

## CONTINUUM DAMAGE MECHANICS: FROM CREEP RUPTURE UNDER HOMOGENEOUS STRESS TO CREEP CRACK GROWTH

D. R. Hayhurst<sup>†</sup>, F. Vakili-Tahami<sup>†</sup> and R. Bradford<sup>\*</sup>

<sup>†</sup> Department of Mechanical Engineering, UMIST,  
PO Box 88, Manchester, M60 1QD, UK.

<sup>\*</sup> Structural Integrity Branch, British Energy, Barnett Way,  
Barnwood, Gloucs., GL4 3RS, UK.

### ABSTRACT

A review is presented of the basic assumptions underlying Continuum Damage Mechanics (CDM) theory. It is shown how CDM may be used in computational mechanics to predict a spectrum of behaviours ranging from rupture at simple stress raisers to creep crack growth. Reference is made to a number of examples which include: stress-state effects, material ductility and creep crack growth. It is shown how damage state variable theories may be used to provide traceability from the macro material behaviour, described by constitutive equations to micro-mechanisms. *micr*

The paper outlines recent developments in the use of CDM to predict creep crack growth in a ferritic steel high-temperature welded branched pressure vessel. The estimates of creep crack growth rates are shown to compare well with those made using  $C^*$  calculational approaches.

Finally, it is argued that with the advent of high speed computer workstations, equipped with low cost large data storage capabilities, the use of finite element CDM computational mechanics, underpinned by appropriate materials databases, will become more attractive to designers than the traditional approach of design by rule or code; and, will provide the vehicle for the reduction of over conservative design margins.

*now ASME NH (1999)*

### 1. INTRODUCTION

High-temperature design in the creep range of metallic components is largely carried out using design by code methods, e.g. BS 806, 1113, 5500 [1, 2, 3], ASME Code Case N47-29 [4] or by design assessment routes, e.g. R5 [5]. However, flexibility is provided within the codes for the designer to use design by analysis methods. The latter are essential in situations where the designer operates on or beyond the boundaries of applicability. It is in these cases where new and more accurate methods of analysis have appeal.

High temperature design has always suffered from uncertainties particularly in three areas: (a) paucity and scatter in creep data, here conservatism has always enforced lower bound data to be used with a superimposed factor of ignorance; (b) the uncertainty of operational conditions, e.g. temperature, stress level and their time histories, here conservatism enforces the use of pessimistic data; and (c) the accuracy of the analysis, this arises from the geometrical and material non-linearities, the complexity of which often necessitate the use of approximate or bounding computations. Scatter in creep data is now well understood and can be controlled if adequate temperature and stress-state control are maintained; however, cost is a limiting factor. The uncertainty of operating conditions is a factor which is always likely to be present to some degree. Lastly, the accuracy of the analysis technique, although strongly

coupled with the need for accurate test data, is an area where rapid change is taking place. This has been enabled by the increasing availability of low cost, high-performance computing and data storage, and the principal theme of this paper is centered around these developments.

The theme of this paper is the use of computational Continuum Damage Mechanics (CDM) with the finite element method to predict a broad range of structural and component behaviours from simple uni-axial creep data; and, to demonstrate that the approach is capable of predicting creep crack growth rates in complex structures. Shown schemetically in Figure 1 is a range of structures of increasing complexity, starting on the left-hand side with a uniformly stressed bar, to a plane stress plate containing a hole, to a Bridgman notched bar with its throat area subjected to complex tri-axial stresses, to a plane strain double-edged notched tension bar, to a Compact Tension Specimen, CTS, and finally on the right to an axisymmetrically welded sphere-cylinder intersection subjected to internal pressure. It will be shown how reference stress techniques can be used to predict lifetimes and deformation behaviour within a CDM framework, and extended to determine creep crack growth in the three remaining structures on the right hand side of the figure. The prediction of the behaviour of the latter has been traditionally regarded as being achievable only by using the non-linear fracture mechanics parameter  $C^*$  [6, 7]. The CDM approach has the advantage of providing traceability from the constitutive equations used through the physics of

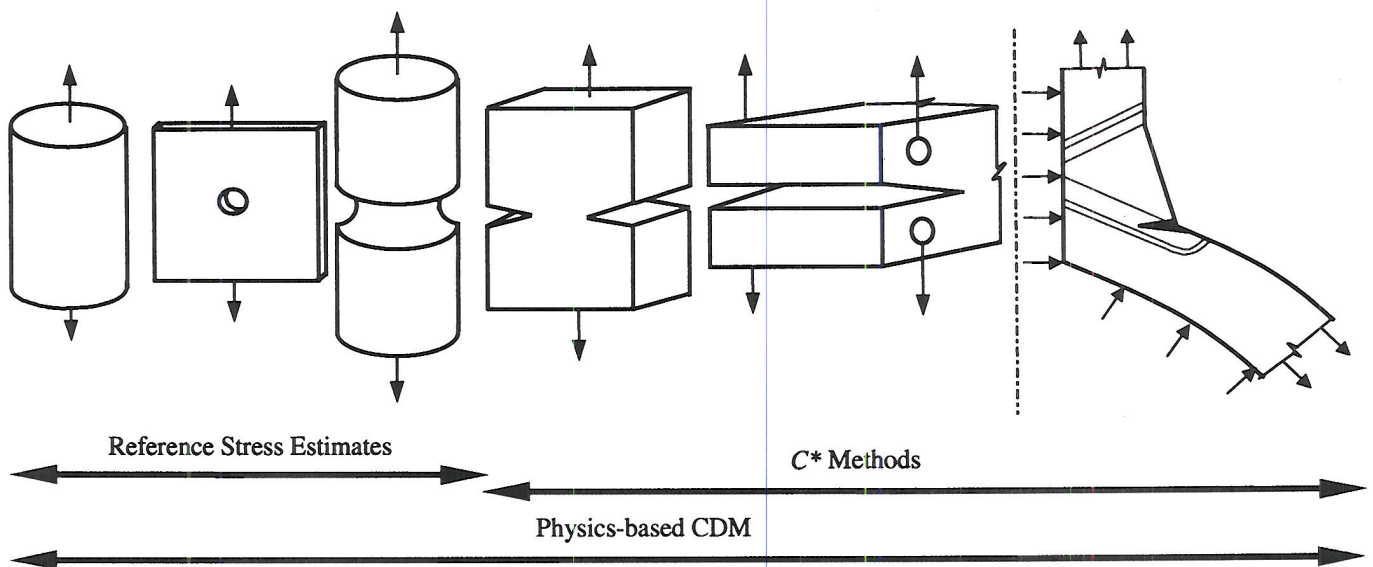


Figure 1. Schematic representation of a range of increasingly complex structures which have been analysed using CDM and C\* techniques.

the deformation, damage and fracture processes involved to the fundamental microstructural behaviour. In this way, starting from the physics of the microstructural processes it is not necessary to specify the type of computation required for each specific category of behaviour, instead all that need be adhered to is the rigour of the generic approach. Emphasis is therefore placed upon the identification of a physical mechanism and upon access to experimental data which both characterises and quantifies the strengths of these mechanisms. The advantage of the approach is that extrapolation from short term to long term behaviour is more reliable, since it is founded upon an accurate description of the physics of all mechanisms involved.

The paper commences with an outline of CDM. This is followed by an overview of the generic approach to design provided by computational CDM using the finite element technique. A review is given of how beneficial stress redistribution can be achieved by permitting widespread continuum damage to evolve; and, of how the multi-axial stress rupture criteria of the material influences component behaviour. It is then shown how the CDM approach can be used, without modification, to predict creep crack growth; and it is also shown how the characteristics of the material ductility may be changed to produce failure of engineering structures by either a widespread growth of damage, or by the growth of a highly localised region of damage to yield creep crack growth.

## 2. WHAT IS CDM?

Hayhurst [8] has shown that macrocracks form in creep rupture from relatively uniform fields of grain

boundary microcracks. What is evident is that a single crack does not predominate and propagate across the section. Hayhurst [9] has shown from this and other studies that provided the stress field is homogeneous then a field of damage nucleates and grows in a uniform way over the same region. The strain rate behaviour and the damage evolution rate behaviour may be described by the following equations:

$$\begin{aligned}\dot{\epsilon}_{ij} &= f(\sigma_{ij}, \omega_1, \omega_2), \quad \dot{\omega}_1 = g(\sigma_{ij}, \omega_1, \omega_2) \\ \dot{\omega}_2 &= h(\sigma_{ij}, \omega_1, \omega_2),\end{aligned}\quad (1)$$

where  $\dot{\omega}_1$  and  $\dot{\omega}_2$  are rates of change of the damage state variables. Each mechanism of damage operates over a given domain of temperature, stress level and stress state. Cocks and Ashby [10] have shown that the micromechanisms can be modelled in this way and that the equations which describe the basic mechanism can be rewritten in the above form to provide an accurate global description. It is this important step that enables one to achieve traceability from the global, or macro scale behaviour of the material back to the behaviour of the material at the micro scale. Guidance on the domains of temperature and stress level over which the mechanisms operate may be obtained from the mechanisms maps of Ashby [11], and on stress state dependence from the work of Cocks and Ashby [10].

Although in Eq. (1) only two damage state variables have been included, in practice there may be more. These could include the following: cavity nucleation and growth; ductile void growth; multiplication of dislocation substructures; and precipitate coarsening [12, 13, 14]. In the work discussed herein, constitutive equations will be used to model most of these mechanisms.

In the next section it will be shown how this background can be used to form a systematic approach to design analysis.

### 3. CDM APPROACH TO DESIGN ANALYSIS

Over the last two and a half decades a number of papers [15-20] have reported contributions to the establishment of Computational Continuum Damage Mechanics as a route to high-temperature design analysis. The design/analysis procedure is set out in the list below. Identify for the structure to be designed:

- (a) Domains of temperature.
- (b) Ranges of stress.
- (c) Stress states.

Then carry out the following:

- (d) Make a preliminary material selection using either Ashby's [10] material selection procedures for conceptual design; or available materials creep data bases.
- (e) Access available and relevant databanks of creep curves at the appropriate temperatures, stresses and stress states.
- (f) Check Ashby's mechanisms maps [11] to provide a steer on the mechanisms of deformation and rupture, in order to identify the appropriate mathematical models.
- (g) Use numerical techniques [21] to fit the appropriate models to available creep data.
- (h) Use the models in conjunction with computational CDM finite element based techniques.
- (i) Determine limiting design factors such as global and local strain histories, damage field evolution, macro crack initiation and failure lifetimes.

In what follows a compendium of design cases and studies is presented to illustrate the power of the above design analysis approach.

### 4. CDM: PREDICTIONS OF STRUCTURAL BEHAVIOUR

The establishment of the design approach set out in Section 3 will now be traced and some of the important results will be illustrated.

#### 4.1 Stress Redistribution due to CDM

Shown in Figure 2 is a mid-thickness micrograph of a region of rectangular cross-section copper beam tested almost to failure at 250°C under a constant bending moment.

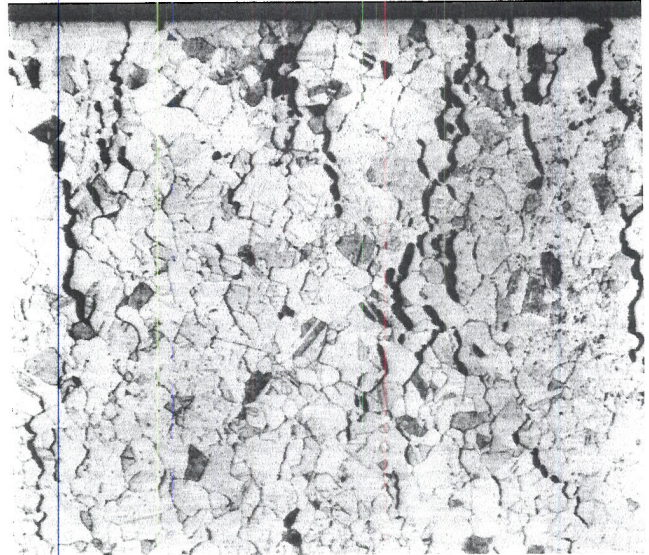


Figure 2. Micrograph of a section of a copper beam subjected to a constant bending moment and tested almost to failure at 250°C.

The region is close to the tension surface (top of figure) and shows two things: firstly, that damage density reduces as one moves down the figure in the direction of decreasing strain; and secondly, as one moves from the left hand side of the figure to the right the damage levels do not vary dramatically. This figure reveals the field property of damage, i.e. where the stress and strain fields vary, then so does the damage field.

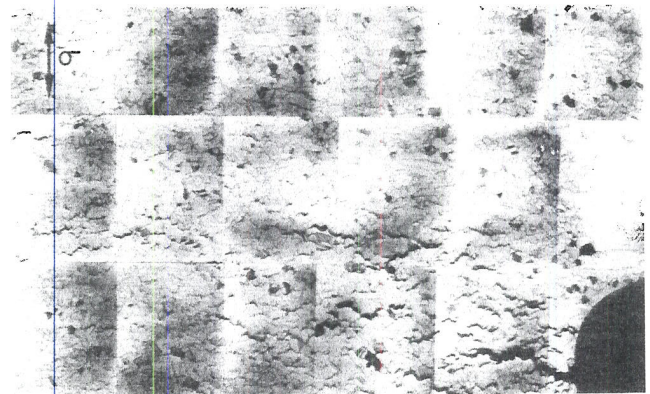


Figure 3. Micrograph of a section of a part quarter-section of a copper tension panel, tested at 250°C, containing a central circular hole. The test was stopped just before failure.

Shown in Figure 3 is a mid-thickness micrograph of a part of a quarter-section of a copper tension panel tested at 250°C immediately prior to failure. The plate contained a central circular hole, part of which is shown in the bottom right-hand corner of the figure. The left-hand boundary of