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The Inevitability of Fine Tuning in a Complex Universe

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ABSTRACT

Why should the universe need to be fine tuned? The thesis is presented that parameter sensitivity arises as a natural consequence of the mathematics of dynamical systems with complex outcomes. Hence, fine tuning is a mathematical correlate of complexity and should not elicit surprise.

Keywords Fine tuning, universal constants, entropy, complexity, multiverse

1 Fine Tuning and the Purpose of this Paper

It is over 40 years since Carter [15] observed that the universal constants of physics appear to be peculiarly fine tuned. Relatively small variations in the universal constants, it is claimed, would produce radical changes in the universe. On first acquaintance this seems remarkable and surprising. It will be argued here that, on the contrary, the fact that the universe contains complex parts makes fine tuning inevitable. Had physicists been foresighted enough, fine tuning could have been anticipated even before physics and cosmology had advanced to the stage where it could be directly demonstrated.

To make clear what we mean by "fine tuning", a few examples are given in section 2. We shall argue that, whilst the tuning of some universal constants may not be as numerically impressive as is sometimes claimed, nevertheless tuning is evident. What has not been brought out clearly before is that fine tuning consists of two separate phenomena: parameter sensitivity and fine tuning itself. This is explained in section 3. In section 4 some toy models are used to explain why parameter sensitivity is to be expected in any universe with a complex outcome. Section 5 then presents our general thesis that parameter sensitivity is a mathematical result of the evolution of complexity, the two issues being linked via entropy interpreted in terms of phase space volume. Section 6 discusses briefly the relevance of this work to the cosmological constant, the causal entropic principle, the creationist argument from design and multiverses.

2 Examples of Fine Tuning

2.1 Varying Single Parameters Illustrates Tuning

A few examples of fine tuning are given below. However, there are many further examples and these matters have been discussed many times before, see for example

[2,3,6,7,13-16,20-24,26,28,29,46,47]. The commentary on each example explains why some of them may not be as impressive as they first appear.

(i) If the neutron were lighter by more than 0.08%, or if it were heavier by more than ~1%, then there would be no stable atomic matter in the universe. This narrow mass range is required to avoid both nuclear capture of the atomic electrons, and also beta decay of the nucleus.

(ii) If the weak nuclear force were sufficiently weaker then the abundance of neutrons and protons after the first few seconds would have been closely matched and Big Bang Nucleosynthesis (BBN) would have resulted in a universe consisting of virtually all helium and very little hydrogen. A universe with no hydrogen would contain no water, no hydrocarbons such as amino acids, and no hydrogen bond chemistry, and hence no life as we know it. However we shall see below that "sufficiently weaker" really means a lot weaker, by at least an order of magnitude.

(iii) If the strong nuclear force (i.e., the effective low energy coupling, g_s) were ~15% weaker then the deuteron would be unbound. The formation of all nuclei would be prevented and there would be no chemical elements other than hydrogen.

(iv) If the strength of the strong nuclear force, g_s, were changed by $\pm 1\%$ the rate of the triple-alpha reaction would be affected so markedly that the production of biophilic[1] abundances of either carbon or oxygen would be prevented.

Example (i) seems impressive, but becomes much less so when it is recalled that the neutron and the proton share a common structure. About 99% of a nucleon's mass is due to the virtual gluons and virtual quarks which comprise the strong nuclear force. This feature is shared by the neutron and the proton, which differ only in regard to the *udd* and *uud* 'valence' quarks which respectively provide the nucleons with their net quantum numbers. Since the *u* and *d* quarks in question have masses of just a few MeV, it is no longer particularly surprising that the neutron-proton mass difference is also of this order. In fact this is to be expected. The moral is that there are structural reasons why the neutron and proton masses should be very close. This is not to say that there is no 'tuning' at all, just that it is not so terribly 'fine' as it first appears. It is more indicative to compare $m_n - m_p$ with the mass of the electron or the mass of the *u* or *d* quarks. On this scale the tuning is at the level of tens or hundreds of percent, rather than less than 1%. Nevertheless, there is *some* tuning. For example, the *d* quark must be heavier than the electron for atomic stability[2].

In Example (ii), the neutron:proton ratio depends upon T, the temperature when the leptonic reactions which interconvert neutrons and protons are frozen out by cosmic

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[1] The term 'biophilic' will be used here to refer to a universe which is sufficiently similar to our universe that conventional biochemistry could potentially arise. Biophilic universes are therefore a sub-set of all complex universes (probably a very small sub-set).

[2] That is, if we make the rather sweeping assumption that $m_n - m_p \approx m_d - m_u$. Far more carefully argued constraints on the *u, d* and *s* quark masses which produce a 'congenial' universe have been discussed recently by Jaffe, Jenkins and Kimchi [31] and by Damour and Donoghue [19]. The term 'congenial' is defined in Jaffe et al as universes with quark masses which allow for certain key nuclei to be stable (so as to make organic chemistry possible).

expansion. This freeze-out temperature depends upon the strength of the weak nuclear force, G_F. Because the neutron:proton ratio is $\exp\{-(m_n - m_p)/kT\}$, it is often claimed that it is highly sensitivity to changes in the freeze-out time, and hence to the strength of the weak force. Actually, a closer examination shows that if the Fermi constant, G_F, were reduced by an order of magnitude, the universe would still be 18% hydrogen (by mass, or ~50% by number of atoms). This would still support hydrogen burning stars with lives in the order of billions of years, long enough for biological evolution. Reducing G_F by a factor of 100 would still leave the universe with ~14% hydrogen by number of atoms. Admittedly if the hydrogen abundance were reduced too much this would ultimately prejudice the formation of the first stars, which is believed to rely on a cooling mechanism via molecular hydrogen. Nevertheless, there is no obvious reason to regard as catastrophic a reduction in G_F by somewhat more than a factor of ten.

Moreover, the constraint on G_F is only single-sided: it must exceed (say) ~10% of its actual value but there is no obvious upper bound resulting from these considerations. If G_F were increased, then there would be less helium in the universe. For example, a factor of 4 increase in G_F results in only ~0.2% helium by mass. But this would seem unimportant. Helium appears to play no essential role in the formation of large scale structure or stellar physics[3]. Although no upper bound on G_F results from these considerations, there are suggestions that Type II supernovae require G_F to lie close to its actual value. This is because crucial aspects of the mechanism of Type II supernovae involve neutrino interactions, i.e., weak-force interactions. The neutrinos seem to be required to interact just weakly enough to escape the core of the collapsed star, but strongly enough to transfer sufficient energy to the mantle to cause the explosion. Unfortunately the quantitative understanding of Type II supernovae is too poor to deduce just how fine tuned G_F must be.

Hence there is a case for considering G_F to be a genuine instance of tuning, but it is not necessarily terribly 'fine', and may only be single-sided.

Example (iii), deuteron stability, does appear to provide a genuine instance of fine-tuning, requiring g_s to exceed ~85% of its actual value. Claims are often made that there is also an upper bound on g_s to avoid diproton stability. If g_s were ~10% larger, then the diproton (${}^2_2He$) would be a bound state[4]. It has frequently been claimed that this would lead to an all-helium universe. The argument is that all the nucleons would end up as helium during BBN, either via the conventional sequence starting with $n + p \rightarrow {}^2_1H$, or via the diproton[5] $p + p \rightarrow {}^2_2He \rightarrow {}^2_1H + e^+ + \nu$. However, this argument is just wrong. The reason is that, even if the diproton were stable, the rate of its formation via $p + p \rightarrow {}^2_2He$ is too slow for any significant number of diprotons to be formed during BBN, [12]. It is

[3] The ppII and ppIII reaction sequences would be slowed by the absence of initial helium, but the ppI sequence is unaffected.

[4] The diproton is not bound in this universe. This is because the spin-singlet nuclear force is weaker than the spin-triplet nuclear force which binds the deuteron. It is *not*, as some authors have claimed, due to electrostatic Coulomb repulsion.

[5] The inverse beta decay which converts the diproton to a deuteron is possible because the binding energy of the deuteron (2.224 MeV) exceeds $m_n - m_p + m_e = 1.804$ MeV.

true that the nuclear physics of stars would subsequently be very different, but there is no obvious reason why biophilic stars would not be stable, [12].

There is possibly a different upper bound on g_s. If g_s were increased sufficiently that deuterium were stable even at the higher temperatures prevailing before 1 second, when the leptonic reactions were still active, then the nucleons could escape into the sanctuary of helium-4 before the protons had gained numerical superiority over the neutrons. This would again lead to a universe with little hydrogen. However, rough estimates suggest that g_s would need to be increased by more than a factor of two for the hydrogen abundance to fall to potentially abiophilic levels.

Hence, deuteron stability provides a case for fine tuning of g_s (a lower bound of ~85% of its actual value) but any upper bound arising from BBN is rather generous in magnitude.

Example (iv) concerns the famous Hoyle [30] coincidence. The instability of beryllium-8 (${}^{8}_{4}Be$) means that carbon (${}^{12}_{6}C$) can be produced only by virtue of the subsequent alpha capture reaction ${}^{8}_{4}Be + {}^{4}_{2}He \rightarrow {}^{12}_{6}C$ being extremely fast due to the existence of a resonance of the carbon nucleus at just the right energy. Moreover, the subsequent burning of all the carbon into oxygen is avoided only by the fortuitous placing of the energy levels of the oxygen nucleus so that resonance is just avoided. Some authors are not impressed by the Hoyle coincidence - for example, Weinberg[6] [53]. But actually quite elementary arguments based on first order perturbation theory are sufficient to show that Weinberg's objection does not stand up to quantitative scrutiny. The triple alpha resonance energies with respect to their reaction thresholds are highly sensitive to changes in the strength of the nuclear force.

A mere 0.4% change in the strength of the nuclear force can produce a change in the C^{12} 0_2^+ resonance energy of up to 38%, [17,18,42,43,48]. Consideration of the detailed stellar models reported in these same references suggests that a reduction in the C^{12} 0_2^+ resonance energy of perhaps ~50% will result in a reduction in carbon production of around two orders of magnitude. Alternatively, an increase in the C^{12} 0_2^+ resonance energy of ~50% will result in a reduction in oxygen production of around two orders of magnitude. These changes in resonance energy would be brought about by a change in the strength of the nuclear force not exceeding +1% or -1% respectively.

Consequently it appears that the Hoyle coincidence is impressively fine tuned, requiring changes in the strong force of less than ±1% to challenge the likelihood of conventional biochemistry by serious depletion of either carbon or oxygen abundance. However, this fine-tuning might only be one-sided. The reason is that an increase in the nuclear force of

6 Weinberg, [40] says, "*I don't set much store by the famous 'coincidence' emphasised by Hoyle, that there is an excited state of* C^{12} *with just the right energy to allow carbon production via* $\alpha - Be^8$ *reactions in stars. We know that even-even nuclei have states that are well described as composites of* α*-particles. One such state is the ground state of* Be^8*, which is unstable against fission into two alpha particles. The same* α*-*α *potential that produces that sort of unstable state in* Be^8 *could naturally be expected to produce an unstable state in* C^{12} *that is essentially a composite of three alpha particles, and that therefore appears as a low-energy resonance in* $\alpha - Be^8$ *reactions. So the existence of this state doesn't seem to me to provide any evidence of fine tuning*."

less than 1% will make Be^8 stable. The triple alpha reaction, via the resonant states Be^8 and $C^{12}(0_2^+)$, would then no longer be necessary. Instead a star might accumulate macroscopic quantities of Be^8 and synthesise subsequent elements in a more conventional manner. Whether the resulting nuclear physics and stellar physics would render this a feasible pathway for biophilic carbon and oxygen production is difficult to judge.

It is concluded that the Hoyle coincidence is an instance of fine tuning, requiring that the nuclear force exceed 99% of its actual strength. It may also need to be less than 101% of its actual value, but this is less clear.

In summary, whilst the tuning of some universal constants may not be as numerically impressive as is sometimes claimed, and might only be single-sided in some cases, nevertheless tuning is evidently a feature of the world.

2.2 Varying Multiple Parameters - Alternative Complex Universes

In section 2.1 we have been guilty of giving the impression that fine tuning requires the affected universal constant to lie within a certain range of values. Indeed many discussions of fine tuning give this impression. But this is quite wrong. In fact, all instances of fine tuning provide relations between two or more parameters. Examples are as follows,

- Consider the bound on the neutron mass discussed in section 2.1. Algebraically this is $m_p + m_e < m_n < m_p + m_e + \Delta B$ where ΔB is the difference in binding energy between the nucleus in question and the nucleus obtained by replacing $n \to p$. So the allowed range for the neutron mass depends upon the other masses, which in turn depend upon the quark masses and other constants of the standard model.
- Reducing the weak coupling constant, G_F, sufficiently could challenge the preservation of hydrogen during the Big Bang. However, the excess of protons over neutrons at the time of the freeze-out of the leptonic reactions depends upon the product $G_F^{2/3}(m_n - m_p)$, so that a reduction in G_F can be compensated by an increase in the nucleon mass difference. (This is likely to involve a reduction in the neutron lifetime, which also influences the final proportion of hydrogen surviving the Big Bang, but the photon:baryon ratio can be re-tuned to negate that effect if necessary).
- The lower bound on the strength of the nuclear force, g_s, to bind the deuteron was stated in section 2.1 as $g_s > 0.85 g_{s,actual}$. But closer inspection reveals that the range of the nuclear force, and the nucleon mass, are also part of this calculation. The combination of parameters which is bounded below is actually $g_s^2 m_n / m_\pi$, where m_π is the pion mass. So the numerical bound on g_s can be changed by varying the nucleon:pion mass ratio.
- The stability of large nuclei requires that the quantum of charge is not too great or else the Coulomb repulsion between the protons will blow the nucleus apart. But a larger quantum of charge can be compensated by also increasing the strength of the

nuclear force. This produces an inequality involving both g_s and α, the electromagnetic fine structure constant.

- One of the original fine-tunings of Carter [15,16] was the requirement that small stars be convection dominated whilst large stars be radiation dominated. This leads to the coincidence $\alpha^{12}\left(\frac{m_e}{m_p}\right)^4 \sim \frac{Gm_p^2}{\hbar c}$. This is a statement about the relative strengths of the gravitational and electromagnetic forces and involves several constants.

Since all known examples of parameter sensitivity are relations between two or more constants, it is clear that each such individual relation can at best give a constraint like that illustrated in Figure 1. Note, however, that there will be many constraints like Figure 1 which must be satisfied simultaneously, potentially up to one for each case of fine tuning. The intersection of these separate constraints might restrict individual parameters more narrowly (see Figure 2 of Bousso et al [9]).

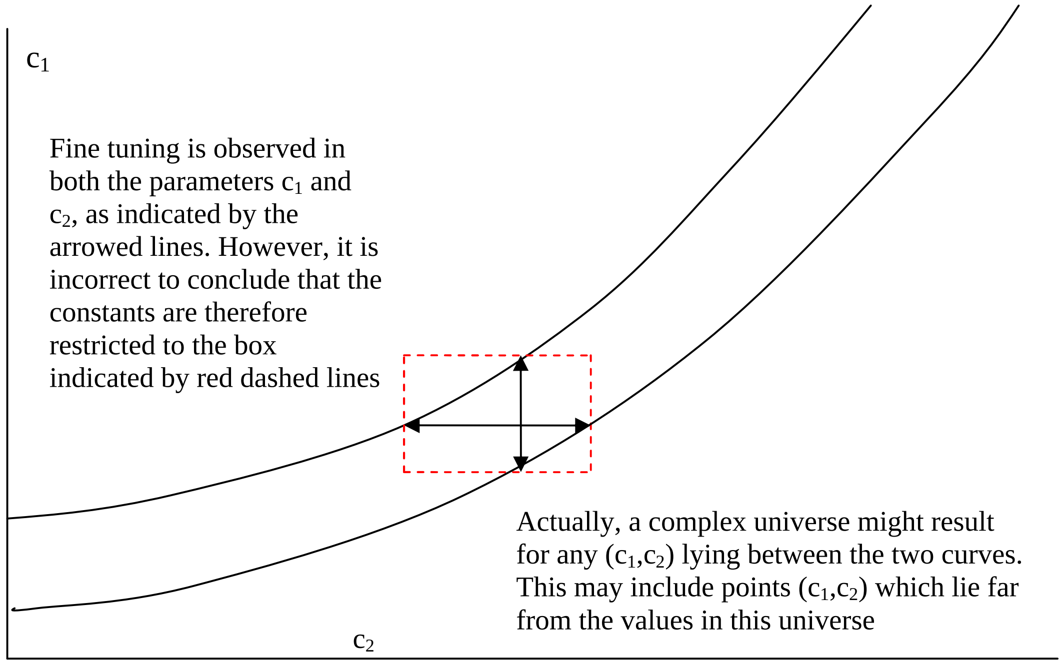

Figure 1: *Illustrating a logical fallacy: the observation of fine-tuning in parameters c_1 and c_2 does not imply that they are confined to the red dashed box.*

The lesson that can be learnt from this simple observation is that complex universes might result for values of the universal constants which depart greatly from the usually claimed fine-tuned bounds, provided that the parameters remain in the 'complex' region, i.e., between the bounding curves in Figure 1. Radically different, but still complex, universes may exist in these directions in parameter space[7]. Support for this contention is provided by a number of radically different model universes which have been constructed

[7] And they will all exhibit parameter sensitivity – see section 5.

by Aguirre [4], by Harnik, Kribs and Perez [27], by Adams [1] and by Jaffe, Jenkins and Kimchi [31]. These are summarised in turn.

Aguirre's Cold Big Bang Universe

On the basis of considering the effect of single parameter variations on structure formation, Carr and Ress [13] have argued that the photon:baryon ratio cannot be less than $\sim 10^6$. Similarly, Tegmark and Rees [51] argue that the magnitude of the primordial density fluctuations, Q, is fine tuned to be within an order of magnitude of its value in this universe. Despite these parameter sensitivities, Aguirre [4] has presented a case for a universe potentially capable of supporting life in which the photon:baryon ratio, ξ , is of order unity, and Q is smaller than its value in this universe by a factor of between a thousand and a million. Aguirre argues that such a cosmology can produce stars and galaxies comparable in size and longevity to our own. As a bonus, a rich chemistry, including carbon, oxygen and nitrogen, can arise within seconds of the Big Bang. The previously claimed single-parameter bounds on ξ and Q are avoided by varying both at once, and by many orders of magnitude.

The Weakless Universe of Harnik, Kribs and Perez (HKP)

Harnik, Kribs and Perez [27] consider a universe which has no weak nuclear force. This is achieved by HKP by simultaneously varying the parameters of the standard model of particle physics and the cosmological parameters. Section 2.1 discusses how reducing the value of the Fermi constant sufficiently would lead to a universe with insufficient hydrogen to support familiar chemistry. The reason is that a smaller G_F produces an earlier freeze-out of the leptonic reactions. Hence the temperature is higher and the abundances of neutrons and protons are closer to equality. However, we have taken for granted that the neutrons and protons achieve their thermal equilibrium densities. This will only be the case if the weak interaction exists, since this provides the mechanism for their inter-conversion. Thus, the situation is entirely different if the weak interaction is effectively absent in the hadron era. In this case, HKP assume that the relative neutron and proton abundance can be fixed by *fiat*, as can the photon:baryon ratio.

HKP found that they could contrive a universe with a similar hydrogen:helium ratio as ours, but with about 25% of the hydrogen being deuterium rather than protons. To do so they chose a photon:baryon ratio of 2.5×10^{11}, i.e., about a hundred times larger than in our universe. HKP argue that galaxies could still form despite the much reduced visible baryon density, but that the number density of stars in the galaxies would be appropriately reduced. They can claim that stars would form, because they have taken the precaution of making the chemical composition of their universe sufficiently similar to ours, thus ensuring that there would be a cooling mechanism to permit gravitational collapse.

In the HKP universe the initial fusion reaction in stars would be the formation of helium-3 from a proton and a deuteron. Note that HKP have cunningly contrived to have substantial quantities of deuterium formed during BBN, so there is no need for the usual weak-force-mediated deuteron formation reaction from two protons. Since the first stellar reaction in HKP stars is very fast compared with the usual weak-mediated deuteron formation reaction, the core temperature of such stars would be lower. It has to be lower to keep the reaction rate down to a level at which the thermal power does not outstrip the available mechanisms of heat transport away from the core.

The moral of both Aguirre's and HKP's model universes is that by varying more than one parameter at once, and by being bold enough to vary them by many orders of magnitude, it is possible to discover distant regions of parameter space which potentially could support a complex universe with a rich chemistry. The key is varying more than one parameter at once. Consistent with Figure 1, the change in one parameter effectively offsets the change in another. In addition, by making very large changes, the nature of the physics involved changes qualitatively.

Adams' Parametric Survey of Stellar Stability

Adams [1] has considered how common the formation of stars might be in universes with different values for the universal constants. The most important quantities which determine stellar properties are the gravitational constant G, the fine structure constant α, and a composite parameter that determines nuclear reaction rates. Adams uses a simple analytical model to determine the region within this 3-dimensional parameter space which permits stellar stability. Using a parameterisation based on the logarithms of the above constants, Adams concludes that about one quarter of the region defined by either increasing or decreasing G or α by ten orders of magnitude supports the existence of stars. Whilst this cannot easily be translated into a statement about probability, nevertheless the requirement that stars be stable is hardly a strong constraint on the universal constants - a dramatically different conclusion from Smolin's [49]. Yet again, so long as more than one parameter is varied, the universe can evolve complexity (in this case, stars) even for parameter values very different from our own.

The Modified Quark Mass Universe of Jaffe, Jenkins and Kimchi (JJK)

Another set of examples of potentially biophilic alternative universes, obtained by varying more than one parameter, has been offered by Jaffe, Jenkins and Kimchi [31-33]. In these examples the parameters which are varied are the masses of the three lightest quarks, *u, d, s*, together with the QCD scale, Λ_{QCD}. JJK consider, for example, a universe in which the proton would be slightly more massive than the neutron. Whilst atomic hydrogen would then be unstable, they argue that deuterium and tritium, as well as some isotopes of carbon and oxygen, could be made stable. It is feasible, therefore, that a rich chemistry could emerge in such a universe. A more radical alternative considered by JJK is to reduce the strange quark mass considerably, so that nuclei become bound states of neutrons and Σ^- rather than protons. Again JJK argue that some isotopes of hydrogen and the surrogates of carbon and oxygen would be stable – and would be expected to possess comparable chemistry. What would happen to stellar physics in these universes, and whether the required elements would actually be formed in abundance, is unknown. But these examples again demonstrate the potential for complexity to arise if multiple parameters are varied to a congenial part of parameter space (using the term 'congenial' as JJK use it).

It will be demonstrated below that all alternative universes which give rise to complexity, such as those suggested by Aguirre, HKP, Adams and JJK, will inevitably display fine tuning.

3 The Two Distinct Fine Tuning Phenomena

How fine is fine tuning? The examples considered in section 2.1 support the existence of some degree of tuning, but the fineness is more debatable. Biophilic abundances of carbon and oxygen appear to require g_s to be fine tuned to within $\pm 1\%$, and nuclear stability requires the neutron mass to be fine tuned to within $-0.08\%/+1\%$ assuming a fixed proton mass. These cases seem quite 'fine', but the latter example is largely, but not completely, 'explained' by the nucleon structure. Klee [35] has pointed out that many of the commonly cited fine tunings are actually not at all fine, and even claimed order-of-magnitude tunings are stretched to cover several orders of magnitude.

The survival of primordial hydrogen may be an example, since G_F could accommodate an order of magnitude reduction without hydrogen abundance being too greatly diminished. On the other hand, in a universe in which the gravitational and electrostatic forces between two electrons differ by 42 orders of magnitude, perhaps a mere factor of ten would count as reasonably fine tuned?

Why are we interested in the degree of fineness of the tuning? It is because the finer the tuning, the more remarkable is the coincidence – or so one is tempted to think. But actually the intuitive notion that a very fine tuning translates into a small *a priori* probability is hard to defend. A number of authors have pointed out that the smallness of the numerical window within which a parameter must lie says nothing at all about its probability (e.g., Manson [38], McGrew et al [40]). Since a probability measure on parameter space is not available this is hard to dispute.

However, the pragmatic approach of this paper is that examples like those of section 2.1, and others in [2,3,6,7,13-16,20-24,26,28,29,46,47], do illustrate a real tuning phenomenon which requires explanation. The degree of tuning may be fine (a few percent or less) or not-so-fine (perhaps an order of magnitude), but the latter is no less in need of an explanation for being coarse-tuning. Both may be regarded as small parameter windows, in some context. And the fact that neither can strictly be claimed to be improbable is not pertinent. It is not the probability that we seek to explain, but the examples of tuning illustrated in section 2.1 and in [2,3,6,7,13-16,20-24,26,28,29,46,47]. An explanation is required for both fine and not-so-fine tunings. To avoid clumsiness of exposition, we continue to use the phrase "fine tuning" to mean both fine and relatively coarse tunings.

We contend that fine tuning actually consists of two distinct phenomena.

The first phenomenon is the parameter sensitivity of the universe. This is the (apparent) property of the universe that small changes in the parameters of physics produce catastrophic changes in the evolved universe. In particular the complexity of the evolved universe, and hence its ability to support life, would be undermined by small changes in the universal constants (in the pragmatic sense of 'small' changes discussed above). Thus, 'parameter sensitivity' is the claim that the target in parameter space which is compatible with a complex universe is small in some sense. The smallness of this target, if true, is a feature which requires explanation.

The second, and quite distinct, phenomenon is that nature has somehow managed to hit this small target – which we will refer to as "fine tuning". The actual constants in our universe have to be fine tuned to coincide with the requirements for a complex outcome. In other words, given that only special values for the parameters will do (i.e., given parameter sensitivity), nature had to contrive to adopt these particular values (i.e., nature is fine tuned).

The present paper is concerned only with the first phenomenon: parameter sensitivity and how it arises. A great deal has been written about the merits, or otherwise, of some sort of Multiverse as the contrivance by which the small target in parameter space is successfully hit (i.e., fine tuning is achieved). It seems to have gone largely unnoticed that an explanation is also required of the distinct phenomenon of parameter sensitivity. The Multiverse postulate does not even attempt to explain parameter sensitivity, i.e., why the target is small in the first place. So, why is the universe parameter sensitive?

There is a danger of misunderstanding this point. Physicists might argue that every instance of fine tuning (e.g., those listed in section 2) constitutes a demonstration, via physical calculation, that the target is small. They might opine that the question "why parameter sensitivity?" is answered by the totality of such calculations. But our calculations are merely observations that parameter sensitivity appears to prevail in *our* universe. They do not provide an explanation of why this should be so, i.e., why this feature should be expected.

The point can be illustrated in the following way. Before one looks into the physics of these things, it is not obvious that there could not be complex universes corresponding to the bulk of the volume of parameter space. Take life as an exemplar of complexity and consider the universes which might result if gradual changes were made to the universal constants. We can imagine, without any nonsense being obvious, that the lifeforms of our universe might give way to a continuous spectrum of morphing lifeforms as the physical parameters varied. Eventually radically different lifeforms would emerge, living in a universe whose physics was also radically different. But this description of the consequences of changing the universal constants is precisely what parameter sensitivity claims is *not* true. But why is this? Specifically, *why could this never be true in any universe which evolves complexity*? This is the question addressed in the following sections.

We shall argue that parameter sensitivity is a mathematical result of the assumed emergence of complexity. We shall argue that parameter sensitivity is inevitable in any complex universe, and hence, as a consequence, so is fine tuning.

4 How Parameter Sensitivity Relates to the Evolution of Complexity: Toy Models

This section illustrates, with the aid of toy models, the way in which parameter sensitivity arises from the assumption that the universe evolves complexity.

What is meant by a complex universe? This is a difficult question, so it is fortunate that a complete answer is not required for the present purposes. We shall see that conditions which are clearly necessary for the emergence of complexity turn out to be sufficient to imply parameter sensitivity.

In considering the meaning of "a complex universe" we generally think of the universe as it is now. The living organisms and the ecosystem of planet Earth are the epitome of complexity. However, all this did not emerge fully formed in a single step from the fireball of the Big Bang. Rather it is the current state of (one part of) a universe which has been evolving for 13.7 billion years. The history of the universe is one of increasing complexity, [24,41,45]. Thus, the formation of helium nuclei after the first few minutes represents an increase in complexity compared with what preceded it. The same is true of the formation of the first neutral atoms at ~360,000 years, and the first stars or galaxies at some hundreds of millions of years. The gravitational congealing of matter provided the opportunity for complex, orderly structures to arise. Despite their gaseous form, stars have a considerable complexity of structure and evolution. The structure of galaxies is vastly more complex still, acting as they do as stellar nurseries. And the solid astronomical bodies, planets and asteroids, provide the opportunity for great complexity on smaller size scales.

From the point of view of the second law of thermodynamics it initially appears curious that the Big Bang fireball, which is generally assumed to have been in local thermal equilibrium, nevertheless gave rise to a universe which spontaneously produced orderly structures. This comes about because the orderly, and complex, structures occur in regions of gravitational collapse. Such regions have shrugged off their excess entropy, using the vast tracts of almost empty universe as a dumping ground. This is the salient fact: inhomogeneity of the entropy distribution is a necessary condition for the emergence of complexity.

This world was not always complex. It became complex. And because, at the fundamental level, "becoming" necessarily entails dynamics, the complexity of the world is a product of dynamics. Most especially the dynamics of gravitational collapse and nuclear fusion are the root cause of the *possibility* of complexity. Both of these processes lead to the reduction of the entropy of (a part of) the baryonic component, the excess entropy being carried away by other, generally lighter, particles, e.g., photons and neutrinos. And when complexity reaches the level of life, it is sustained against the tendency to increase its entropy (decay) by the flux of free energy from its parent star, Egan [25], Lineweaver and Egan [37], Michaelian [41], Wallace [52].

It may be unusual to speak of stellar nuclear reactions as "dynamics", but the reacting core of a star is comprised of a myriad of individual particle dynamics. So if complexity emerges from dynamics, and parameter sensitivity is defined in reference to complexity, it follows that parameter sensitivity should also be understood as a property of dynamics. Fortunately *thermo*dynamics, the statistical properties of many body dynamics, suffices.

The key is to recognise that only a small portion of the mass of the universe will end up complex. We do not need to consider the whole observable universe, but just some comoving region, $\Im$, which is large enough to approximate to a closed system. Within $\Im$ the sub-system which will end up complex will be called $\aleph$. It is necessarily an *open* sub-system and may be regarded as a fixed inventory of baryonic matter. It is claimed that the sequence of processes leading to complexity must include steps in which $\aleph$ reduces its entropy and hence becomes more ordered. We claim that this is sufficient to produce parameter sensitivity. Note that, in accord with the second law of thermodynamics, the entropy of $\Im$ can only increase. In fact, the irreversible processes involved in forming and sustaining $\aleph$ implies that the entropy of $\Im$ *will* increase.

The relationship between entropy and complexity is not an easy one. It is certainly not the case that unfettered entropy minimisation leads to complexity. Arranging atoms in a perfect crystal structure minimises their entropy but is the antithesis of complexity. On the other hand, the maximal entropy of a gas in thermal equilibrium is also the antithesis of complexity. These observations are consistent with the contention that complexity appears to exist at the boundary between order and disorder (Prigogine [44], Kauffman [34]). Many definitions of complexity have been offered, but there is no universal agreement. Fortunately the only claim we need make is that some of the evolutionary steps, starting from a universe devoid of structure, must involve the reduction of the entropy of $\aleph$. If true complexity is to be the outcome, then such processes will not proceed to minimisation of the entropy of $\aleph$, since, as noted already, $\aleph$ would not then be complex. Nor is it necessary for the reduction in the entropy of $\aleph$ to be monotonic. A certain number of entropy reducing processes is claimed to be necessary, but certainly not sufficient, for the emergence of the highest degrees of complexity.

Our contention is that this holds for any set of universal constants, and any replacement of the standard model of particle physics, which result in a complex outcome.

A simple toy model will help illustrate a sequence of entropy reducing steps. We deliberately choose a form of particle physics which differs from the standard model in order to emphasise the generality of the argument. However, it reads across to similar processes in our universe.

Let us suppose that complex structures are to be built out of combinations of two types of 'matter' particle: *a* and *b.* These particles are assumed to be present in a free state in some primordial, chaotic epoch of the universe. The first step in reducing the entropy of the 'matter' component of the universe may consist of a reaction $a+b \to c+d$, where *c* is to be regarded as a bound state of *a* and *b*. The reaction is assumed to be exothermic. Almost all the rest mass of *a* and *b* ends up in *c*, whilst particle *d* is relatively light and hence will carry away the bulk of the energy released. So *a*, *b* and *c* are the analogues of baryonic matter, whereas *d* may be the analogue of a photon or neutrino. The reaction produces a reduction of the entropy of the 'baryonic' matter simply because, if attention is paid to the baryonic components alone, the reaction is $a+b \to c$. So one particle results where there were previously two. Other things being equal, entropy becomes simply particle counting, so this represents a reduction in the baryonic entropy by a factor of about 2.

Of course the reaction proceeds because it causes an *increase* in the total entropy of the universe. There is no deficit in the number of *all types of* particle in the reaction $a+b \to c+d$. Moreover, the energy released leads to an entropy increase. The energy released by the reaction is the binding energy, B_c, of *a* and *b* in the bound state *c*. For the reaction to make net progress in the face of potential thermal dissociation (i.e., despite the reverse reaction $c+d \to a+b$), the binding energy must be large compared with the typical prevailing thermal energy, kT. Hence the energy of the 'photon', *d*, is much larger than kT. Presuming that these *d* particles have some mechanism available by which they can come into thermal equilibrium with the surroundings, it follows that after thermalisation the initial energy of the *d* particle will be spread over $\sim B_c / kT >> 1$ different particles. Increasing the energy of a large number of particles constitutes an entropy increase. In summary, the reduction of the baryonic entropy is bought at the cost of an overall increase in entropy which is born by the other particles (e.g., particles *d*).

Any reasonable measure of the strength of the interaction which causes the binding of *a* and *b* will be such that the binding energy B_c increases monotonically with the strength. Hence we can use B_c itself as a measure of the strength of this interaction. The reaction will proceed only if the interaction is strong enough to overcome thermal dissociation, which we can write symbolically as $B_c > kT$ (whilst accepting that there will really be phase space factors and so on which complicate the exact expression). There is nothing in the least surprising about this: the interaction must be strong enough to make the reaction go. But note that $B_c > kT$ is a constraint on the 'universal constant', B_c, which measures the strength of the interaction, and this constraint is necessary in order to achieve a reduction of the 'baryonic' entropy via $a + b \to c + d$. This is an example of how entropy reduction of $\aleph$ results in a constraint on the universal constants, and ultimately parameter sensitivity.

Our toy model has not yet achieved any significant complexity, and could not be expected to do so in a single step. A succession of entropy reducing steps of differing kinds is to be expected, and a complex outcome must at least include a great diversity of compound particles. So we suppose that subsequent reactions can occur which result in compound particles of the form $a_n c_m$. We may write this as $na + mc \to a_n c_m + Ne$, it being understood that this is shorthand for a sequence of two-body reactions. There are *N* light particles, *e*, which carry away the bulk of the binding energy released in forming the composite particle $a_n c_m$. For the same reasons as before, such a reaction involves an overall increase in entropy, as it must, but a reduction in the 'baryonic' entropy because $n + m$ particles are replaced by just one compound baryonic particle. Also for the same reason as before, the interaction which binds the compound particles $a_n c_m$ must have a certain minimum strength to overcome thermal dissociation (the reverse reaction).

However, there is now an important additional feature. Reactions of the form $na + mc \to a_n c_m + Ne$ can occur only if the initial reactions $a + b \to c + d$ do not consume all the a particles. But this leads to an *upper bound* on the strength of the interaction measured by B_c, in addition to its lower bound established previously. This can be seen as follows.

Firstly let us assume that $a + b \to c + d$ is far faster than $na + mc \to a_n c_m + Ne$. If $a + b \to c + d$ were to proceed to completion, reactions $na + mc \to a_n c_m + Ne$ could not occur. For $na + mc \to a_n c_m + Ne$ to occur, there would need to be some means of terminating the $a + b \to c + d$ reactions whilst some a particles remained. This may arise due to $a + b \to c + d$ being frozen-out by falling temperature or by cosmic expansion. Or it may be that some other reactions are also occurring which result in copious production of *d* particles. A very large density of *d* particles will favour the reverse reaction $c + d \to a + b$ and might lead to a dynamic equilibrium at some non-zero density of a particles. Whatever the mechanism, a non-zero density of a particles can result only if the strength of the interaction, B_c, is insufficient to drive the reaction against the countervailing effects. For example, $a + b \to c + d$ would be frozen-out by cosmic expansion if the reaction rate falls below the universal expansion rate (i.e., the Hubble parameter). But the reaction rate will increase if the interaction strength, B_c, were increased. So freeze-out whilst a reasonable abundance of a particles remains requires that the interaction strength, B_c, is less than some bound. Too large a value for B_c would result in a late freeze-out, when the a particle density had already fallen too far.

Secondly, what happens if we assume that $a+b \to c+d$ is far *slower* than $na+mc \to a_n c_m + Ne$? In this case the burden of achieving a balance of compound particles $a_n c_m$ shifts to the relative rates of the various reactions implicit in the sequence leading to $na+mc \to a_n c_m + Ne$. We shall see in the toy model below that this also requires the interaction strength, B_c, to be bounded both above and below.

And finally, what if we assume that the rates of $a+b \to c+d$ and $na+mc \to a_n c_m + Ne$ are comparable? Well, in that case, we have assumed fine tuning from the start – namely that the strength of the interaction driving the first reaction is closely matched by the strength of the interaction driving the second.

So there is an upper bound to B_c as well as the lower bound established previously. Only if the strength of the 'force' is bracketed between a lower and an upper bound can both reactions $a+b \to c+d$ and $na+mc \to a_n c_m + Ne$ actually happen. This is the origin of parameter sensitivity, as illustrated by Figure 1, for this illustrative example.

The point is that parameter sensitivity (in the sense of a parameter being bounded both above and below) is necessary in order that the 'baryonic' matter can undergo *sequential* entropy reductions through both $a+b \to c+d$ and $na+mc \to a_n c_m + Ne$. The upper and lower bounds are *both* required in order to produce the compound particles $a_n c_m$.

The prize is to achieve a rich variety of suitable chemistry, not merely a monoculture of a single nuclear variety. In our universe, nucleosynthesis in stars enriches the ISM with a very broad range of nuclei in several ways. Some lighter nuclei are ejected as the star evolves, in the form of stellar winds and various instability phenomena. In fully evolved stars, elements up to iron are preserved by the shell structure in which there is a gradation of temperature, pressure and density conditions. Finally, elements beyond iron are made by the complex physics occurring during supernovae. It is not trivial to ensure a balance of this kind in the abundance of the products of nucleosynthesis.

In our universe, the synthesis of the chemical elements inside stars is immensely complicated. So to illustrate how the production of a balance of elements requires fine tuning we opt for a simple toy model. We return to Gamow's original idea in which the chemical elements are made during BBN. To permit this to happen we change particle physics and nuclear physics drastically. There is no neutron in this universe; the only nucleon is the proton. Protons in this universe can form bound states of two, three, four, and up to ten nucleons via some analogue of the strong nuclear force. The time-temperature relation is assumed equal to that in our universe. The nuclear reaction rates are contrived to give the more highly charged nuclei a reasonable chance of being formed, despite the falling temperature and density. This means radically altering the Coulomb barrier (by *fiat*, there is no underlying theory here). The strength of the nuclear force is measured by the binding energy per nucleon, *B*, which is assumed to be the same for all nuclei.

The absolute number density of nucleons is found from an assumed photon:baryon ratio, ξ, and the usual black-body photon density in terms of temperature. The absolute number densities are also adjusted due to cosmic expansion following the usual time dependence. The reaction network consists of 25 different reactions. These reactions are eventually frozen-out by cosmic expansion. The parameters *B* and ξ are the universal constants whose fine tuning we wish to demonstrate.

The specific algebraic and numerical assumptions of the model are given in the Appendix, and are essentially arbitrary. It is rather obvious what the outcome will be. If the fusion reactions are sufficiently fast compared with the cosmic expansion rate, the reaction sequence will proceed to completion before freeze-out. The universe will then contain only the highest mass nuclei and no lighter nuclei. This will occur if either the nuclear force is sufficiently strong (large *B*) or the nucleon density is sufficiently high (small ξ). Conversely, if the fusion reactions are sufficiently slow compared with the cosmic expansion rate, even the first compound nucleus (pp) will not have time to form before freeze-out. The universe will be all hydrogen (perhaps permanently so if this universe remains star-free). This will occur if either the nuclear force is sufficiently weak (small *B*) or the nucleon density is sufficiently low (large ξ).

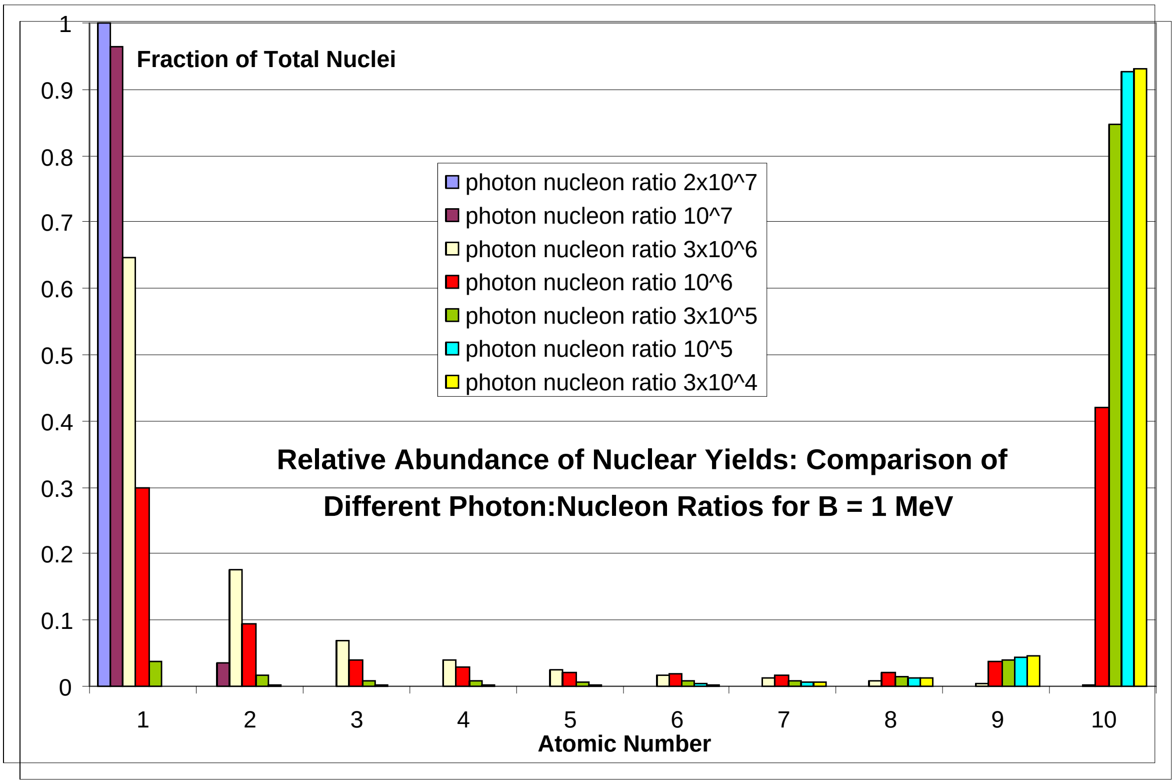

Figure 2: *Relative abundance of nuclei for B=1MeV: Comparison of different* ξ

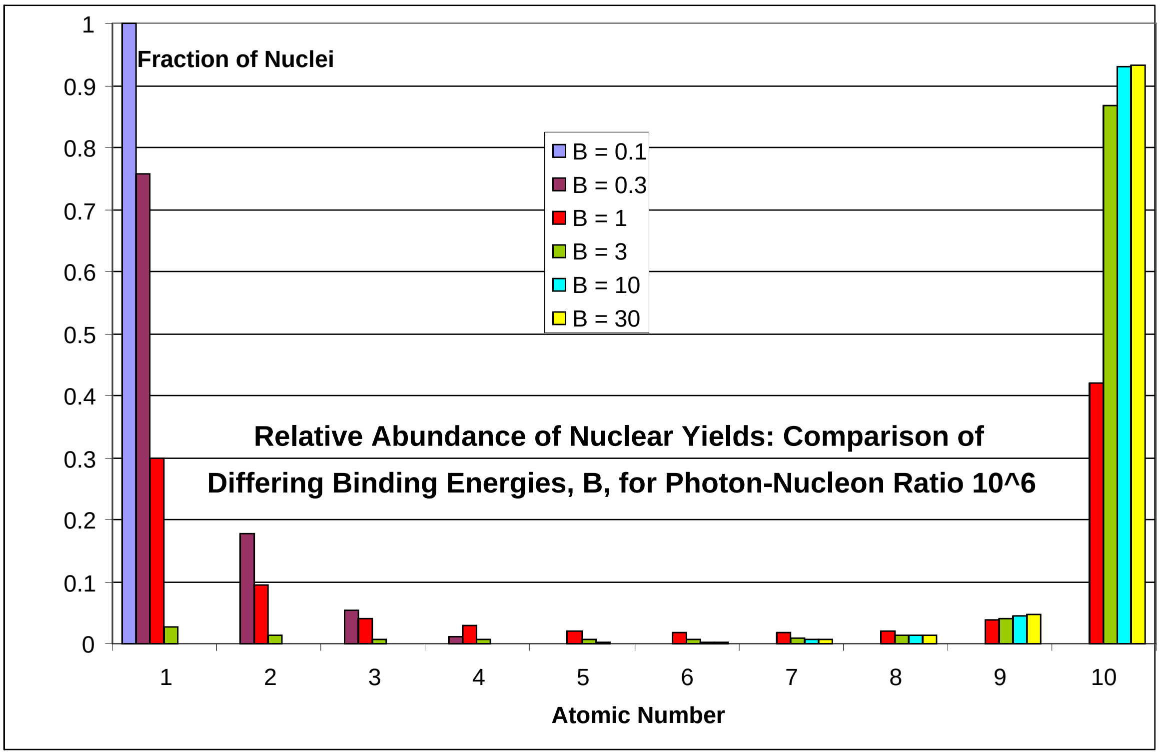

Figure 3: *Relative abundance of nuclei for $\xi=10^6$: Comparison of different B*

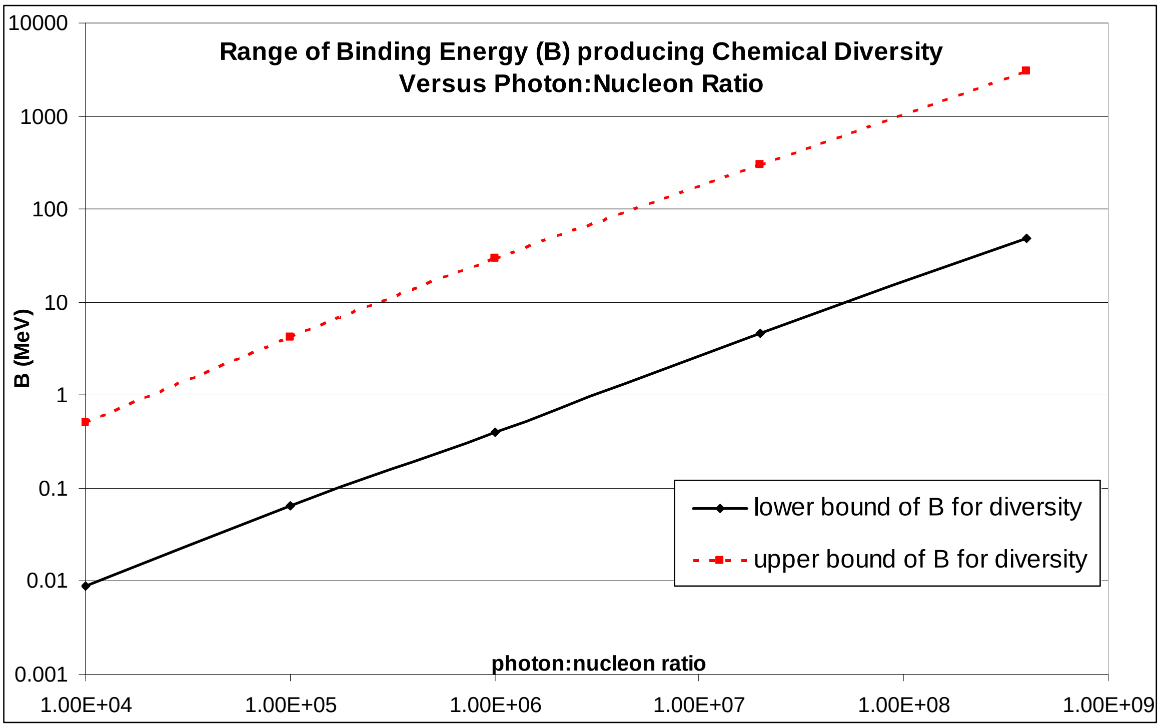

Figure 4: *Ranges of B and ξ (between the lines) producing chemical diversity*

The numerical results of the model are in accord with these expectations, Figures 2 and 3. The optimal balance of light and heavy nuclei is obtained for B = 1 MeV and $\xi = 10^6$. Suppose a 'chemically diverse' universe has at least a fraction 10^{-4} of each element. To produce such a chemically diverse universe it is necessary that B and ξ fall within the range shown in Figure 4. This is the realization of Figure 1 for this model. Outside these bounds the universe would be virtually all hydrogen or virtually all "neon" (Z=10). The universal constants must therefore be fine tuned to produce a complex outcome, i.e., a diversity of chemical elements. The tuning is not actually terribly fine, but then many of the instances of tuning in our universe are not so terribly fine either.

Suppose we seek to circumvent this conclusion. We could make the fusion reactions very rapid compared to cosmic expansion, but protect the lighter nuclei from being consumed in some way. For example, we could assume the presence of some other type of particle which could combine with protons or nuclei such that, after combination, the resulting compound object was immune to further fusion reactions. Call this reaction X.

If reaction X is too fast compared with the fusion reactions then we will end up with an all hydrogen universe again. Conversely, if reaction X is too slow we will end up with a universe containing only the heaviest nuclei. So this contrivance fails: we will again require a fine tuning relating *B* and ξ to whatever universal constant controls reaction X.

It will be apparent from these examples that the connection between parameter sensitivity and a complex outcome is quite elementary in nature. On the other hand, a few specific examples cannot establish the general truth of the assertion that parameter sensitivity is *always* a concomitant property of a universe which evolves complexity. This is addressed next.

5 The General Dynamical Explanation of Parameter Sensitivity

We wish to demonstrate that *any* sequence of entropy reductions of some subsystem, $\aleph$, must always require tunings of the universal parameters. Such a demonstration is required for any system of the sort considered in the preceding examples, in which complexity is achieved by binding fundamental particles in ever more intricate ways. But the proof should also accommodate any other form of entropy reduction, not just that arising from particle binding. For example, volume contraction due to gravitational collapse is a particularly important cause of entropy reduction in our universe.

Actually the demonstration should be more general still, since it is conceivable that complexity could arise in quite a different manner. For example, structure might not reside in binding particles (i.e., in identifying their spatial degrees of freedom) but in a correlation or coherence between spatially separated particles. Also, the fundamental degrees of freedom may not be those of particles, but of fields. Most physicists believe that the number of field degrees of freedom is actually finite rather than a continuum, limited to a spatial resolution at around the Planck scale. It is even possible (according to the holographic hypothesis) that the resulting spatial lattice is only two dimensional, rather than three dimensional, [11,50]. It is desirable to formulate the argument in terms which are sufficiently general to embrace all these possibilities and more.

The natural arena for a generalised exposition is phase space. Phase space comprises the totality of degrees of freedom of the system in question, both generalised coordinates (or fields), q_i, and generalised momenta, p_i. A point in phase space specifies a unique microstate of the system. Studies of bulk behaviour consider, not individual microstates, but macrostates corresponding to large numbers of possible microstates, and hence to large volumes of phase space. For example, the pressure and temperature of a gas may specify its macrostate, but there are many corresponding microstates, each of which uniquely specifies the position and velocity of every molecule at a given time. The volume of phase space within which a system might lie is a measure of the number of possible microstates, and hence is related to the entropy of the macrostate.

The evolution of complexity is thus related to the variation over time of the phase space volume occupied by $\aleph$. Specifically, it is claimed that there are necessarily physical processes which reduce the phase space volume of $\aleph$, since this is equivalent to reduction of the entropy of $\aleph$.

Now the development over time of a given microstate is described by a trajectory through phase space. For deterministic physics, this trajectory is uniquely determined by the initial state (the starting point in phase space) plus the physical laws and the values of the universal constants. It follows that the development of the extended phase space region corresponding to a macrostate is also uniquely determined by the starting region, the laws of physics and the universal constants, $\{c_j\}$. In particular this applies to $\aleph$. It is claimed that there must be processes which sequentially reduce the volume of the phase space region occupied by $\aleph$. Suppose these processes are labelled 1, 2, 3…, in chronological order. The phase space volumes of $\aleph$ at these times are $V_1 > V_2 > V_3 > \ldots$. But each volume is calculable from the initial state for a given set of universal constants, $\{c_j\}$. Taking the initial state and the laws of physics as understood, we can thus write $V_1(\{c_j\}) > V_2(\{c_j\}) > V_3(\{c_j\}) > \ldots$. This is meant to emphasise that each phase space volume can be calculated in terms of the assumed universal constants. This sequence of inequalities between *calculable* functions of $\{c_j\}$ therefore implies constraints upon the universal constants themselves. This is the origin of parameter sensitivity.

It would not be reasonable to expect complexity to arise from a single physical process. A sequence of processes is to be expected. The particular sub-set of universal constants which is important will differ from one process to another. So the sequence of processes gives rise to different tunings, possibly of different sets of parameters.

It is important to recognise that the dynamics determines not just the change of volume of the phase space region representing $\aleph$ but also its position within phase space. This is crucial to the next process in the sequence. As we saw in the toy model above, moving to a phase space region with no a particles may prevent further entropy reduction. Hence, the *path* taken by $\aleph$ through phase space is crucial. Achieving *sequential* entropy reductions is a strong constraint upon the *global* path, not just upon the 'current' process. For example, the entropic benefit to $\aleph$ of preserving hydrogen in the first minutes after the Big Bang is realised only a billion years later with the formation of hydrocarbon chemistry.

The phase space algebra can be made more explicit, and this has the advantage of displaying the crucial path dependence more clearly. For any deterministic physics the phase space trajectory $\{x_k(t)\}$ is uniquely defined by the initial microstate, $\{x_k(0)\}$. This statement can be written in differential form as,

$$\frac{dx_i}{dt} = f_i\left(\{x_k\}, \{c_j\}\right) \tag{1}$$

The functions f_i specify the physics which determines the system's evolution, and depend both upon the current state, $\{x_k(t)\}$, and upon the universal constants, $\{c_j\}$. We assume that the phase space is sufficiently generally formulated that it can address particle reactions. Thus, if $a + b \to c + d$ occurs, the phase space includes regions representing both $a + b$ and $c + d$. Suppose, for convenience of exposition, that significant physical events occur at the discrete times $t_1, t_2, t_3, ...$ when the region of phase space occupied by $\aleph$ is $\wp_1, \wp_2, \wp_3, ...$ respectively. Consider a small element of volume of $\wp_L$ at time t_L and location $\{x_k\}$, $dV = \prod_i dx_i$. A time δt later the evolved system effectively defines a new set of coordinates $x_i' = x_i(t + \delta t) = x_i + \delta t \cdot f_i$. The evolved volume $dV' = \prod_i dx_i'$ is related to the initial volume by the Jacobian determinant: $dV' = \left\| \left\{ \frac{\partial x_i'}{\partial x_j} \right\} \right\| dV$. And since $\frac{\partial x_i'}{\partial x_j} = \delta_{ij} + \delta t \cdot \frac{\partial f_i}{\partial x_j}$ we find that $\left\| \left\{ \frac{\partial x_i'}{\partial x_j} \right\} \right\| = 1 + \delta t \cdot \sum_i \frac{\partial f_i}{\partial x_i}$ to first order in δt. Re-arranging and integrating over the region $\wp_L$ yields the evolution of the phase space volume, $V_{\wp_L}$, corresponding to $\aleph$, to be,

$$\frac{dV_{\wp_L}}{dt} = \int_{\wp_L} \left(\sum_i \frac{\partial f_i}{\partial x_i} \right) dV \tag{2}$$

In the case of a conservative system defined by Hamiltonian mechanics, and if $\aleph$ were replaced by the approximately closed system $\Im$, the RHS of (2) would be identically zero by virtue of Hamilton's equations[8]. Hence, the phase space volume does not change for conservative Hamiltonian systems (Liouville's theorem).

But, by hypothesis, $\aleph$ is a dissipative sub-system, necessarily ejecting energy and entropy into its surroundings, so that its phase space volume is reducing, $\frac{dV_{\wp_L}}{dt} < 0$. This is the familiar behaviour of dissipative dynamic systems, whose phase space volume tends to shrink asymptotically onto some attractor, typically of lower dimension than the phase space (albeit probably fractal). Thus, the phase space volume of $\aleph$ might shrink asymptotically to zero. In fact, the dimension of $\aleph$ will reduce each time particles bind to form composite baryonic particles. However, an equation like (2) will continue to hold with the volume reinterpreted as being of reduced dimension.

[8] The 'velocity field' f_i in phase space has zero divergence because of Hamilton's equations.

The entropy of $\aleph$ need not reduce monotonically. It is sufficient that there are times when it does so. By definition, these are the times $t_1, t_2, t_3, ...$ Consequently our thesis is that,

$$\frac{dV_{\wp_1}}{dt} = \int_{\wp_1(\{c_j\})} \left(\sum_i \frac{\partial f_i(\{c_j\})}{\partial x_i} \right) dV < 0 \tag{3a}$$

$$\frac{dV_{\wp_2}}{dt} = \int_{\wp_2(\{c_j\})} \left(\sum_i \frac{\partial f_i(\{c_j\})}{\partial x_i} \right) dV < 0 \tag{3b}$$

$$\frac{dV_{\wp_3}}{dt} = \int_{\wp_3(\{c_j\})} \left(\sum_i \frac{\partial f_i(\{c_j\})}{\partial x_i} \right) dV < 0 \qquad \text{etc.} \tag{3c}$$

In (3a-c) the dependencies upon the universal constants, $\{c_j\}$, have been explicitly displayed. Not only do the phase space velocity functions, f_i, depend upon the universal constants, but so do the changing phase space regions, $\wp_L$, occupied by $\aleph$ over which the integrals are carried out. The 'path' through phase space taken by the evolving region $\wp_L$ depends upon the preceding region from which it derives, $\wp_{L-1}$, and also upon the physics encoded in the functions f_i, and also upon the universal constants, $\{c_j\}$.

For a conservative Hamiltonian system, (3a-c) would be replaced by equalities. Moreover these equalities would be algebraic identities. But (3a-c) are not algebraic identities. They are dynamic constraints on any subsystem whose entropy is successively reducing. And note that all the dynamic variables, $\{x_i\}$, vanish from (3a-c) by virtue of the volume integrations. Hence, (3a-c) are actually constraints on the universal constants, $\{c_j\}$, which must be fulfilled if $\aleph$ is to have sequential reductions in entropy. These constraints constitute fine, or possibly not-so-fine, tuning. These relations are the algebraic embodiment of Figure 1.

The importance of the path, $\wp_L$, taken through phase space must be emphasised. The path determines the potential for continuing entropy reduction in later steps. The opportunity for entropy reduction in, say, step 2, would be lost if step 1 resulted in a $\wp_2$ which is not conducive to further entropy reduction. A phase space path which permits continuing entropy reductions over many sequential processes might appear highly contrived. Indeed, it may *be* highly contrived. However, it is not our concern here to explain how complexity comes about (i.e., how such a path may arise spontaneously). It is sufficient to note that complexity requires this, and that relations like (3a-c) must follow. This is the origin of parameter sensitivity.

There are a number of potential challenges to the generality of the above argument. Firstly, it is possible to question whether (3a-c) do actually constrain the parameters $\{c_j\}$. After all, if we replace $\aleph$ with the closed system $\Im$ the second law of thermodynamics tells us that,

$$\int_{\Im} \left(\sum_i \frac{\partial f_i(\{c_j\})}{\partial x_i} \right) dV \geq 0 \tag{4}$$

and this must hold for *any* values of the parameters $\{c_j\}$. So, despite appearances, (4) is *not* a constraint upon $\{c_j\}$. Why should (3a-c) not also be of this type? To refute this consider a set of parameters, $\{c_j^0\}$, which correspond to switching off all inelastic interactions. Under suitable circumstances the sub-system $\aleph$ can now be considered as effectively closed. It is no longer exporting its entropy to $\Im$. In which case we have,

$$\int_{\aleph}\left(\sum_i \frac{\partial f_i\left(\{c_j^0\}\right)}{\partial x_i}\right) dV \geq 0 \qquad (5)$$

But comparison of (3a-c) with (5) shows that (3a-c) clearly *do* constraint the parameters $\{c_j\}$ since there is a set of values, $\{c_j^0\}$, for which (3a-c) are *not* true.

The second objection is that it is not entirely clear, in the general case, what is meant by the distinct "processes", enumerated above as 1, 2, 3… At the risk of being tautological, these relate to the discrete fine tunings. The limiting curve in parameter space which is defined by the inequalities (3a-c) may be identified with what Bousso et al [9] refer to as a "catastrophic boundary".

The third objection may be that "not everything is dynamics". But actually, everything that involves change *is* dynamics. It may be unfamiliar to formulate some physical processes in terms of dynamics, simply because the dynamic perspective may not be helpful in performing calculations. But the physics underlying any change in any system must be dynamical when considered at the microstate level. The formulation in terms of phase space is a convenient means of linking the entropy reduction to the evolution as determined by the universal constants. It does not matter that it may not be a practical means of carrying out a calculation.

The fourth objection is the assumption of deterministic (classical) dynamics. This does not address the potential indeterminism due to quantum mechanical behaviour. However, we are concerned with physics on an astronomical scale, with correspondingly large phase space volumes. It is true that quantum calculations are essential in providing much of the input to astrophysical calculations (e.g., reaction rates). However, there is no compelling reason to be concerned that this translates into any significant indeterminism at the scale of interest. As far as bulk behaviour is concerned, and providing we do not attempt to address what happens at t = 0, it is likely that the classical formulation is sufficient to establish the principle.

The final objection is that we have been cavalier in comparing phase space 'volumes' when the dimension of the sub-space occupied by $\aleph$ is reducing. In a true continuum this might indeed be a problem. But really, talking about phase space 'volumes' is just a shorthand for counting the number of degrees of freedom which are all ultimately integers, whatever the dimension may be in the continuum description.

6 Interpretation of Equations (3a,b,c…) in Our Universe

There is a danger that the very generality of equations (3a,b,c…) obscure what is actually a simple message: every instance of fine, or not-so-fine, tuning in our universe is actually an instance of entropy reduction.

Consider the examples of section 2.1. The fine-tuned nucleon mass range (or alternatively, the coarsely tuned quark mass range) relates to nuclear stability, and hence to the potential for entropy reduction via nuclear fusion. The same applies to the lower bound on the strong nuclear coupling of $0.85g_s$. The case of the Hoyle coincidence (the production of biophilic abundances of carbon and oxygen) is different. In this case the entropy reduction does not occur at the time of stellar nucleosynthesis. Indeed, the star would reduce its own entropy more if a long sequence of alpha capture reactions occurred producing some very heavy nuclei, with little remnant carbon and oxygen. The entropic benefit of biophilic production of carbon and oxygen, ***to sub-system $\aleph$ only***, occurs later with the formation of organic molecules in the giant gas clouds and with planetary formation and subsequent planetary chemistry. Similarly, the constraint on the Fermi constant arising from survival of hydrogen during BBN does not reduce entropy at that time. The entropy would be lower if the result of BBN were all helium. The benefit to sub-system $\aleph$ is realised only with the formation of main sequence stars and with the occurrence of hydrocarbon chemistry after a billion years or so. So the complexity-benefit of one of equations (3a,b,c…) may be long delayed, and is conferred on system $\aleph$ alone.

Whatever examples of fine (or coarse) tuning are chosen, the relation to entropy reduction can always be made. This is simply because all tunings are identified as being required to permit the formation of structure and complexity – and this implies entropy reduction of the open sub-system, $\aleph$, in question. But the complexity-benefit might occur much later than the process giving rise to the numerical fine tuning. For example, Carter [15,16] has argued that planetary formation is facilitated due to typical stars lying near the boundary of convection and radiation dominance. This gives rise to the tuning $\alpha^{12}\left(\frac{m_e}{m_p}\right)^4 \sim \frac{Gm_p^2}{\hbar c}$ which, whilst derived from stellar physics, does not relate to increased complexity in the stars concerned. Instead the complexity benefit occurs only later upon planetary formation. Planets are system $\aleph$ in this case.

A further example is that, in our universe, there must be stable stars to forge the chemical elements beyond lithium. The formation of stars requires a cooling mechanism, this being the mechanism by which entropy is exported away from the collapsing material. Assuming that cooling is dominated by photon emission, this must require some lower bound on the fine structure constant. Despite the fact that star formation is difficult to analyse algebraically, it is clear that cooling must require a lower bound on α since there would be no cooling in the limit $\alpha \to 0$. The lower bound on α is required both to bring about the immediate entropy reduction of the collapsing gas cloud, and also to permit the formation of the chemical elements thereafter. The complexity-benefit of the formation of the chemical elements occurs later still.

Where the above examples involve a *delayed* entropy reduction in system $\aleph$, this emphasises the importance of the path taken through phase-space in the general formulation of section 5. Thus, whilst equations (3a,b,c…) express our thesis in general form, the interpretation for any specific model universe is very simple. Indeed it is essentially merely a re-statement of the manner in which the fine tunings are usually derived.

Finally, what about fine, as opposed to not-very-fine, tuning? It is not clear whether equations (3a,b,c…) produce particularly *fine* tuning. But this does not matter. Equations (3a,b,c…) are precisely the degree of tuning required to permit the associated complexity to arise. Whether this tuning happens to be fine or coarse is unimportant. Consequently, referring back to section 3, we now see that it is inappropriate to be overly concerned about the actual degree of fineness of the tunings evident in our universe. They are as fine as they need to be.

7 Relation of these Observations to Other Issues

What is and what is not explained

The hypothesis of this paper does not address how the universe becomes fine tuned. An explanation of parameter sensitivity alone has been offered, i.e., why the target in parameter space is necessarily small. No comment is made regarding how the universe contrives to hit this small target. The existing scientific responses to this question have involved one of the many variants of the Multiverse. The present work leaves these offered explanations for fine tuning untouched, neither supporting nor refuting them. However one thing is achieved: it is inappropriate to be surprised by fine tuning. Fine tuning is inevitable given the existence of complexity.

The Cosmological Constant

Not all apparent fine tunings need necessarily arise in the manner proposed in section 5. At one time the extreme flatness ($\Omega = 1$) required at early epochs in order that the universe be even approximately flat now might have been regarded as fine tuning. But its very fineness spoke of a mechanistic explanation, such as is provided by inflation theory. It is possible that a mechanistic explanation may ultimately be forthcoming for the apparent extreme tuning of the cosmological constant, Λ, although a revolution in fundamental physics is probably required to achieve it, Albrecht et al [5]. Alternatively, the fine tuning of Λ may be an observer selection effect, Martel et al [39], Weinberg [53], Lineweaver and Egan [36], Egan [25]. The present work offers no enlightenment on this issue.

Relationship to the Causal Entropic Principle

The Causal Entropic Principle has been proposed by Bousso et al [8-10] as a criterion for the abundance of observers in anthropic predictions. These authors define the causal entropic principle thus, "*the principle asserts that physical parameters are most likely to be found in the range of values for which the total entropy production within a causally connected region is maximized*". They argue that, "*the formation of any complex structure (and thus, of observers in particular), is necessarily accompanied by an increase in the entropy of the environment. Thus, entropy production is a necessary condition for the existence of observers. In a region where no entropy is produced, no observers can form. The 'entropic principle' is the assumption that the number of observers is proportional, on average, to the amount of entropy produced*". Bousso et al are careful to emphasise that it is the matter entropy only to which they refer. The Bekenstein-Hawking entropy associated with black hole or cosmological horizons should not be included.

The causal entropic principle is therefore closely related to the arguments of this paper, though not the same. We have not claimed that entropy is maximised, within any particular region. Our assumption is weaker, namely that $\aleph$ reduces its entropy. This irreversible process necessarily leads to an increase in the entropy of $\Im$. This is consistent with the spirit of the causal entropic principle, but attaining a maximum has not been assumed here.

8 Conclusion

Parameter sensitivity and fine tuning are two separate phenomena. The problem of elucidating why the universe is parameter sensitive has not previously been addressed, despite much effort expended on fine tuning.

The thesis has been presented that parameter sensitivity arises as a natural consequence of the mathematics of dynamical systems with complex outcomes. The argument is as follows: the emergence of complexity in a sub-system, $\aleph$, requires a sequence of entropy reductions of $\aleph$, which can be interpreted as a sequence of reducing phase space volumes. This leads, via a very general formulation of system evolution, to constraints on the set of universal constants. This is the origin of parameter sensitivity.

Hence, if a universe contains complex parts then parameter sensitivity is inevitably observed, and therefore fine tuning will always be required to produce a complex world. In other words, complex universes are generically fine tuned. The fine tuning of our universe should therefore not elicit surprise. Moreover, any alternative universe which gives rise to complexity, such as the model universes of Aguirre [4], Harnik, Kribs and Perez [27], Adams [1] or Jaffe, Jenkins and Kimchi [31-33], would inevitably display fine tuning. This answers the question raised earlier: why could varying the universal constants never give rise to a continuous morphing of lifeforms in any universe? It is because any complex universe will be fine tuned.

This paper has not addressed how the universe achieves its fine tuning, only that fine tuning is an inevitable requirement for a complex outcome. Consequently the extent to which postulating a Multiverse may be motivated is unchanged by the present work.

Appendix: Details of the Toy Model for Gamow-style BBN Nucleosynthesis

The model does not bear any relationship to the physics of our universe. It is intended only as an illustration of how fine tuning arises if a balance of chemical elements is to be the outcome. A nucleus of N nucleons is assumed to have a binding energy of (N-1)B, so that any reaction between a nucleus of N nucleons and a nucleus of M nucleons to create a nucleus of N + M nucleons involves an increase in binding energy by B. All reactions of this form $n_N + n_M \to n_{N+M} + \gamma$ are permitted to occur, up to $N + M = 10$. The rate of these reactions is set to,

$$R[n_N + n_M \to n_{N+M} + \gamma] = NMC\sqrt{B}\exp\left\{-3\left(\frac{B}{kT}\right)^{1/3}\right\} \quad \text{(A.1)}$$

where C = 22,000. Equ.(A.1) gives the reaction rate in $s^{-1}(\text{mole/cm}^3)^{-1}$ when B is in MeV. In our universe the exponential in (A.1) would represent the Coulomb barrier which would be of increasing height for nuclei of increasing charge. In the alternative universe the potential barrier has been made the same height for all nuclei.

It is important to include a barrier term so that BBN ceases virtually completely after a few minutes or hours. The nuclei formed during BBN are then permanent features of the universe. But we wish to give the heavier elements a chance of forming during BBN and hence the barrier is made independent of atomic number. (We make no comment as to whether this would be possible by adjusting the parameters of the standard model – quite possibly not, but such is not the intention).

The universal temperature is taken to be,

$$T = \frac{10^{10}}{\sqrt{t}} \tag{A.2}$$

where T is in K and t is in seconds. The universe is assumed to contain an equilibrium number density of zero-mass, spin one particles (photons), given in mole/cm^3 by,

$$\rho_\gamma = \frac{0.2436}{A}\left(\frac{kT}{\hbar c}\right)^3 \tag{A.3}$$

where A = 6 x 10^{29} m^{-3}. The total number density of nucleons is given in terms of an assumed photon:nucleon ratio, ξ,

$$\rho_n = \frac{\rho_\gamma}{\xi} \tag{A.4}$$

The size scale of the universe varies as $R \propto \sqrt{t}$ in the radiation dominated era being assumed here, and hence the above relations are consistent with a constant number of photons and a constant number of nucleons, though the mean energies of both are reducing.

Since all reactions of the form $n_N + n_M \to n_{N+M} + \gamma$ are assumed to occur, and since we have assumed that there are no stable nuclei beyond n_{10}, there are thus 25 contributing reactions. The effect of these reactions on both particle creation and particle consumption must be addressed. The original supply of nucleons is only consumed and not replenished by any of these reactions (since we do not model photodisintegration directly). Conversely, n_{10} is only created and not consumed by any reactions. Writing the rate of a reaction $n_a + n_b \to n_{a+b} + \gamma$ as $[ab]$, an example equation - for the rate of change of the number density of n_6 nuclei - is,

$$\frac{d\rho_6}{dt} = [15]\rho_1\rho_5 + [24]\rho_2\rho_4 + [33]\rho_3\rho_3 - [16]\rho_1\rho_6 - [26]\rho_2\rho_6 - [36]\rho_3\rho_6 - [46]\rho_4\rho_6 \tag{A.5}$$

Ten equations of this form, one for each $\dot{\rho}_n$, comprise the complete reaction network. These ten equations constitute the macroscopic description of the dynamical system, together with Equs.(A.1-4). Numerical integration by time stepping from some starting time, t_s, to some finishing time, t_F, is straightforward. Hence, the final abundance of each of the nuclei is found.

It is necessary, of course, to take account of universal expansion in reducing the absolute number densities as time proceeds. This was done by multiplying all the particle densities by a factor $\left(\frac{t}{t+\delta t}\right)^{3/2}$ at the end of each time increment, δt.

It remains only to define how the starting and finishing times for the integration, t_s and t_F, are determined. The former is defined by photodisintegration, and the latter by cosmic expansion freezing-out the reactions. A full dynamical treatment would model the photodisintegration reaction rate $n_{N+M} + \gamma \to n_N + n_M$. However, at high enough temperatures there is such a numerical preponderance of photons with energies in excess of B that this photodisintegration reaction is far faster than the rate of formation of compound nuclei. At such temperatures we can assume as a working approximation that there are no compound nuclei present. The simulation of the nuclear reactions is therefore started at the earliest time that the nuclei become stable against photodisintegration. By integration of the black body spectrum, the fraction of photons with energies in excess of B is given by,

$$Photons(E > B) = 0.417\left(2 + 2x_1 + x_1^2\right)e^{-x_1} \tag{A.6}$$

where, $x_1 = B / kT_1$ and T_1 is the highest temperature for nuclear stability. This occurs when the fraction of photons with E > B is less than the nucleon:photon fraction, so that T_1 is found by setting (A.6) equal to $1/\xi$, i.e.,

$$0.417\left(2 + 2x_1 + x_1^2\right)e^{-x_1} = \frac{1}{\xi} \tag{A.7}$$

The time at which this occurs is then found from (A.2) and defines t_s. However, the net rate of production of nuclear species will be virtually zero at t_s, since nuclear stability is only marginal at that time. To account for this in a very crude fashion we arbitrarily factor the rate of nucleus formation (i.e. the ‘forward’ reaction rate) by $(t - t_s)/t_s$ for times between t_s and $2t_s$.

Reactions cease when their rate falls below the Hubble parameter. Hence, the freeze-out of the nuclear reaction $n_a + n_b \to n_{a+b} + \gamma$, whose rate is $[ab]$, is assumed in our simple treatment to occur when,

$$[ab]\sqrt{\rho_a \rho_b} \le \frac{1}{2t} \tag{A.8}$$

Integration continues until all reactions are frozen out, which defines time t_F. In cases where the freeze-out time for the first reaction, $n + n \to n_2 + \gamma$, is earlier than the time t_s at which stability against photodisintegration occurs, then no nuclear reactions will take place and there will be no nucleosynthesis at all.

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