International Journal of Fracture 25 (1984) R7–R9

J-ESTIMATION FORMULAS FOR TOUGHNESS MEASUREMENTS ON STRAIN HARDENING MATERIALS

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In deriving toughness values from tests in the post-yield regime, use is often made of J estimation formulas which correlate J with the area U under the load-displacement curve, e.g.,

$$J = \eta U/A \tag{1}$$

where A is the uncracked ligament area and η is a dimensionless factor. In general, the assumptions of elastic or perfectly plastic behaviour will give different values for η. To account for this fact, a common procedure is to split the area U, in some way, into an "elastic" and a "plastic" part, $U = U_e + U_p$. The two different η factors are then incorporated into the estimate by putting, for example,

$$J = (\eta_e U_e + \eta_p U_p)/A \tag{2}$$

In addition, U_e may be defined by having a "blank" (uncracked) energy subtracted from it. Eqn. (2) is essentially just an interpolation between the elastic and extreme plastic regimes. In this report we suggest an alternative technique of interpolation which involves the use of the empirical load-displacement curve to derive η in (1) as a function of the applied displacement. This smoother interpolation is less arbitrary than that of (2) and may be important for strongly strain hardening materials. In addition, it is probably better suited to empirical derivations of J at loads less than those required to cause crack growth, e.g., for use in deducing "assessment" diagrams. The method is described below.

Firstly, the experimental load-displacement curve is approximated by the form

$$\begin{aligned} F &= SD & D &< D_L \\ F &= F_M[1 - (1 - F_L/F_M)(D_L/D)^n] & D &> D_L \end{aligned} \tag{3}$$

where F = load, D = displacement, and S = elastic stiffness. The parameters F_M, F_L, and n are chosen so as to fit the observed curve at the crack length of interest (D_L is defined by F_L/S). The fitted curve deviates from linearity at displacement D_L and tends asymptotically to a maximum load of F_M as $D \to \infty$.

Secondly, we assume that the maximum load can be written

$$F_M = CB(W - a)^{\alpha} \tag{4}$$

where W - a = the uncracked ligament length; B = the uncracked ligament thickness; and C, α = some constants.

Lastly, we assume that the ratio F_M/F_L, as well as the parameters n, α, and C, are independent of the crack length a. Identifying J with the energy release rate,

$$J = -(1/B)\ \partial U/\partial a \tag{5}$$

(the derivative being carried out at constant D), and noting that this, with (1), leads to

$$dS/da = -\eta_e\ S/(W - a) \tag{6}$$

in the LEFM regime, it is a matter of simple algebra to derive η from (1) for $D > D_L$, as

$$\eta = \frac{X_1 + X_2}{X_3 - X_4}$$

where

$$X_1 = (F_M - \frac{F_L}{2})(\eta_e - 2\alpha) + F_M/F_L\ S\alpha D$$

$$X_2 = \frac{(F_M/F_L) - 1}{1 - n}\left\{[n\eta_e - (n+1)\alpha]F_L^{\ n}(SD)^{1-n} + (2\alpha - \eta_e)\ F_L\right\} \tag{7}$$

$$X_3 = \frac{F_L}{2} - F_M + F_M/F_L\ SD$$

$$X_4 = \frac{(F_M/F_L) - 1}{1 - n}\left\{F_L^{\ n}(SD)^{1-n} - F_L\right\}$$

Eqns.(7) thus give η as a function of the displacement D in terms of the shape of the observed load-displacement curve as represented by the parameters F_M, F_L, and n, and also the elastic stiffness S. Putting $D = D_L$ in (7) gives $\eta = \eta_e$, as it should, and similarly letting $D \to \infty$ gives $\eta \to \alpha$, which is the usual value adopted for η_p in (2). For intermediate displacements, (7) interpolates η between these extremes. The alternative J estimation scheme proposed here will therefore be of most relevance when η_e and $\alpha(= \eta_p)$ are substantially different. The author has found this to be the case for some mode II (shear) specimens but, not, for example, for compact tension specimens.

In practice, η_e is determined from an LEFM stress analysis (either via dS/da or via K), whereas α is determined from the limit behaviour. It should be emphasised that η_e is NOT the same as the eta factor occurring in, say, (2) when subtraction of the blank energy is included in the definition of U_e, since no subtraction has been used in (1). Care should be taken to remove any extraneous displacements from the load-displacement curve before fitting to the form of (3). Preferably, agreement with a theoretical (LEFM) value of S should be obtain... Also, we note that the area U entering (1) should be the area under the empirical curve rather than the fit, the latter being used only to determine η. Finally in (7) we have assumed $n \neq 1$. For n = 1, integration gives terms in log D in place of $D^{1-n}/1-n$.

Acknowledgement: The author is grateful to the Director General of the Central Electricity Generating Board, South Western Region, for permission to publish this paper.

19 January 1984