How Far Can You Go?

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1. The Question and the Wrong Answer

Given the finite lifespan of a human, how far can one person explore the universe? Let us say for sake of argument that the lifespan in question is the biblical 70 years and our intrepid traveller will spend the whole of this life on his journey. Let us dismiss immediately this incorrect answer: 70 light-years. If you have not learned your relativity sufficiently well, and recall only that nothing can travel faster than light, then 70 light-years would be your answer. The reason why this is wrong - and really extremely wrong, as we shall see - is time dilation. As we Earth bound observers see our astronaut disappear into the distance we also see him age increasingly slowly as he accelerates to ever greater speeds. It is true that when an observer on Earth experiences the passage of 70 years, the astronaut will have travelled rather less than 70 light-years - according to us Earth bound observers. But our astronaut still appears to be youthful and has plenty of travelling left to do after our demise, and that of our children and our children's children.

In fact there is no purely *kinematic* limit to how far the astronaut may go in his (local biological) 70 years of life. We need only subject him initially to extremely severe acceleration so that he reaches as close to the speed of light as we might wish in as small a period of time as we want. His time dilation can therefore be as extreme as we might require almost straight away - hence allowing him to reach an arbitrarily distant object before his allotted span expires. The snag is clear. Such extreme acceleration would kill our traveller. The limit on a single person's exploration of the universe is therefore not kinematic but dynamic: it results from the human frame being able to tolerate only a limited "g-force". We want to get our astronaut up to the greatest speed we can, so we want to sustain the acceleration throughout his whole journey. So we would not be talking of accelerations of 4g or 5g, or whatever it is that jet pilots might transiently experience. This level of acceleration would not be tolerable if persistent. An acceleration we know to be tolerable long term is simply g, 9.81 m/s^2. And it is reasonable to expect somewhat more than this to be tolerable also, perhaps 1.4g - since this is equivalent to being 40% over-weight. For now we shall assume a constant acceleration of g and see what it implies for the distance the astronaut might cover.

2. Reminder of Cosmic Distance Scales

Before doing that, a brief reminder of cosmic distances, working in light-years. In this unit the distance to the Sun is a trivial 0.000016 lyr (i.e., 8.3 light-minutes), whilst the distance to the outermost reaches of the solar system (defined by the heliopause) is a mere 0.002 lyr or less. In contrast the distance to either the centre or the edge of our Galaxy is ~30,000 lyrs. The size of the solar system compared with the size of the Galaxy is less than the size of a flea compared with the size of Mount Everest. If we believed the (incorrect) claim that a single person could explore at most a region of ~70 lyrs, then this would be a very tiny fraction of our Galaxy in the vicinity of the solar system. Moreover, exploring other galaxies would be entirely out of the question. We shall see that, in fact, this is not the case. Superclusters of galaxies, the largest structures in the universe which depart from statistical homogeneity, are typically of size ~ 200 Mlyrs, around 2000 times larger than our Galaxy. Finally the current distance to the edge of the observable universe is ~47 Blyrs. We shall see that even this extraordinary distance could be achieved, in principle, by a single human. However the technological difficulties are enormous.

3. Calculation of the Maximum Exploration Distance

Suppose our astronaut has travelled a distance x in a time t, both as measured from Earth. The relation,

$$x = \sqrt{\alpha^2 + (ct)^2} - \alpha \tag{1}$$

represents motion which accelerates at a constant rate with respect to the instantaneous co-moving inertial frame. This will be shown shortly. For now note that (1) gives $x = 0$ when $t = 0$ and $x \to ct$ when $ct >> \alpha$, and hence the velocity tends towards c at late times. Moreover, for $ct << \alpha$, (1) approximates to $x = \frac{c^2 t^2}{2\alpha}$ which is what we would expect in the non-relativistic limit for constant acceleration at a rate $g = c^2 / \alpha$, i.e., we can identify α with c^2 / g, where g is the constant acceleration with respect to the instantaneous co-moving inertial frame. Differentiating (1), and then again, gives the velocity and acceleration with respect to the Earth an any Earth-time, t,

$$v = \frac{gt}{\sqrt{1 + \left(\frac{gt}{c}\right)^2}} \tag{2}$$

$$a = \frac{g}{\gamma^3}, \tag{3}$$

where, $$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = \sqrt{1 + \left(\frac{gt}{c}\right)^2} \tag{4}$$

Now the period of biological time experienced by the astronaut if a time interval δt passes on Earth is,

$$\delta\tilde{t} = \frac{\delta t}{\gamma} \tag{5}$$

So the total ageing experienced by the astronaut over an Earth-time of t from the start of his journey is,

$$\tilde{t} = \int_0^t \frac{dt}{\gamma} = \int_0^t \frac{dt}{\sqrt{1 + \left(\frac{gt}{c}\right)^2}} \tag{6}$$

Carrying out the integral gives,

$$\frac{g\tilde{t}}{c} = \sinh^{-1} \frac{gt}{c} \tag{7}$$

or, equivalently, $$\frac{gt}{c} = \sinh \frac{g\tilde{t}}{c} \tag{8}$$

Substitution of the time from (8) into (1) gives the distance travelled in terms of the astronaut's own biological time, $\tilde{t}$,

$$x = \frac{c^2}{g}\left(\cosh\left(\frac{g\tilde{t}}{c}\right) - 1\right) \tag{9}$$

The maximum journey length the astronaut can make can be defined in three distinct ways,

[1] The maximum distance he can go on a one-way trip, ignoring the need to stop (i.e., to "get off") at the other end. Equ.(9) is the answer to this case. Putting $\tilde{t}$ equal to 70 years (2.2 x 10^9 s) in (9) and assuming constant acceleration at one-g, i.e., $g = 9.81$ m/s^2, gives a travel distance of 10^{31} lyrs, massively greater than the size of the observable universe.

[2] The maximum distance he can go on a one-way trip, but taking into account the need to stop in order to "get off" at the other end. In this case each half of the journey is symmetrically equivalent and so the total distance is twice the distance given by (9) in half the time, i.e.,

$$x = 2\frac{c^2}{g}\left(\cosh\left(\frac{g\tilde{t}}{2c}\right) - 1\right) \tag{10}$$

which gives a travel distance of ~4 x 10^{15} lyrs. Though radically shorter than the first case, this is still enormous compared with the current size of the observable universe (4.7 x 10^{10} lyrs).

[3] The third case takes account of the desire to make the return journey, which necessarily means decelerating to halt at the furthest point before accelerating back, and decelerating again for the last quarter of the trip. Consequently the maximum extent of his journey is just twice the distance gone in one-quarter the time, i.e.,

$$x = 2\frac{c^2}{g}\left(\cosh\left(\frac{g\tilde{t}}{4c}\right) - 1\right) \tag{11}$$

which gives a maximum distance gone from earth of 6.7 x 10^7 lyrs. The desire to make the return trip reduces the exploration distance by a massive factor of about 100 million. The returned astronaut would be able to boast only of having gone about 0.1% of the way to the edge of the currently observable universe. But suppose our astronaut can tolerate 1.4g (13.7 m/s^2). He could then travel beyond the edge of the currently observable universe ($x \sim 6$ x 10^{10} lyrs) and return to boast about it. Unfortunately the planet Earth would no longer exist having long since been consumed in the expanding red giant that the Sun will become in a few billion years from now.

Thus these examples show that it is possible in principle for a single human to get to any object in the currently observable universe, and even to make the return trip in the last example (at ~1.4g). Of course, by then the observable universe will also be larger. It will never be possible to have visited an object near the prevailing observational horizon.

4. The Technological Problem: The Propulsion Device?

These conclusions have given no thought to the achievability of a spacecraft and an engine which could provide the required thrust for the length of time necessary, not to mention the necessary provisions for 70 years! One can show quite quickly that a conventional rocket drive which works by ejecting at high speed part of the initial mass of the spacecraft (the fuel) is not feasible. Suppose the exhaust is ejected at the speed of light. The thrust is then $\dot{m}c$ where $\dot{m}$ is the rate at which mass is being ejected. This must balance with mg, so integration gives the initial total mass of the rocket, fuel, supplies and astronaut, m_0, in terms of its final mass, m_f, as,

$$m_o = m_f \exp\left\{\frac{g\tilde{t}}{c}\right\} \tag{12}$$

For a 70 year journey the exponential factor is a massive e^{72} = 2.2 x 10^{31}. The final mass must at the very least include the astronaut, and hence must be at least ~70 kg. Hence the initial mass of our expeditionary craft must be at least 1.6 x 10^{33} kg, or about 1000 times the mass of the Sun. Not a practical possibility, I think you'll agree - and one heck of an outlay for one passenger.

However, it may not be necessary to achieve forward acceleration by chucking stuff out of the back of the craft. There may be alternatives. One is to sail on stellar winds. Here the craft would capture momentum which is already present in the winds. This is probably quite feasible in the vicinity of the Sun, or sufficiently close to any star. But such winds will presumably become very weak between stars, and even weaker between galaxies. Are there great cosmic ocean currents? It's a nice thought, but probably rubbish. Nevertheless this example establishes the principle that it is not essential to carry vast amounts of mass as part of the initial cargo.