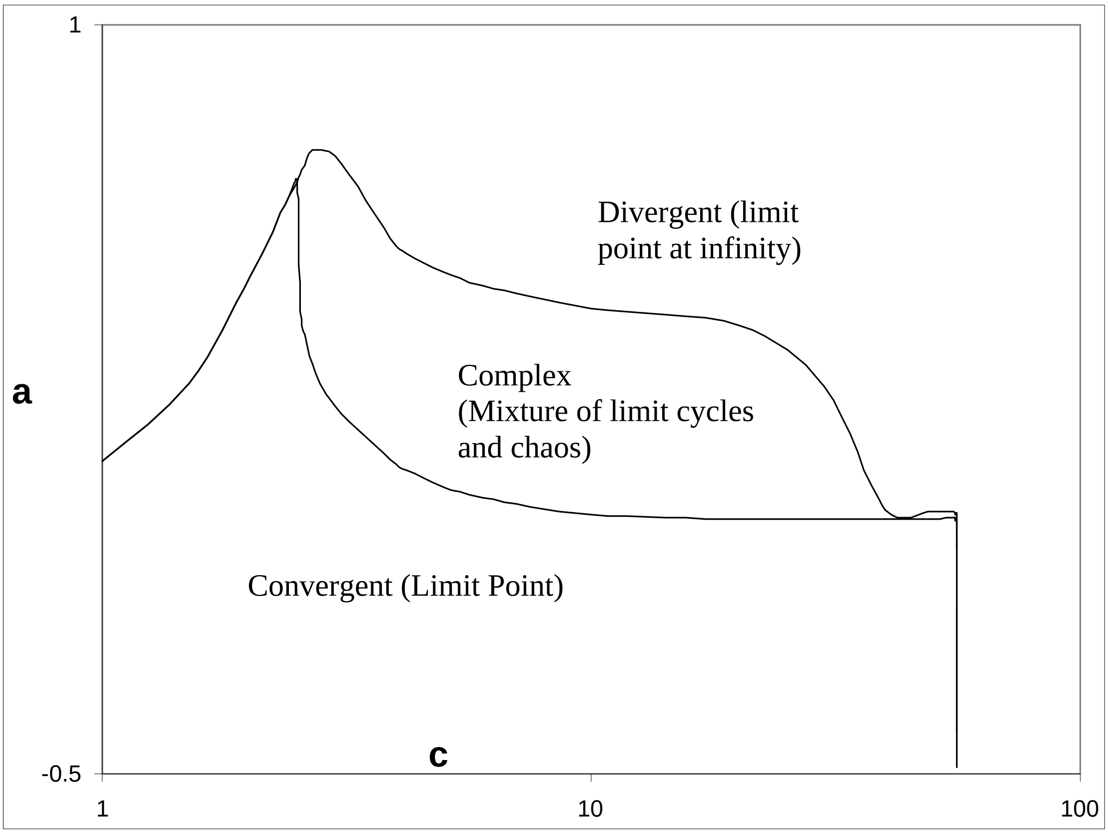

Figure 2: *Regions of parameter space producing complex behaviour for the Rossler system*

5 Illustrations of Dynamical Tuning

Having illustrated our position with two very simple examples, both far removed from cosmology, astrophysics or biology, the obvious question is, "how general is the connection between complexity and a finite range of tuneable parameters, and is this a relevant observation for the physical systems of interest?" We contend that it is both very general and of great relevance. We suggest that there is a strong tendency for any dynamical system to exhibit interesting (complex) behaviour only for a restricted range of its parameters. If true, this suggests that the observation of 'fine tuning' in this world is not only unsurprising but is a necessary mathematical correlate of the fact that the world is complex. To illustrate this further we offer two more examples which can be interpreted in cosmological terms whilst being both sufficiently simple to analyse easily and also very different from our universe. The point is to illustrate that 'fine tuning' (i.e. not-so-fine tuning) tends to occur whenever complexity occurs, and that it is not confined to our particular universe.

The first illustration is a model of gravitational collapse in one spatial dimension with a circular topology. This circle starts at radius R_0 at time zero, but expands according to,

$$R = \left[R_0^{1/\zeta} + \alpha t \right]^{\zeta} \tag{4}$$

where α and ζ are universal constants, and t is time. We assume there are N particles, with masses m_i, $i \in [1, N]$, whose instantaneous positions around the circle are $x_i \in [0, 2\pi R]$. At time zero these positions are chosen at random from a flat distribution. Similarly, the initial velocities, $\dot{x}_i$, are chosen randomly from a flat distribution with an amplitude, V_0, i.e. $\dot{x}_i \in [-V_0, V_0]$. An initial "Hubble velocity" is also added, equal to $(x_i / R)\dot{R}$. The gravitational force of attraction between every pair of particles is,

$$g_{ij} = s_{ij} G m_i m_j \left[\frac{1}{\left|\varepsilon + x_{ij}\right|^{\mu}} - \frac{1}{\left|\varepsilon + x'_{ij}\right|^{\mu}} \right] \tag{5}$$

where, $$x_{ij} = x_j - x_i \quad \text{and} \quad x'_{ij} = 2\pi R - x_{ij} \tag{6}$$

and s_{ij} is the sign of x_{ij}. This form of the law ensures that the gravitational force is exactly zero when particles are diametrally opposite, as should be the case by symmetry. It is necessary to insert a small positive value for ε in (5) to avoid the force becoming divergent at zero distance. This is because the only way for two particles to pass by each other in 1D is via zero distance (in contrast to 2D or 3D, where conic section orbits are the rule, at least when $\mu = 2$).

In order for gravitational collapse to be possible, there must be some 'cooling mechanism', i.e., some means by which a particle can loose energy. For this purpose we introduce a damping force which becomes active only when two particles are sufficiently close. The form chosen is,

$$B_{ij} = \beta s_{ij} \left|\dot{x}_{ij}\right|^{\lambda} \exp\left\{ -\left(\frac{\tilde{x}_{ij}}{D} \right)^2 \right\} \tag{7}$$

where, $\tilde{x}_{ij} = x_{ij}$ if $x_{ij} < \pi R$, but $\tilde{x}_{ij} = 2\pi R - x_{ij}$ if $x_{ij} > \pi R$. The parameters β, λ and D are universal constants. The equation of motion for each point mass is,

$$m_i \ddot{x}_i = m_i \frac{x_i}{R} \ddot{R} + \sum_{j \neq i} \left(g_{ij} + B_{ij} \right) \tag{8}$$

The first term on the RHS is a purely geometrical term which ensures that, in the absence of any gravitational or damping forces (e.g. if all the particles are a long way apart) then the solution becomes a uniform dilation.

We have solved the system of equations (4-8) numerically for 10 particles of equal unit mass, using parameter values G = 20, $\mu = 1$, $\lambda = 2$, D = 50, $R_0 = 100$, $\zeta = {}^2/_3$, $V_0 = 40$ and $\varepsilon = 0.1$. We explored the qualitatively different behaviours which arise by varying the values of the parameters α and β, which relate to the expansion rate of the universe and the strength of damping respectively.

Two extremes of behaviour can occur. The first occurs if damping is insufficient to cause any gravitational collapse. The universe then remains homogeneous (albeit a rather crude approximation to homogeneity for just 10 particles). This corresponds to a simple universe devoid of the structure which would arise through the formation of galaxies, stars and planets. The other extreme occurs if gravity and damping are too strong, in which case all the matter collapses into one giant clump. This would correspond, roughly, either to a rapidly re-collapsing universe or a universe which contained just one giant black hole and nothing else. Both these extreme behaviours lack complexity. However complex behaviour can occur in-between these extremes. A complex universe will contain a large number of independent collapse centres, each of which consists of a large number of particles. Whilst our simple model can hardly reproduce this with a total of just 10 particles, it is, in fact, very easy to distinguish the patterns of particle trajectories which are qualitatively intermediate between the extremes. This is good enough for our purposes.

We ran the simulation ten times for each of many pairs of (α, β) values, and thus produced an estimate of the (α, β) values which demarked the onset of simple behaviour of either of the above types. This is illustrated by Figure 3. The lower bound curve in Figure 3 is such that values of β below the curve virtually always produce a trivial homogeneous universe. Conversely, the upper bound curve in Figure 3 is such that larger values for β virtually always produce a trivial universe with all the matter collapsed into one clump. Only for values of β between these two curves do we find interesting behaviour. In a larger simulation, these intermediate values for β could be expected to produce a large number of substantial clumps of matter, analogous to stars and galaxies.

Thus there is only a restricted range of β which produces a complex universe for any given α. Despite the fact that this 1D "circle-world" is very different from our universe, it clearly exhibits 'fine tuning' of the universal constants α and β. In our universe these parameters would be related to the universal gravitational constant (G) and the quantum of charge (e) respectively.

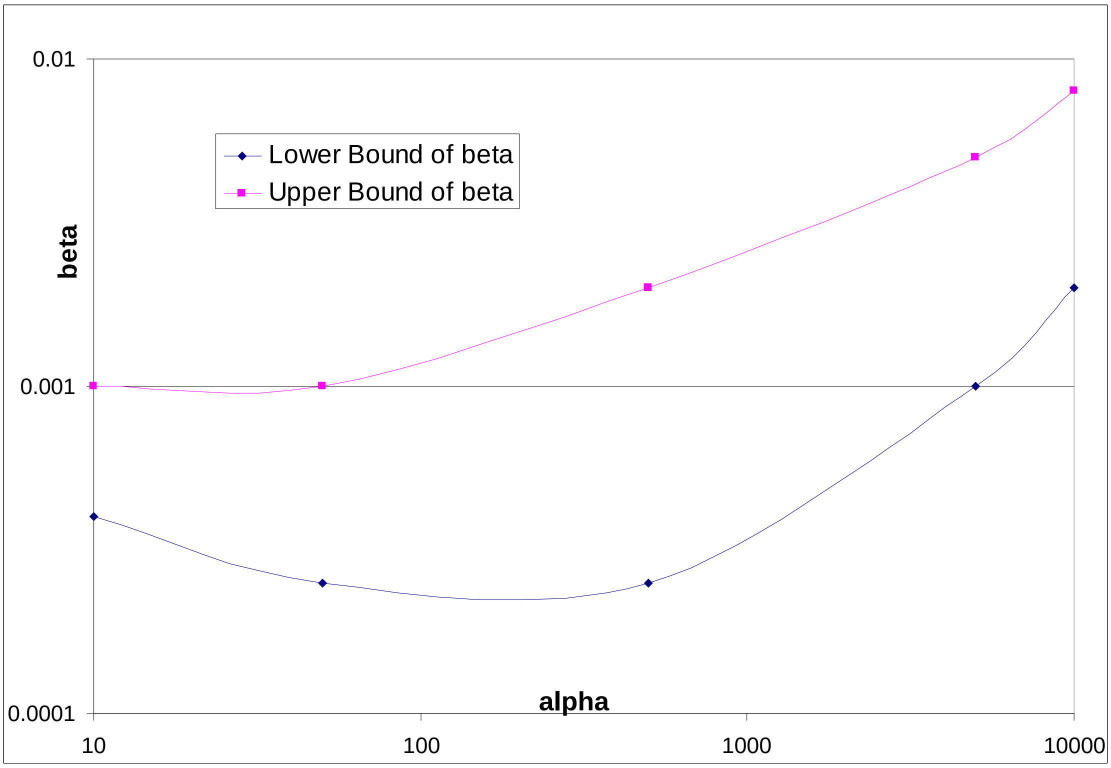

Figure 3: *"CircleWorld" - The region of α,β parameter space which can give rise to a complex universe by gravitational collapse is confined between the lower and upper curves*

The advantage of the preceding example is that gravitational collapse is overtly a dynamical issue. A possible objection to our thesis relates to whether all the instances of fine tuning can be regarded as dynamical. We hold that they can because all the complexity in our universe is a result of evolution, and this necessarily implies changes in time, which implies dynamics. However, the dynamics in question might be quantum chromodynamics, for example.

Our second example addresses nucleosynthesis. In our universe, the synthesis of the chemical elements inside stars is immensely complicated. Instead we return to Gamow's original idea that the chemical elements are made during the Big Bang. To permit this to happen we change particle physics and nuclear physics rather drastically. We deploy just one species of nucleon, rather than two. This particle can form bound states of two, three, four, and up to ten nucleons via some 'strong nuclear force'. There will be an equal (net) number of electrons also, which will be electromagnetically attracted to the nucleons. As in our universe, the electromagnetic energies will be assumed negligible compared with typical thermal energies at BBN temperatures, so that the universe remains fully ionised until much later than we are concerned with here.

A nucleus of N nucleons is assumed to have a binding energy of (N-1)B, so that any reaction between a nucleus of N nucleons and a nucleus of M nucleons to create a nucleus of N + M nucleons involves an increase in binding energy by B. All reactions of the form $n_N + n_M \rightarrow n_{N+M}$ are permitted to occur. The rate of such a reaction is taken to be,

$$R[n_N + n_M \rightarrow n_{N+M}] = NMC\sqrt{B}\exp\left\{-3\left(\frac{B}{kT}\right)^{1/3}\right\} \tag{9}$$

where C = 22,000. Equ.(9) gives the reaction rate in $s^{-1}(\text{mole/cm}^3)^{-1}$ when B is in MeV. The magnitude of the C coefficient is inspired by values in the true universe. However, the exponent is very different. In the true universe, the exponential would represent the Coulomb barrier which would be of increasing height for nuclei of increasing atomic number (i.e., increasing charge). In the alternative universe we have made the potential barrier the same height for all nuclei.

It is important to include such a barrier term so that BBN ceases virtually completely after a few minutes or hours. The nuclei formed during BBN are then permanent features of the universe. However, in the true universe, the increasing height of the Coulomb barrier for heavier nuclei means that increasingly high temperatures are needed to forge the heavier elements (e.g. $\sim 10^7$K for helium, $\sim 10^8$K for carbon and oxygen, and $\sim 10^9$K for silicon and iron). This is crucial to the strategy adopted for nucleosynthesis in stars in the true universe. Were it otherwise, little of the lighter elements would survive, since stellar nuclear burning would proceed straight to the 'ash' of the iron group elements. As it is, the shell structure of stars in the later stages of evolution permits a whole range of different nuclei to survive by virtue of the different physical conditions in each shell.

But our alternative strategy is different. We wish to give the heavier elements a chance of forming at the same temperatures during BBN as the lighter elements. Since the heavier elements can form only after the lighter elements, by which time the temperature will have reduced, an exponential barrier of greater height would effectively prevent their formation. Consequently we decree that the barrier should be of the same height for all nuclei. The particular algebraic form of (9) is essentially arbitrary.

The binding energy per nucleon, B, is related to the low energy coupling constant of the strong nuclear force, g_s, by $B \propto g_s^4$. Hence, $g_s \propto B^{1/4}$ is one of the universal constants – the strength of the nuclear force – whose value we seek to show is fine-tuned.

The universal temperature will be taken to be,

$$T = \frac{10^{10}}{\sqrt{t}} \tag{10}$$

where T is in K and t is in seconds. The value of the numerator is unimportant since it merely scales the unit of time. The universe shall be assumed to contain an equilibrium number density of zero-mass, spin one particles (photons),

$$\rho_\gamma = \frac{0.2436}{A}\left(\frac{kT}{\hbar c}\right)^3 \tag{11}$$

where A = 6 x 10^{29} m^{-3} is Avogardro's number, thus giving the number density in units of mole/cm^3. The total number density of nucleons is given in terms of an assumed photon:nucleon ratio, ξ,

$$\rho_n = \frac{\rho_\gamma}{\xi} \tag{12}$$

This is the number of free nucleons prior to any nuclear reactions, and also the total number of nucleons, bound and free, at all times. Note that the size scale of the universe varies as $R \propto \sqrt{t}$ in the radiation dominated era being considered here, and hence the above relations are consistent with a constant number of photons and a constant number of nucleons, though the mean energies of both are reducing. The assumed photon:nucleon ratio, ξ, is another universal constant whose value we seek to show is fine tuned.

Since all reactions of the form $n_N + n_M \to n_{N+M}$ are assumed to occur, and since we have assumed that there are no stable nuclei beyond n_{10}, there are thus 25 contributing reactions. The effect of these reactions on both particle creation and particle consumption must be addressed. The original supply of nucleons is only consumed and not replenished by any of these reactions (since we shall not model photodisintegration directly). Conversely, n_{10} is only created and not consumed by any reactions. In a notation in which the rate of reaction $n_a + n_b \to n_{a+b}$ is written $[ab]$, an example equation, for the rate of change of the number density of n_6 nuclei, is,

$$\frac{d\rho_6}{dt} = [15]\rho_1\rho_5 + [24]\rho_2\rho_4 + [33]\rho_3\rho_3 - [16]\rho_1\rho_6 - [26]\rho_2\rho_6 - [36]\rho_3\rho_6 - [46]\rho_4\rho_6 \tag{13}$$

Ten equations of this form, one for each $\dot{\rho}_n$, comprise the complete reaction network. These ten equations constitute the dynamical system, together with Equs.(9-12). Numerical integration by time stepping from some starting time, t_s, to some finishing time, t_F, is straightforward. Hence, the final abundance of each of the nuclei is found.

It is necessary, of course, to take account of universal expansion in reducing the absolute number densities as time proceeds. This was done by multiplying all the particle densities by a factor $\left(\frac{t}{t+\delta t}\right)^{3/2}$ at the end of each time increment, δt.

It remains only to define how the starting and finishing times for the integration, t_s and t_F, are determined. The former is defined by photodisintegration, and the latter by cosmic expansion freezing-out the reactions. A full dynamical treatment would model the photodisintegration reaction rate $n_{N+M} + \gamma \to n_N + n_M$. However, at high enough temperatures there is such a numerical preponderance of photons with energies in excess of B that this photodisintegration reaction is far faster than the rate of formation of compound nuclei. At such temperatures we can assume as a working approximation that there are no compound nuclei present. We thus start our simulation of the nuclear reactions at the earliest time that the nuclei become stable against photodisintegration. By integration of the black body spectrum, the fraction of photons with energies in excess of B is given by,

$$Photons(E > B) = 0.417\left(2 + 2x_1 + x_1^2\right)e^{-x_1} \quad (14)$$

where, $x_1 = B / kT_1$ and T_1 is the highest temperature for nuclear stability. This occurs when the fraction of photons with E > B is less than the nucleon:photon fraction, so that T_1 is found by setting (14) equal to $1/\xi$, i.e.,

$$0.417\left(2 + 2x_1 + x_1^2\right)e^{-x_1} = \frac{1}{\xi} \quad (15)$$

The time at which this occurs (t_s) is then found from (10) and defines the time at which we start the integration of the reaction rate equations. However, the net rate of production of nuclear species will be virtually zero at t_s, since nuclear stability is only marginal at that time. To account for this in a very crude fashion we arbitrarily factor the rate of nucleus formation (i.e. the 'forward' reaction rate) by $(t - t_s)/t_s$ for times between t_s and $2t_s$.

Reactions cease when their rate falls below the Hubble parameter. This is because particles cease to collide, their distance apart increasing faster due to universal expansion than their thermal velocities can counter. Hence, the freeze-out of the nuclear reaction $n_a + n_b \to n_{a+b}$, whose rate is $[ab]$, is assumed in our simple treatment to occur when,

$$[ab]\sqrt{\rho_a \rho_b} \leq \frac{1}{2t} \quad (16)$$

Integration continues until ***all*** reactions are frozen out, which defines time t_F. In cases where the freeze-out time for the first reaction is earlier than the time t_s at which stability against photodisintegration occurs, then no nuclear reactions will take place and there will be no nucleosynthesis at all.

A complex universe requires a diversity of chemical elements in sufficient abundance. What constitutes 'sufficient abundance'? The only guide we have is the actual universe. Just six elements make up 98.7% of the mass of animals' bodies. These elements are, in order of abundance of atoms: H, O, C, N, Ca and P. The least common of these six elements is phosphorus which accounts for a fraction 0.2 x 10^{-6} of the atoms in the universe (although it is less uncommon in the Earth's crust, ~0.4 x 10^{-3}). However, this rather sparse fraction of phosphorus is skewed by the overwhelming preponderance of H and He in the true universe, as a consequence of the rather slow synthesis of other elements via stars. It may be more indicative to consider the abundance of phosphorus as a proportion of all atoms from lithium onwards. On this basis phosphorus comprises a fraction ~10^{-4} of the atoms with $Z \geq 3$ in the universe as a whole.

Consequently, our alternative universe will be regarded as sufficiently 'complex' as regards diversity of elements if all ten elements have a relative abundance of at least 10^{-4}. However, we would also require stars to be a possibility in a biophilic universe and this requires that significant quantities of unburnt nuclear fuel should survive the big bang.

Figure 4 shows the relative abundances of the ten elements as a histogram, for an assumed binding energy of B = 1 MeV. The different coloured histogram bars refer to different photon:nucleon ratios, ξ , from 3 x 10^4 to 2 x 10^7. For a photon:nucleon ratio of 2 x 10^7 no compound nuclei are formed at all. The reactions are frozen-out before they start. For $\xi = 10^7$ only the first compound nucleus, n_2, is formed before freeze-out sets in. There is no yield of n_3 or heavier nuclei. Hence $\xi \geq 10^7$ fails to provide a complex universe, for lack of diversity of elements. However, with $\xi = 3x10^6$ we find that all ten elements are formed, the least abundant being n_{10} which accounts for 0.2% of the nuclei in this universe. This is sufficient for a complex universe according to the above criterion.

Decreasing the photon:nucleon ratio further to $\xi = 10^6$ provides the optimal yield across all the nuclei. About 30% of the nucleons remain (providing plenty of 'hydrogen') and n_{10} accounts for 42% of the nuclei. All the other nuclei are also formed in sufficient abundance, the least being n_7 (1.8%). This is a particularly well balanced yield compared with either other values of ξ, or, for that matter, compared with the actual universe.

Decreasing the photon:nucleon ratio further still results in a rapidly diminishing amount of 'hydrogen' remaining unreacted, whilst the bulk of the initial nucleons fuse fully to n_{10}. With $\xi = 10^5$ the universe is 93% n_{10}. However all the other nuclei remain in sufficient quantities at freeze-out, the least abundant being 'hydrogen' (n_1) at ~0.1%. In terms of chemical diversity this is still plenty for a sufficiently complex universe. However, we note that nuclear fuel is scarce in this universe, since it is almost fully burnt in the big bang. Consequently there may be no stars in this universe, which might prevent the nurturing of life. Consequently it is unclear whether $\xi = 10^5$ is viable as a complex universe – perhaps not.

In any case, with $\xi = 3x10^4$ or $\xi = 10^4$, the proportion of remaining 'hydrogen' (n_1) has reduced to only $0.8x10^{-5}$ and $0.2x10^{-6}$ respectively, with only a little more of n_2 and n_3. This fails our chemical diversity criterion, particularly since hydrogen is probably most important.

In summary, a viable complex universe results for B = 1 MeV only if ξ lies in the range from about $5x10^4$ to about $5x10^6$, but the lower limit may be rather greater. A range of between one and two orders of magnitude is not especially ***finely*** tuned. But this is partly due to the particular parameter employed. Consider how the very large photon:nucleon ratio might come about. It may be that in some earlier epoch the nucleons underwent an exponential decay from an initial abundance which was comparable with that of the photons. If this exponential decay lasted for a period equal to τ nucleon 'half-lives', the photon:nucleon ratio which would result is such that $1/\xi = \exp\{-\tau\}$, i.e., $\tau = \log\xi$. So, it may be more natural to employ $\tau = \log\xi$ as the universal constant. In this case the biophilic range is from about 10.8 to about 15.4. This seems like a much clearer case of fine tuning.

Figure 5 plots the histogram of nuclear abundances for a range of different B values, for a fixed photon:nucleon ratio $\xi = 10^6$, noting that this was optimal for B = 1 MeV. Reducing B to 0.1 MeV results in no nuclei being formed, and most nuclei are still not formed with B = 0.3 MeV. Conversely, if B is increased to 30 MeV, only 10^{-6} of the 'hydrogen' survives (and little more n_2 and n_3). The biophilic range is encompassed by these extremes, i.e. between B = 0.3 MeV and B ~ 30 MeV. Again, a range of two orders of magnitude might not seem especially ***finely*** tuned. But again this depends largely on the universal constant employed. If we express the result in terms of a nuclear coupling constant defined by $g_s = B^{1/4}$ then the biophilic range is from 0.74 to 2.3, which appears much more finely tuned.

Figure 6 summaries the biophilic ranges of the parameters on a plot of B versus ξ. Biophilic conditions pertain only between the two lines. Hence, we have again found that a complex outcome from a dynamical system, this time producing a diversity of chemical elements, is naturally related to fine tuning of the universal constants, B and ξ.

The reader might be suspicious that this outcome has been contrived by a careful choice of dynamical equations. This is not so. Both in the BBN example, and in the case of gravitational collapse in 'circle-world', the equations were quite arbitrary, beyond having some features that were expected to permit complex behaviour for some values of the parameters. Both examples exhibited fine-tuning on the first attempt without any adjustment being necessary. The reader is advised to try similar numerical experiments himself.

We conclude that fine tuning, or rather not-so-fine tuning, is likely to be a generic correlate of complexity, and not peculiar to our own universe. Fine tuning is a mathematical property of complex universes. It requires neither God nor a multiverse to explain it, which postulates do *not* explain it anyway.

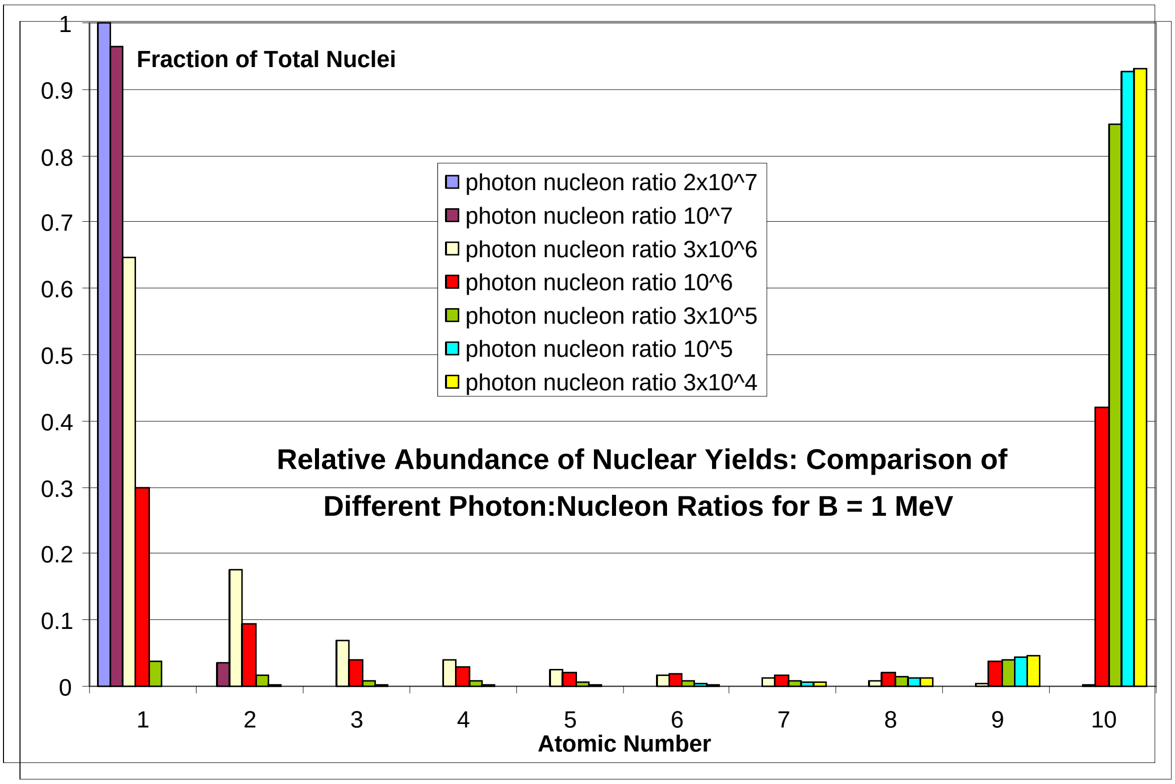

Figure 4: *Relative abundance of nuclei for B=1MeV: Comparison of different ξ*

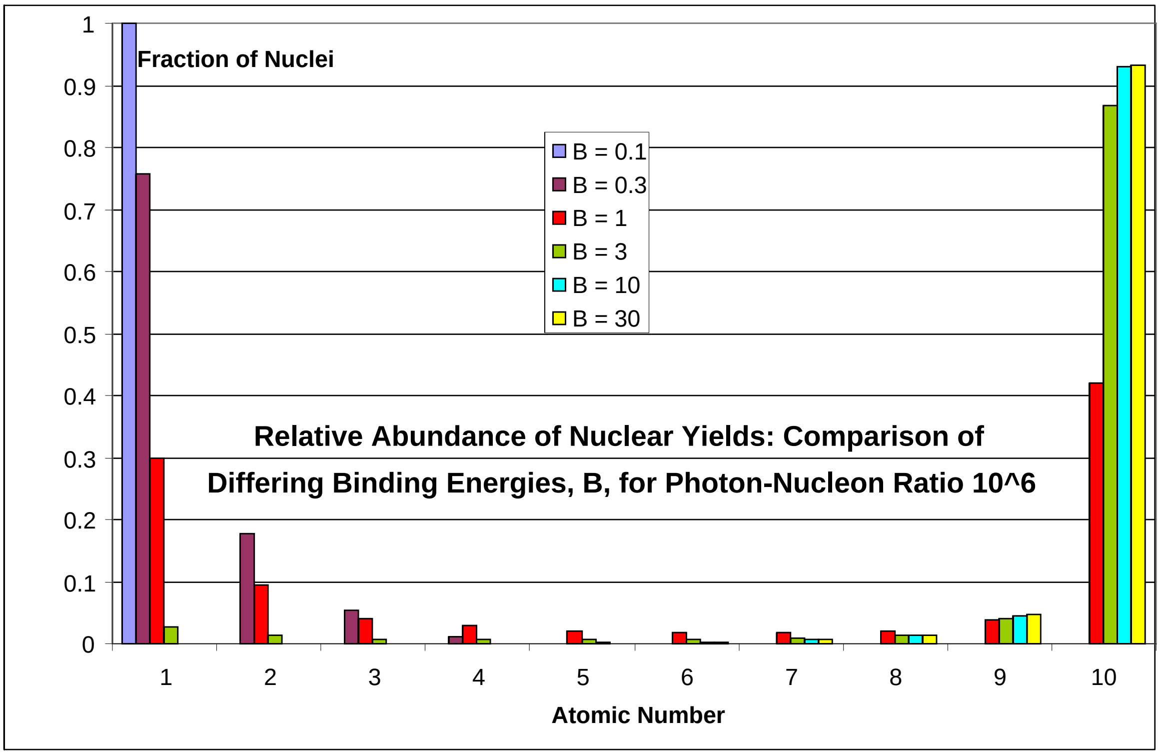

Figure 5: *Relative abundance of nuclei for $\xi=10^6$: Comparison of different B*

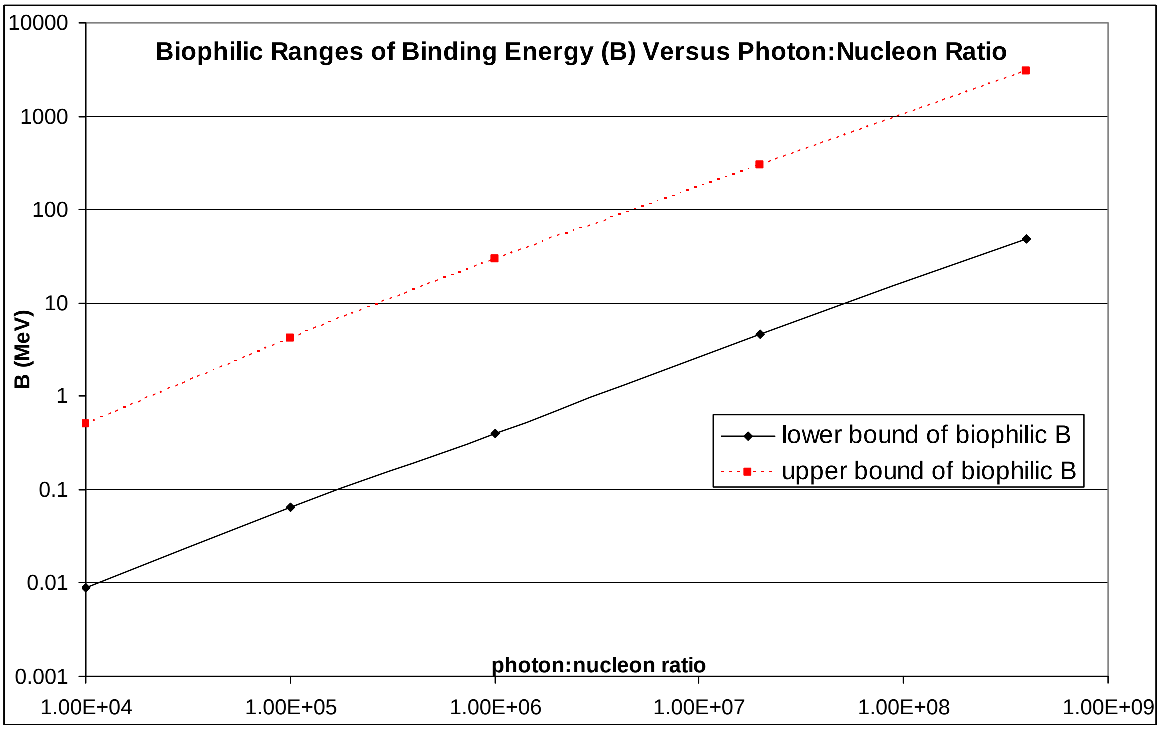

Figure 6: *Biophilic ranges of the parameters B and ξ (between the lines)*

6 Alternative Biophilic Universes and the Congenial Parameter Surface

It might be imagined that fine tuning consists of identifying an approximate range within which each universal constant must lie. Suppose we have N universal constants, $\{c_i\}$. Fine tuning might be assumed to imply $c_1 \in [c_1^L, c_1^U]$, and $c_2 \in [c_2^L, c_2^U]$, and $c_3 \in [c_3^L, c_3^U]$, etc., so that the 'allowed' region of parameter space is an N-dimensional rectangle. This is not the case. Let us suppose that varying the constant c_1 alone does indeed imply that it must lie in the range $c_1 \in [c_1^L, c_1^U]$ to produce a complex universe. Let us suppose also that varying the constant c_2 alone implies that it must lie in the range $c_2 \in [c_2^L, c_2^U]$. A corresponding statement is found to hold for each constant varied individually. But it is an elementary logical error to suppose that this implies that the permissible points in parameter space lie with the rectangular block specified by $c_1 \in [c_1^L, c_1^U]$, $c_2 \in [c_2^L, c_2^U]$, $c_3 \in [c_3^L, c_3^U]$, etc., all holding true simultaneously. The fallacy is exposed most simply by the following illustration.

Suppose that, within the N-dimensional C-space, a complex universe will result if and only if the constants, $\{c_i\}$, lie on a particular N-1 dimensional hyper-surface. Suppose that our universe lies on a typical point of this surface, and hence that the surface is not parallel to any of the c_1, c_2,… axes at this point. It follows that varying any single c_j, i.e. moving parallel to axis j, will take us off the 'magic surface' and into the region of non-complex universes. Hence, the theoretical observation that each c_j is fine tuned when varied alone does not imply that we are restricted to a hyper-cube in C-space. Most generally, it implies only that we are restricted to an N-1 dimensional sub-space (hypersurface) of C-space. This crucial point is often overlooked. It means that, despite demonstrating fine tuning in each of N individual universal constants, nevertheless we can only claim, so to speak, a reduction by one degree of freedom in our choice of the set of constants, $\{c_i\}$. The point is illustrated by Figure 7.

Figure 7 is typical of fine tuning, showing a lower bound and an upper bound curve in the 2- parameter subspace illustrated. Rather than an N-1 dimensional biophilic surface, the coarseness of not-so-fine tuning leads to a 'thick surface' defined by the region between the curves. By varying just one parameter at a time, tuning is observed in both the parameters c_1 and c_2, the extent of which is indicated by the arrowed lines. However, it is incorrect to conclude that the constants are therefore restricted to the box indicated by the red dashed lines. By assumption they are actually restricted to the more extensive region defined by the lower and upper bound curves. This means that there are parameter values far distant from the red dashed box which give rise to a complex universe.

Figure 7 is the characteristic fine-tuning result, as illustrated by the examples of Section 5, Figures 3 and 6. This behaviour appears to be typical also of fine tuning in the true universe. That is, the required range of a parameter to produce complexity will vary depending upon the values of other parameters. A specified variation in a given constant may destroy complexity, but complexity can generally be restored by a compensating change in another parameter.

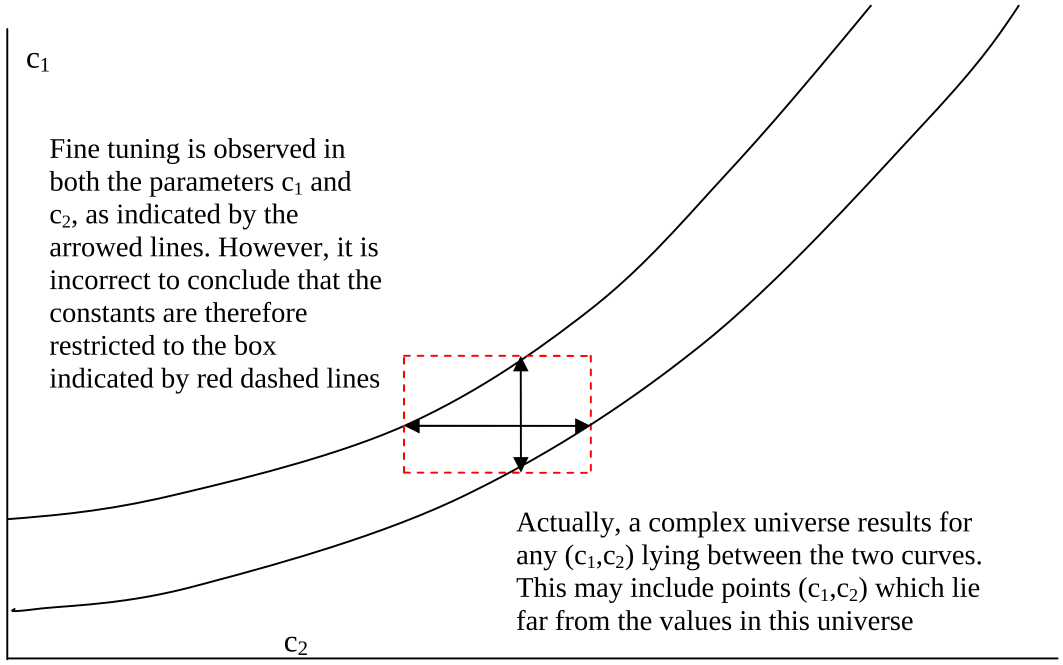

Figure 7: *Illustrating a logical fallacy: the observation of fine-tuning in parameters c_1 and c_2 does not imply that they are confined to the red dashed box.*

For example, the stability of nuclei requires a sufficiently strong nuclear force or else the Coulomb repulsion will blow them apart. However, a weaker nuclear force can be compensated by a reduced quantum of charge, and stability restored.

A second example is the bound on the neutron mass discussed in Section 2, i.e. $M_p + m_e < M_n < M_p + m_e + \Delta B$. The LH inequality can be undermined by an increase in the electron mass. However, by increasing the quark masses and the strong nuclear coupling constant appropriately we can contrive to ensure that $M_p, M_n, \Delta B$ all increase roughly in proportion, thus preserving the inequalities.

Another example is the preservation of hydrogen during the Big Bang. We have seen in Section 2 that reducing the weak coupling constant, G_F, by a factor of ~100 could challenge this. However, the excess of protons over neutrons at the time of the freeze-out of the leptonic reactions depends upon the product $G_F^{2/3}(M_n - M_p)$, so that a reduction in G_F can be compensated by an increase in the nucleon mass difference. (This is likely to involve a reduction in the neutron lifetime, which also influences the final proportion of hydrogen surviving the Big Bang, but the photon:nucleon ratio can be used to negate that effect if necessary).

Whether it is possible to make the changes to multiple parameters suggested above and still preserve other effects such as supernovae or the production of carbon in stars is very difficult to determine. However, it may not matter. What matters is attaining complexity, rather than preserving the particular strategies employed in this universe for achieving specific outcomes. Hence, preserving the Hoyle effect is not necessary if carbon has already been produced in sufficient quantities during the Big Bang, as in our example of Section 5. Once we move significantly away from the red box in Figure 7 we must accept that the universe will probably be qualitatively very different indeed, despite still being complex provided we stay within the bounding curves. In short, fine tuning does not apply if we move in the congenial direction, parallel to the bounding curves. Radically different, but life supporting, universes may exist in these directions. And recall that this might mean a "thick" N-1 dimensional subspace of the N dimensional parameter space. There is potentially plenty of room in parameter space for qualitatively different complex universes. But they will all exhibit not-so-fine tuning.

Support for this contention is provided by a number of radically different universes which have been constructed by Aguirre (2001), by Harnik, Kribs and Perez [2006] and by Adams [2008].

Consider firstly Aguirre's "cold big bang" universe. If the photon:baryon ratio were less than $\sim 10^6$ then structure formation would be prevented because the universe would be permanently opaque and hence small density fluctuations would be supported against gravitational collapse by radiation pressure. Also, Tegmark and Rees [1997] argue that the magnitude of the primordial density fluctuations, Q, is fine tuned to be within an order of magnitude of its value in this universe. Despite this, Aguirre [2001] has presented a case for a universe capable of supporting life in which the photon:baryon ratio is of order unity, and Q is smaller than in this universe by a factor of between a thousand and a million. Aguirre argues that such a cosmology can produce stars and galaxies comparable in size and longevity to our own. As a bonus, a rich chemistry, including carbon, oxygen and nitrogen, can arise within seconds of the Big Bang.

The moral of Aguirre's work is that by varying more than one universal constant at once, and by being bold enough to vary them by many orders of magnitude, it is possible to discover distant regions of parameter space which appear to support a complex, biophilic universe. The key is varying more than one parameter at once, the change in one parameter effectively offsetting the change in the other. In terms of Figure 7, this is a change in the congenial direction parallel to the bounding curves. In addition, by making very large changes, the nature of the physics involved changes qualitatively.

Harnik, Kribs and Perez [2006] (HKP) consider a universe which has no weak nuclear force. In Section 2 we discussed how reducing the value of the Fermi constant sufficiently would lead to a helium dominated universe. The reason is that the smaller G_F, the earlier the freeze-out of the leptonic reactions, and hence the higher the temperature, and hence the closer to equality is the density of neutrons and protons. However, we have taken for granted that the neutrons and protons achieve their thermal equilibrium densities. This will only be the case if the weak interaction exists, since this provides the mechanism for the inter-conversion of neutrons and protons. Thus, we have an entirely different situation if the weak interaction does not exist at all. In this case, the relative abundance of protons and neutrons (in the primordial universe prior to BBN) would be determined by whatever CP symmetry violating mechanism gives rise to baryogenesis. In other words, we can presumably fix the relative neutron and proton abundance by *fiat*.

This was the line taken by HKP. So there is no reason to assume equal numbers of protons and neutrons, and hence an all-helium universe does not result.

Moreover, the same argument applies to the baryon:photon ratio, which HKP also adjusted at will. HKP found that they could contrive a universe with a similar hydrogen:helium ratio as ours, but with about 25% of the hydrogen being deuterium rather than protons. To do so they chose a baryon:photon ratio of 4×10^{-12}, i.e., about a thousand times smaller than in our universe. HKP argue that galaxies could still form despite the much reduced visible baryon density, but that the number density of stars in the galaxies would be appropriately reduced. They can claim that stars would form, because they have taken the precaution of making the chemical composition of their universe sufficiently similar to ours, thus ensuring that there would be a cooling mechanism to permit gravitational collapse.

The main difference for stars in the HKP universe would be that the initial fusion reaction would be the formation of helium-3 from a proton and a deuteron. Note that HKP have cunningly contrived to have substantial quantities of deuterium formed during BBN, so there is no need for the usual weak-force-mediated deuteron formation reaction from two protons. Since the first stellar reaction in HKP stars is very fast compared with the usual weak-mediated deuteron formation reaction, the core temperature of such stars would be lower. It has to be lower to keep the reaction rate down to a level at which the thermal power does not outstrip the available mechanisms of heat transport away from the core.

The moral once again is that by varying more than one universal constant at once, and by being bold enough to vary them by many orders of magnitude, it is possible to discover distant regions of parameter space which appear to support a complex, biophilic universe. The key is varying more than one parameter at once, as illustrated by Figure 7. By making very large changes, the strategies adopted by the universe to achieve its complexity may change qualitatively.

Finally, Adams [2008] has considered how common the formation of stars might be in universes with different values for the universal constants. The most important quantities which determine stellar properties are the gravitational constant G, the fine structure constant α, and a composite parameter that determines nuclear reaction rates. Adams uses a simple analytical model to determine the region within this 3-dimensional parameter space which permits stellar stability. The result is about one-quarter of it. So the requirement that stars be stable is hardly a strong constraint on the universal constants, a dramatically different conclusion from Smolin's [1997]. Yet again, so long as more than one parameter is varied, complexity is obtained even for parameter values very different from our own.

7 Conclusions

We have pointed out that there are two distinct fine-tuning problems. Fine-tuning itself is the supposed property of the universe that small changes in the parameters of physics produce catastrophic changes in the evolved universe, in particular that its complexity, and hence its ability to support life, is undermined. The first question is, "why should the universe be fine tuned?" The second question is, "given that fine-tuning presents a small target in parameter space, how does the universe contrive to hit it?" Previous endeavours have addressed only the second question.

The physics of our universe does indeed exhibit tuning of the universal constants. However, this tuning is generally not-so-fine. Moreover, in an N-dimensional parameter space, the observation of tuning in all N parameters need only reduced the number of degrees of freedom by one. Hence, there may be no tuning in N-1 directions in parameter space (the congenial directions). Alternative complex universes might therefore exist (at least mathematically) with values of the universal constants well outside the fine-tuned bounds that are usually claimed. This is supported by the explicit construction of such universes by Aguirre [2001], Harnik, Kribs and Perez [2006] and Adams [2008].

Our contention is that the tuning of the parameters which does exist arises as a natural consequence of the mathematics of dynamical systems. A complex outcome for a dynamical system will tend to occur only for finite ranges of its tuneable parameters. This is offered as the correct explanation for the first question, together with the observation that tuning is generally not terribly fine. The contention has been illustrated by the examples of Sections 4 and 5. Tuning is an inevitable property of any complex universe.

God and the multiverse provide no explanation for fine tuning, i.e., the first question. They have been proposed as explanations for the second question: how to hit a small target. However, it would appear that the size of the target is not so small after all. In the N-1 congenial directions in parameter space the target is indeterminately large. Even in the tuned direction it is not as small as is sometimes claimed. Consequently this second question becomes a non-question. The motivation for the God and multiverse hypotheses disappears. The argument from design reverts to its 19th century form, fine tuning having nothing to add to it.

However, there is still the physicists' long standing question, "what determines the values of the universal constants?" This third question is distinct from the second because it would appear that there is great scope for radically different, but still complex, universes. The constructions of Aguirre [2001], Harnik, Kribs and Perez [2006] and Adams [2008], together with the logical fallacy illustrated by Figure 7, imply that the anthropic argument has no power to discriminate this universe from competing complex universes. Neither string theory nor the anthropic argument can provide an explanation for the values of the universal constants. They may be irreducibly contingent, or they may be determined by some physical theory yet to be devised, or they may be a consequence of an evolutionary scenario such as envisaged by Smolin [1997, 2004, 2006]. In the latter case, the multiverse would re-enter the picture, but in a non-anthropic guise.

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