The Fine Tuned Weak Force? - (1): The Hydrogen:Helium Ratio

Last Update: 3 July 2006

TRACK 1

We have explained in Chapter ? how the freeze-out of the lepton-nucleon reactions by cosmic expansion, together with deuterons becoming stable in the cooling universe, lead to a correct 'prediction' of the observed cosmic hydrogen:helium ratio, i.e. about 75%:25% by mass. The most sensitive feature of this calculation is the time (or temperature) at which the lepton-nucleon reactions freeze out. The weaker the weak force, the earlier this freeze-out will occur and the higher will be the temperature. A weaker weak force would therefore lead to a more nearly equal number of protons and neutrons. In the limit, if the number of protons and neutrons at freeze-out is virtually identical, the primordial universe would become virtually all helium after the first few minutes. There would be no excess protons left over. There would be no hydrogen.

This would be anthropically disastrous. An all helium universe does not preclude the possibility of stars forming, though they would be of a very different kind. There is little point in speculating on the properties of helium stars, however, since the game is, anthropically speaking, already lost. Such stars would not manufacture hydrogen where there was none to start with. The universe would therefore be without water, without hydrocarbons, without amino acids, without hydrogen bonds.

By how much is it necessary to reduce the strength of the weak force to bring this scenario about? In Track 2 the calculation is carried out. The results are given in the following graph which shows the helium percentage by mass against Fermi's constant (G_F) on a logarithmic scale.

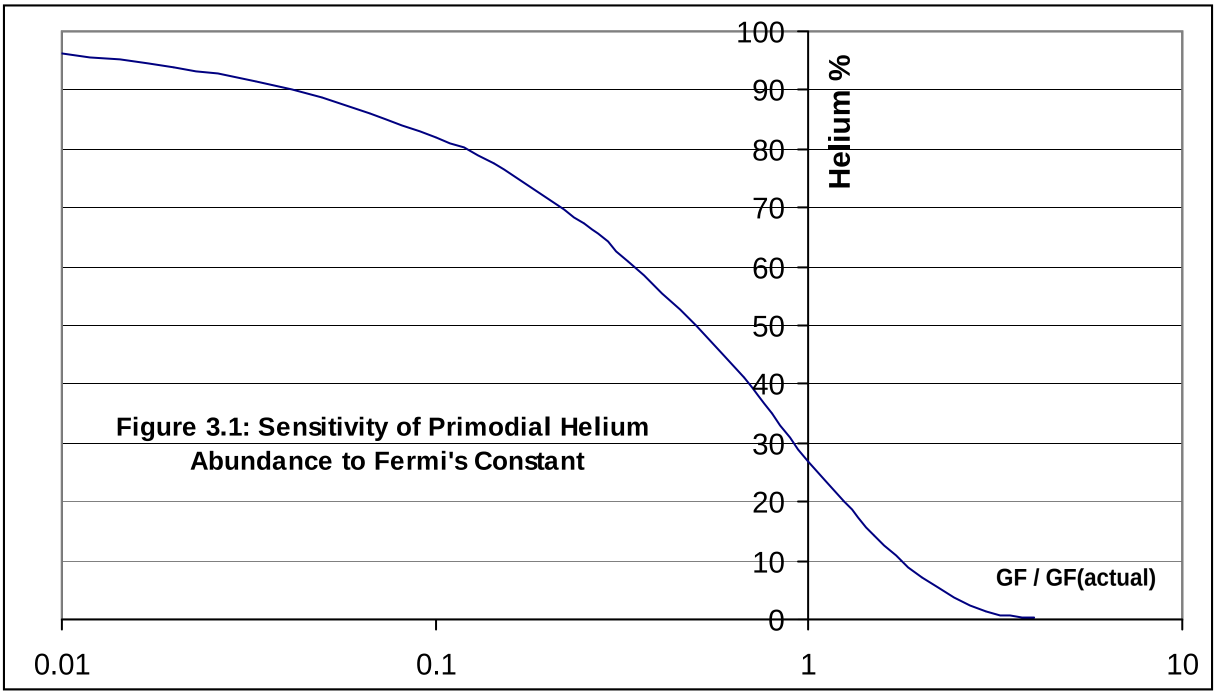

Figure 3.1: Sensitivity of Primodial Helium Abundance to Fermi's Constant

In the case of a stronger weak force it is necessary also to take account of the reduced half-life of the neutron (see Track 2).

A low hydrogen abundance would mean that the initial, hydrogen burning, phase in a star's life would be shortened in proportion. Since the life of a star is dominated by the hydrogen burning phase, it follows that stars would have lives reduced in proportion to the hydrogen abundance. If the typical stellar lifetime (say, for a solar mass star) drops below the order of a billion years it becomes questionable whether life would have time to evolve.

Having made these apocalyptic remarks, we now examine the actual sensitivity of the hydrogen abundance to $\tilde{G}_F$. Reducing $\tilde{G}_F$ by an order of magnitude results in the hydrogen abundance reducing from 75% to 18%. This is still a very substantial amount of hydrogen. It would certainly lead to a hydrogen burning phase in stars. It also provides plenty of raw material for the eventual formation of water, hydrocarbons and all the usual chemistry. The main change from our universe is that the hydrogen burning phase in stars would be briefer. Naively assuming stars of the same luminosity, their lifetime would be reduced in the ratio 18:75, i.e. to about a quarter. Thus a solar mass star would have a lifetime of around 2 or 3 billion years. Whether this is long enough for life to evolve no one knows, but it passes our suggested criterion of exceeding 1 Byrs. Hence, a factor of ten reduction in $\tilde{G}_F$ is not anthropically ruled out by the primordial hydrogen abundance.

Whether we can stretch this further to a reduction in $\tilde{G}_F$ of two orders of magnitude becomes less clear. There would then be just 4% hydrogen. Solar mass stars would live for around 500 million years. This is becoming borderline for the emergence of life to be credible, but who knows? Perhaps it is not entirely out of the question. Nor is the 'mere' 4% of hydrogen obviously insufficient for subsequent chemical needs. After all, it is about three times greater than the abundance of all the elements above helium in this universe put together.

One further issue is worth noting in the context of a primordial universe consisting of almost all helium. The formation of the first stars might be made quite problematical. It is essential for star formation that there is an effective cooling mechanism. If energy is not removed, the gas cloud cannot collapse. Within mature galaxies, mechanisms exist to enhance cooling. Galaxies act as nurseries for new stars. Knowledge of how the first stars formed is poor and speculative. It is thought that cooling mechanisms may have been provided by emission modes of molecular hydrogen. There is no equivalent for helium, since helium is monotonic. The production of molecular hydrogen is catalysed by the free electrons and protons that remain after the freeze-out of hydrogen recombination. Consequently, the cooling rate will be reduced at least in proportion to the abundance of hydrogen, and possibly as the square of the hydrogen abundance. The latter possibility arises because the residual free electron and proton densities may also be reduced. Consequently, stable, long-burning, stars might never be formed in a low-hydrogen universe due to the absence of an adequate cooling mechanism, but knowledge of these matters is too poor to be sure.

Now consider increases in $\tilde{G}_F$. This would lead to a reduced primordial helium abundance. It appears that this would not matter very much at all. It would have little effect on the formation of the first stars and little effect on their opacity. Most of the life of a main sequence star is spent burning hydrogen, and this would be somewhat more abundant in such a universe. Hydrogen burning occurs via three main nuclear reaction sequences, ppI, ppII and ppIII. The first of these, ppI, is dominant, accounting for 85% of the hydrogen burnt in a solar mass star. The ppI sequence does not require helium-4 as a reactant. Rather it is a product of the ppI sequence. In contrast, the ppII and ppIII sequences do require helium-4. In a low-helium universe, these sequences would therefore be suppressed until such a time as sufficient helium-4 had accumulated via the ppI reactions. However, this would reduce the nuclear power density (other things being equal) only by a factor of 0.85 at worst – and this would be offset by the greater hydrogen abundance. Overall there might be virtually no change in nuclear power density. The next stage in the life of a main sequence star involves helium burning. This is crucial to the eventual chemical composition of the universe because the so-called triple-alpha reaction, in which three helium-4 nuclei form carbon-12, is key to the formation of all the heavier elements. However, by this time it would appear that the star would be virtually identical to a star in this universe. This is because the helium burning phase does not start until virtually all the hydrogen in the core has been exhausted. In which case, the core will be virtually all helium by this stage in either universe. Hence, we conclude that there is no anthropic upper bound to $\tilde{G}_F$ implied by the primordial helium abundance.

Conclusion

The primordial composition of the universe does not provide an anthropic upper bound to $\tilde{G}_F$, but does provide a 'soft' lower bound to $\tilde{G}_F$. It would appear that $\tilde{G}_F$ could be an order of magnitude smaller without fatally compromising the evolution of a universe very similar to our own, and possibly two orders of magnitude might be accommodated. The most serious objection to reducing G_F by two orders of magnitude is that the formation of the first stars might be compromised due to the reduced abundance of molecular hydrogen, essential for cooling. A value of $\tilde{G}_F$ smaller than $\tilde{G}_F^0/100$ would probably not result in a universe conducive to life – at least any form of life requiring hydrogen, water, water based chemistry or hydrocarbons.

As regards the primordial hydrogen:helium ratio, the conclusion is that $\tilde{G}_F$ provides only the weakest of coincidences – classified as $\tilde{D}$ - a one-sided, order-of-magnitude, coincidence only. It barely rates as a coincidence at all.

Admittedly, the fact that $\tilde{G}_F \sim 0.887\alpha_G^{1/4}\left(\frac{M_p}{\Delta M}\right)^{3/2}$ (see Track 2) is true to within about a factor of 2 is indeed striking. But as regards an anthropic constraint on $\tilde{G}_F$, the chemical composition of the primordial universe is not a particularly impressive example.

TRACK 2

The equations from Chapter ? from which the hydrogen:helium ratio can be derived are,

Freeze-Out 1: The temperature, T_{fo1}, at which the reactions $n + \nu_e \Leftrightarrow p + e$, $n + \bar{e} \Leftrightarrow p + \bar{\nu}_e$ freeze out due to universal expansion is given by,

$$kT_{fo1} = 0.923 G_F^{-2/3} G^{1/6} \hbar^{11/6} c^{7/6} \qquad (3.1)$$

where $G_F = \tilde{G}_F \hbar^3 / M_p^2 c$, where $\tilde{G}_F = 1.03\text{x}10^{-5}$ (the actual value of Fermi's constant – in this universe!). The neutron:proton ratio at the time of this freeze out is given by,

$$N_n / N_p = \exp\{-(M_n - M_p)/kT_{fo1}\} \qquad \tilde{N}_n^{fo1} = \frac{N_n}{N_n + N_p} \qquad (3.2)$$

Freeze-Out 2: The temperature, T_{fo2}, at which deuterons become stable, and hence the cosmic neutrons becomes stabilised inside helium nuclei is given by,

$$(2 + 2x_1 + x_1^2)e^{-x_1} = \tilde{N}_n^{fo1} / (0.416 \xi_{\gamma N}) \qquad (3.3)$$

where $x_1 = B_D / kT_{fo2}$, B_D is the deuteron binding energy and $\xi_{\gamma N}$ = photon:nucleon ratio. The neutron abundance at this time is evaluated from that at the time of the nucleon-lepton reaction freeze-out from,

$$N_n(t_{fo2}) = N_n(t_{fo1}) e^{-(t_{fo2} - t_{fo1})/\tau_n} \qquad (3.4)$$

where, t_{fo1}, t_{fo2} are the times corresponding to the freeze out temperatures T_{fo1}, T_{fo2} above. Thus, $N_n(t_{fo2})$ is the cosmic neutron abundance.

These times can be found from the time-temperature relation,

$$T = \frac{0.258\phi_1}{\varepsilon} \left(\frac{c^3}{G\sigma t^2} \right)^{1/4} = \frac{1.33 \text{ x } 10^{10}}{\sqrt{t}} \qquad (3.5)$$

where $\varepsilon = 1.4$ before positron annihilation, and unity thereafter. In this universe, the positron annihilation time is 14 sec (50% remain) or 3 mins (1% remain), but see Section 3 for how this varies with the values of the fundamental constants. In this formula ϕ_1 = 0.768 is an empirical correction to improve agreement with the measured CMB temperature.

The neutron lifetime can be estimated from,

$$\tau_n \approx \frac{1}{2}\frac{192\pi^3}{G_F^2(M_{down} - M_{up})^5} \quad (3.6)$$

where M_{down}, M_{up} are the down/up quark masses. The correct lifetime, 886 sec, results if M_{down} - M_{up} is taken as 1.766 MeV. This is consistent with current quark mass estimates. Note that this mass difference is fairly close to the neutron/proton mass difference (1.2934 MeV). It is not clear why this should be the case given that ~99% of the nucleon mass is due to the quark/gluon kinetic energy. Nevertheless, to avoid introducing the quark masses as additional independent variables, we assume that the neutron lifetime can be expressed for our purposes in terms of the neutron/proton mass difference,

$$\tau_n \approx \frac{1}{2}\frac{192\pi^3}{G_F^2[\phi_2(M_n - M_p)]^5} \quad (3.7)$$

where $\phi_2 = 1.365$ is an empirical correction.

From (3.2) we see that the exponent on the RHS must be of order unity if the amounts of hydrogen and helium in the primordial universe is to be similar. If kT_{fo1} is very small compared with the mass difference $\Delta M = M_n - M_p$ then the number of neutrons will shrink to virtually nothing. This results in a primordial universe which is all hydrogen. Conversely, if kT_{fo1} is very large compared with the mass difference ΔM, then the number of neutrons and protons is the same – resulting in an all-helium universe. To obtain a universe with substantial quantities of both hydrogen and helium thus requires that kT_{fo1} be of a similar order of magnitude to ΔM. Inserting the dimensionless form of Fermi's constant in (3.1), and also substituting the dimensionless gravitational 'fine structure constant' in place of G, this results in,

$$\tilde{G}_F \sim 0.887\alpha_G^{1/4}\left(\frac{M_p}{\Delta M}\right)^{3/2} \quad (3.8)$$

The RHS evaluates to 0.48 x 10^{-5}. This is indeed in order of magnitude agreement with Fermi's (dimensionless) constant, i.e. 1.03 x 10^{-5}.

We now investigate just how sensitive is the predicted hydrogen:helium ratio to the strength of the weak nuclear force. Note that it is quite arbitrary to choose to vary $\tilde{G}_F$ whilst keeping the other constants, such as M_p, ΔM and α_G fixed. We could equal well investigate the sensitivity of the chemical composition of the primordial universe to these other parameters. It is a simple matter to repeat the calculations based on equations (3.1) to (3.7) for increased or decreased values of $\tilde{G}_F$. The result is,

$\tilde{G}_F$ **factored by…**	**% He^4**
0.01	96.1
0.02	93.9
0.033	91.4
0.05	88.7
0.067	86
0.1	82
0.143	77.4
0.25	67.5
0.33	61
0.5	50
0.67	41
0.8	35
1	27*
1.25	20
1.5	14.4
2	7
3	1.36
4	0.18

*NB: The best value for the actual primordial helium abundance is 24-25%, but we have retained 27% for consistency with the sensitivity cases.

The results are plotted in Figure 3.1. The dominant cause of the change in helium fraction is a consequence of equations (3.1,2). However, for *increases* in $\tilde{G}_F$ the change is exacerbated by the reduced neutron half-life, as given by Equ.(3.7). Note that the time at which deuterons become stable, and the neutron:proton ratio is finally frozen (t_{fo2}), is insensitive to $\tilde{G}_F$, lying in the range 125 sec to 160 sec for the range of $\tilde{G}_F$ considered above.