The Effect of Hypothetical Diproton Stability on the Universe

R. A. W. Bradford
Bennerley, 1 Merlin Haven, Wotton-under-Edge, Gloucestershire, England, GL12 7BA
email: RickatMerlinHaven@hotmail.com
12 pages (single spaced); 2 Figures; 3 Tables.

Abstract. By calculation of the proton-proton capture cross section it is shown that the existence of a bound diproton state would not lead to significant production of diprotons during big bang nucleosynthesis, contrary to popular belief. In typical stellar interiors, the stability of diprotons would lead to a reaction pathway for converting protons to deuterons perhaps $\sim 10^{10}$ times faster than the usual weak capture reaction. This would prevent stars of the familiar hot, dense type from occurring in the universe. However, if diproton stability is achieved by an increase in the low-energy strong coupling, g_s, then stars with temperatures and densities sufficiently reduced so as to offset the faster reaction pathway to deuterium appear to meet elementary stability criteria. The claim that there is a fine-tuned, anthropic upper bound to the strong force which ensures diproton instability therefore appears to be unfounded.

Keywords: big bang nucleosynthesis, star formation, anthropic cosmology

1. Introduction

The standard models of particle physics and cosmology involve around 31 dimensionless universal constants (Tegmark *et al* 2006). It has long been the dream of physicists to derive the numerical values of these constants from underlying mathematical principles. The early hopes that string theory might achieve this goal have been dashed by the plethora of possible string theories which are now known to exist (Susskind 2003). Nevertheless, many of the universal constants cannot be varied greatly from their actual values without fatally compromising the production of a universe within which highly complex structures, including life, can evolve (Dyson 1971; Carter 1974; Carr & Rees 1979; Davies 1982; Barrow & Tipler 1986). This has led to a resurgence of interest in the anthropic constraint, expressed in recent years in terms of our location on the string landscape (Susskind 2003; Dine 2003; Davies 2004; Hogan 2006; Linde 2007a). The anthropic perspective fits well with the naturally occurring multiverse scenarios envisaged in eternal inflation cosmologies (Davies 2004; Linde 1994; Linde 2007b; Guth 2007). Some success can be claimed in regard to specific applications of anthropic selection, including calculations of: constraints on the magnitude of the cosmological constant, consistent with structure formation (Martel, Shapiro & Weinberg 1998; Weinberg 2007); and calculations of constraints on dark matter density, consistent with solar system stability (Tegmark *et al* 2006).

The application of anthropic reasoning works by considering the implications of varying a chosen universal constant, or constants, and determining what magnitude of variation has catastrophic consequences for some key feature of the universe. An example is some change which would result in a universe without hydrogen. Whatever other elements may exist, the absence of hydrogen would be sufficient to radically alter the available chemistry. There would be no water, no hydrocarbons, no proteins, and no hydrogen bonds. There would be no life as we know it. Whether any form of life could exist without hydrogen is unknown. But, from the point of view of the multiverse, any such universes are de-selected as candidates for our universe by our own water-and-protein based existence.

The challenge implicit in such anthropic reasoning is that we must be confident in our ability to calculate reliably the consequences of specified changes in the magnitudes of the universal constants. By definition, we are required to assess the properties of a universe that is not our own. This is intrinsically hazardous. Unlike real-world physics, we no longer possess the advantage of having the answer at the end of the book (that is, at the end of an experiment or observation). Placing reliance purely upon theory which is unverifiable even in principle must be treated with the greatest circumspection. This paper addresses a salutary example, namely the much quoted implications of diproton stability.

Dyson (1971), Davies (1972), Carr& Rees (1979), Davies (1982), Barrow & Tipler (1986), Rees (1999), Davies (2004) and Tegmark *et al* (2006) all state that diproton stability would lead to a universe devoid of hydrogen, since all the hydrogen would be burnt to helium, via diprotons, during big bang nucleosynthesis (BBN). We claim that this is untrue. This scenario would be realised only if the proton-proton capture reaction were sufficiently fast to ensure that virtually all the free protons were captured before the diminishing temperature and density in the minutes following the big bang led to the reaction being frozen-out. We shall show that, on the contrary, the pp capture reaction is not sufficiently fast.

This oft-quoted "diproton disaster" appears to have been based on a false analogy with neutron-proton capture to form deuterium during BBN. In this paper we shall show that the proton-proton capture reaction rate is suppressed with respect to proton-neutron capture as a consequence of the former involving identical fermions. Such a universe therefore remains unaffected by diproton stability until the first stars form.

There are at least two ways in which diproton stability can be contrived. One is by an increase in the strength of the strong force (g_s), as envisaged in (Dyson 1971, Davies 1972, Carr& Rees 1979, Davies 1982, Barrow & Tipler 1986, Rees 1999 and Davies 2004). The other is by a decrease in the Higgs vacuum expectation value (v), and consequently a reduction in the quark masses and a reduction in the pion mass, and thus an increase in strong binding as a result of the increased range of the nuclear force (Tegmark *et al* 2006; Hogan 2000; Agrawal *et al* 1998a; Agrawal *et al* 1998b). In both cases the percentage change required (in g_s or v) is quite modest. Either scenario is equivalent as regards what happens during BBN, provided they correspond to the same diproton binding energy (B). However, they differ as regards their implications for stars.

If a sufficiently large change in the Higgs vacuum expectation value, v, is considered, then there are profound effects on the cooling mechanisms for star formation and the heat transfer properties of the stellar medium, due to the changes in lepton masses. This has been discussed in (Agrawal *et al* 1998a). Here we shall consider the alternative: that diproton stability has resulted from an increase in the low-energy effective strong coupling constant, g_s. This is a simpler scenario for understanding stars in that only the nuclear heating is affected. The heat transfer properties, which depend upon the particle masses and electromagnetic interactions, are unchanged. We shall show that elementary stability criteria can be met by 'biophilic' stars in such a universe; that is, by stars with lifetimes of the order of billions of years, and with luminosities and surface temperatures appropriate for the nurturing of biological life based on conventional molecular chemistry.

There is a danger that this exercise might be regarded as falling into the same trap as the false claims of a "diproton disaster". The trap is one of hubris. One needs to bear in mind that the notional variation of a universal constant may be intrinsically nonsensical. This would be the case if, after all, the numerical magnitudes of the universal constants are fixed by pure mathematics. But even if different values of the universal constants do make sense, and perhaps are actually realised within some multiverse, we fall victim to the sin of Prometheus in imagining that our understanding is sufficient to embrace its full implications. But this exceeds our far more modest objective. We claim only to shift the burden of proof back to anyone claiming that diproton stability is anthropically de-selected.

2. The proton-proton capture cross-section

In what follows we assume g_s to have been increased sufficiently for the diproton to be bound. (Note that this g_s is the old-fashioned, low-energy effective strong coupling, as opposed from the running coupling of QCD determined perturbatively at higher energies). For definiteness we shall consider increases in g_s of 20%, 30% and 40%. The proton-proton capture cross section (σ_{pp}^{cap}) is smaller than that for neutron-proton capture (σ_{np}^{cap}) for three reasons:-

1) Most obviously, the Coulomb barrier reduces σ_{pp}^{cap}. This is a small effect at BBN temperatures (~10^9K), but accounts for several orders of magnitude reduction in the corresponding reaction rate at, say, central solar temperatures (~14 x 10^6K).
2) At the non-relativistic energies of interest (<0.1 MeV), the neutron-proton capture cross section can be estimated simply from Schrödinger matrix elements, see for example (Blatt & Weisskopf 1952; Evans 1955). The dominant contribution arises from the magnetic dipole term, i.e. the coupling between the nuclear spins and the magnetic component of the electromagnetic field. However, this matrix element is proportional to the difference between the magnetic dipole moments of the two incident particles, and is therefore zero for identical particles, e.g. for pp capture.
3) The second order term contributing to neutron-proton capture is the electric dipole interaction, i.e. the coupling between the charge and the electric component of the electromagnetic field. (The electric dipole cross-section is about an order of magnitude smaller than the dominant magnetic dipole cross-section at ~10^9K, and about three orders of magnitude smaller at ~10^7K). Because the deuteron is a spin triplet (^{3}S), and because the electric dipole interaction Hamiltonian (H_I^D) is proportional to r.cosθ and does not affect the spin, the only non-zero matrix element is for an initial spin-triplet P-wave state, i.e. $< {}^3S(\text{bound}) \left| H_I^D \right| {}^3P(\text{free}) >$. In contrast, the diproton is a spin singlet state (^{1}S), as required by the exclusion principle. The relative weakness of the singlet nuclear force is the reason why, in this universe, the diproton does not exist. The electric dipole matrix element for proton-proton capture would be $< {}^1S(\text{bound}) \left| H_I^D \right| {}^1P(\text{free}) >$, but of course the singlet P-state cannot exist for identical fermions. Hence, the second order term contributing to σ_{np}^{cap} is also zero for σ_{pp}^{cap}.

We conclude that the lowest order non-zero term contributing to σ_{pp}^{cap} must be the electric quadrupole term, i.e. $< {}^1S(\text{bound}) \left| H_I^Q \right| {}^1D(\text{free}) >$, where, $H_I^Q \propto r^2 P_2(\cos\theta)$. Thus, it is clear that $\sigma_{pp}^{cap} << \sigma_{np}^{cap}$.

Standard methods, as in Blatt & Weisskopf (1952) and Evans (1955), may be used to derive an analytic expression for the cross section if the Coulomb interaction is ignored, and in the zero-range approximation,

$$\sigma_{pp}^{cap} = \frac{64\pi\alpha(\hbar c)^2 B^{1/2} E^{3/2}}{15\left(M_p c^2\right)^3 (E+B)} \qquad (\textit{no Coulomb barrier}) \qquad (1)$$

where B is the postulated diproton binding energy and E is the sum of the two incident protons' kinetic energies in the centre-of-mass system. When the Coulomb barrier is included, the cross section diminishes at sufficiently low energies proportionally to,

$$\sigma_{pp}^{cap} \propto \exp\left\{-\pi\alpha\sqrt{M_p c^2 / E}\right\} \quad (\textit{E} \sim \textit{1 keV or smaller}) \qquad (2)$$

in the usual way. We have chosen to evaluate numerically the Schrödinger wavefunctions including the Coulomb potential, using a nominal singlet nuclear potential 'square well' with $a = 2.4$ fm and $V_0 = 16.1$ MeV (Blatt & Weisskopf 1952; Evans 1955) noting that great accuracy is not necessary. The potential well depth is increased proportionally as g_s^2, i.e., by factors of 1.2^2, 1.3^2 or 1.4^2, to derive the cross section for diproton formation. (These correspond to diproton binding energies of 0.6, 2 and 4 MeV respectively). The numerical results have the low energy behaviour required by equation (2) and reproduce equation (1) when the Coulomb interaction is removed. The cross section (in barns) is plotted against E in Figure 1, and as a fraction of the neutron-proton capture cross section in Figure 2.

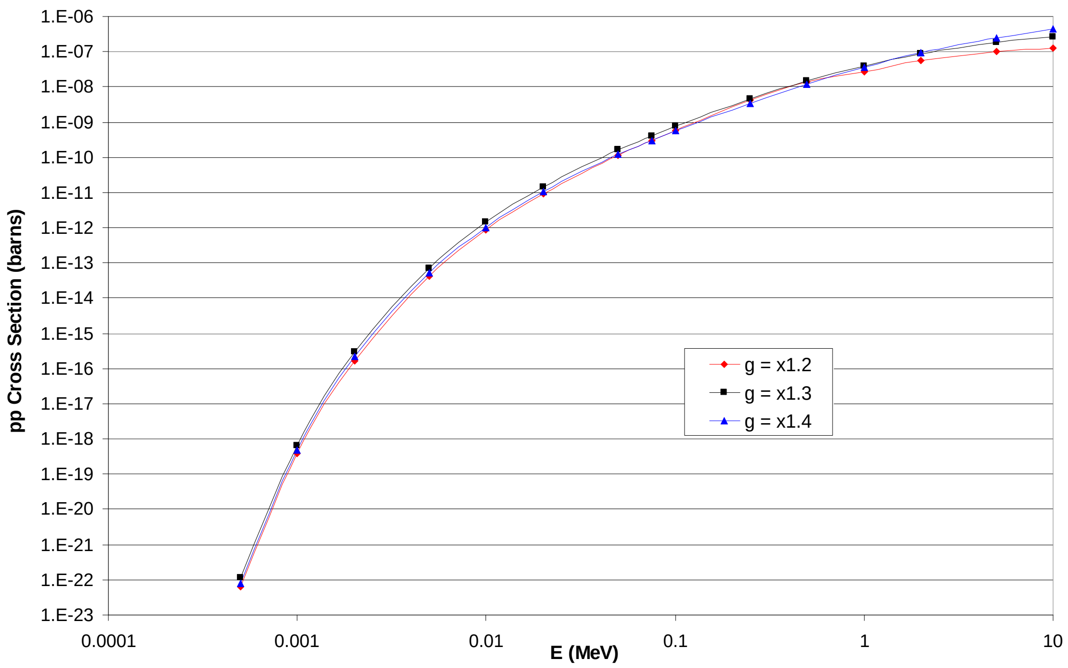

Figure 1. Numerical results for the pp capture cross-section (barns) versus energy.

At ~0.1MeV, the proton-proton capture cross section is about 5 orders of magnitude smaller than the neutron-proton capture cross section. At ~1keV the difference is about 15 orders of magnitude. The latter is due largely to the Coulomb barrier. However, the smaller proton-proton capture cross-section compared with that of neutron-proton capture at ~0.1MeV, a typical BBN energy, is mostly due to the fact that the former involves identical particles, and hence a quadrupole interaction rather than a dipole interaction.

A reasonably good closed-form approximation to the numerical cross-section results (within an order of magnitude for 0.5keV < E < 5MeV) is,

$$\sigma_{pp}^{cap} = \frac{64\pi\alpha(\hbar c)^2 B^{3/2} E^{1/2}}{15(M_p c^2)^3 (E+B)} \exp\left\{-\pi\alpha\sqrt{M_p c^2 / E}\right\} \tag{3}$$

Note that equation (3) differs from the simple product of equations (1) and (2) by an additional factor of B/E, motivated simply to improve agreement with the numerical results shown in Figure 1. Equation (3) tends to over-estimate the cross-section in the energy range of interest, and hence any inaccuracy does not detract from the arguments which follow.

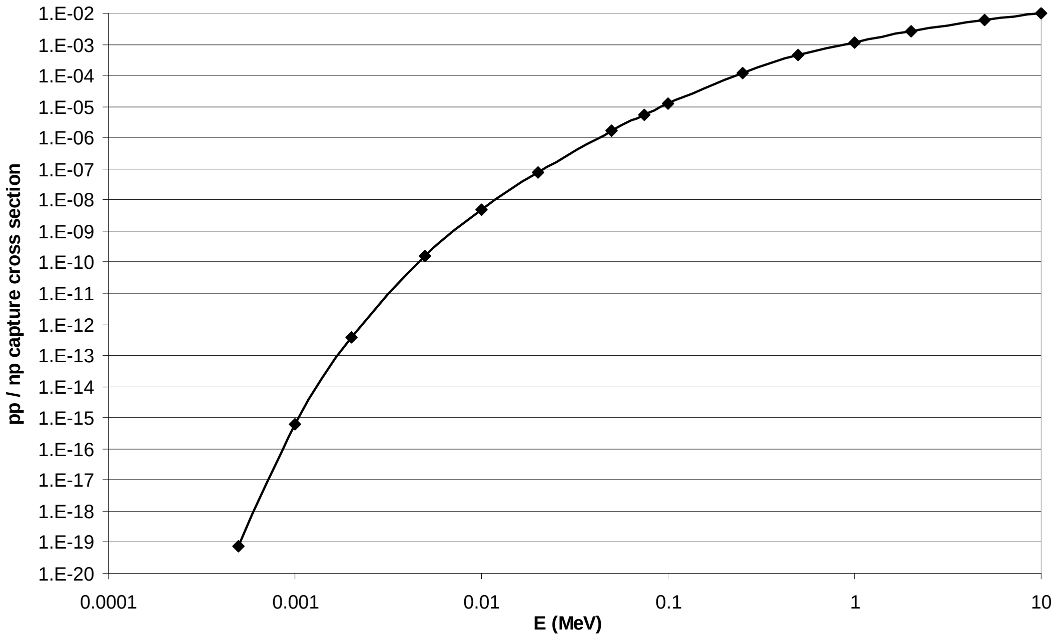

Figure 2. Numerical results for the pp capture cross-section as a fraction of the pn capture cross-section versus energy.

The overall reaction rate for a thermal distribution of proton energies is found by integrating the monochromatic rate, determined from equation (3), appropriately weighted by the Maxwell distribution. Using a first order approximation for the resulting 'Gamow peak' integral yields a reaction rate at temperature T given by,

$$R[T] \approx \frac{8 \cdot 2^{2/3}}{15} \pi b^{7/3} \alpha A \left(\frac{\hbar}{M_p c} \right)^2 \left(\frac{c}{M_p c^2} \right) \cdot \sqrt{\frac{B}{M_p c^2}} \cdot \frac{\exp\{-f_{min}\}}{(kT)^{1/6}} \tag{4}$$

where, $$f_{\min} = 3\left(\frac{b}{2\sqrt{kT}} \right)^{2/3} \quad \text{and,} \quad b = \pi\alpha \cdot \sqrt{M_p c^2} \tag{5}$$

and where E << B is assumed. A is the proton number density corresponding to one mole/cm^3 (i.e., 6 x 10^{29} /m^3). Equation (4) gives the reaction rate in s^{-1}(mole/cm^3)$^{-1}$ for kT in MeV. Using B = 2 MeV for illustration (i.e. for a 30% increase in g_s), we deduce an approximate reaction rate at temperature T,

$$R[T] = 3.5 \frac{\exp(-f_{min})}{[kT(\text{MeV})]^{1/6}} \ \text{s}^{-1}(\text{mole/cm}^3)^{-1} \tag{6}$$

Again, any inaccuracies due to the approximations inherent in the derivations of Equs.(4,6) lead to an *over*-estimate of the reaction rate, and hence do not detract from the arguments which follow.

3. Are diprotons formed during BBN?

It is convenient to express results in terms of time (t), taken as correlated with temperature according to $T(K) = 10^{10} / \sqrt{t(\text{sec})}$. The photon-baryon ratio is taken to be 2 x 10^9. The diproton reaction times over the first hour following the big bang, derived from equation (6), are given in Table 1.

Table 1. Diproton formation reaction times.

t (s)	kT (MeV)	Reaction rate $s^{-1}(mole/cm^3)^{-1}$	proton density $/m^3$	Reaction time (s)	Reaction time / t
1	0.86	0.75	7.5E+27	100	100
10	0.27	0.43	2.3E+26	6,000	600
30	0.16	0.30	4.9E+25	41,000	1,400
50	0.12	0.24	2.1E+25	1.2E+5	2,400
100	0.086	0.18	7.5E+24	4E+5	4,500
200	0.061	0.13	2.7E+24	2E+6	9,000
300	0.050	0.10	1.5E+24	4E+6	13,000
500	0.039	0.074	7.0E+23	1E+7	24,000
1000	0.027	0.044	2.3E+23	6E+7	60,000
2000	0.0193	0.026	8.5E+22	3E+8	140,000
3000	0.0157	0.018	4.6E+22	7E+8	240,000
5000	0.0122	0.011	2.2E+22	2E+9	500,000

The condition for reaction freeze-out by cosmic expansion is that the reaction time exceed 1/H ~ 2t, i.e. that the last column in Table1 should exceed ~2. Hence we see that the diproton formation reaction is frozen out at all times after ~1 second, and indeed somewhat before that. The situation contrasts with that for neutron-proton capture. Consistent with actual big bang nucleosynthesis, Figure 2 implies that the reaction times for the latter are shorter than t during this period, as they must be.

To complete the argument that diprotons would not be a product of big bang nucleosynthesis we now demonstrate that diprotons would photodisintegrate prior to 1 second. All neutrons will be assumed to have combined as deuterons before the time at which diprotons become stable against photodisintegration. (The increased strength of the nuclear force will increase the binding energy of the deuteron, which will thus always be stable at higher temperatures than the diproton or the dineutron). The maximum possible diproton to photon ratio is thus $0.75 / (2 \times 2 \times 10^9) = 2 \times 10^{-10}$, noting that the proportion of remnant protons (~75%) is not affected by the change in g_s.

The simplest estimate of the temperature at which diprotons will be stable against photodisintegration is obtained by equating the maximum possible diproton:photon ratio to the fraction of photons with energies sufficient to cause photodisintegration, i.e. greater than B. This fraction is, from the blackbody photon spectrum,

$$0.416\int_{x_1}^{\infty}\frac{x^2dx}{e^x-1} \approx 0.416\left[2+2x_1+x_1^2\right]e^{-x_1} \quad \text{where,} \quad x_1 = B/kT >> 1 \qquad (7)$$

Thus we find $x_1 = 28.22$ and the earliest times at which diprotons are stable are as given in Table2.

Table 2. Times and temperatures for diproton stability for various g/g_{actual}

g / g_{actual}	B (MeV)	T (K)	t (s)
1.2	0.6	2.5×10^8	1,600
1.3	2	8.2×10^8	150
1.4	4	1.65×10^9	37

Thus, diprotons do not become stable until well after ~1 second for increases in g_s of up to a factor of ~1.4. The proton-proton capture reaction is therefore frozen out before diprotons become stable. In other words, proton-proton capture freezes-out when the diproton density is still negligible. There would therefore be no large scale production of diprotons during BBN. The existence of a bound diproton state would not affect the chemical constitution of the primordial universe, which would remain about 75%:25% hydrogen:helium.

In passing we note that if g_s were increased by more than a factor of ~1.4, then deuterons would be stable before ~1 second. At this time the leptonic reactions which interconvert neutrons and protons would still be active. The neutron:proton ratio is determined by thermodynamic equilibrium at such times. Hence, a substantial suppression in the hydrogen inventory of the universe would result if g_s were increased sufficiently to stabilise the deuteron at a small fraction of a second. For example, increasing g_s by a factor of ~4 would result in a universe with only ~3% hydrogen. However, the diproton is irrelevant in this scenario.

4. What effect does diproton stability have on stars?

It would appear that the stability of the diproton makes no difference to the primordial universe, which would retain its approximately 75%:25% ${}^1_1\mathrm{H} : {}^4_2\mathrm{He}$ constitution (by mass). However, the effect on star formation would obviously be dramatic. For example, at a solar central temperature of ~15 x 10^6 K, the pp capture reaction rate would be about 10^{-5} $s^{-1}(\mathrm{mole/cm}^3)^{-1}$, and a solar central proton density of 4 x 10^{31} m^{-3} would lead to a pp reaction time of less than an hour. Thus, the rate of production of deuterons would be controlled by the rate of the weak decay of the diproton. Even if this were the order of a year, the overall deuteron production rate would be ~10^{10} times faster than the usual weak capture reaction $p + p \rightarrow D + e^+ + \nu_e$ under the same conditions.

However, the diproton reaction is so rapid under solar temperature and density conditions that it would be explosive. Stars of this type could not form. It is clear, therefore, that the universe would be radically different from the actual universe once star formation started. What is not clear is whether stable, long lived, stars would form with temperatures and densities suitably reduced so as to offset the faster reaction pathway to deuterium. We do not pretend to provide a definitive answer to this question here. However, the answer appears to be less clear cut than is often implied. In particular, the fact that the first, and rate controlling, nuclear reaction is intrinsically faster – even if it be 10^{10} times faster - does not preclude the possibility of stable, long lived stars. The reason is that nuclear fusion reaction rates between charged reactants are exponentially sensitive to temperature. Consequently, even enormous intrinsic reaction rate differences can be tamed by relatively modest changes in core temperature. Thus, it is possible to have a strong-force mediated fusion reaction as the rate controlling step in stellar heat production, as was also envisaged by Harnik, Kribs & Perez (2006). To see this, consider an hypothetical star with a central temperature of 10^6 K in a universe with stable diprotons.

There are several elementary constraints which a stable star must respect. For example, dynamical stability requires that the radiation pressure within the star should not be too much larger than the gas pressure. This constraint leads to the familiar upper bound on stellar masses, in the order of ~100 solar masses, a limit which will also apply in our alternative universe. This constraint can also be written as a lower bound on the gas density required for stability, namely $\rho > 0.1 M_p (kT/\hbar c)^3$, which is 0.015 kg/$m^3$ for our example. (M_p is the proton mass).

The star must also be able to transport heat efficiently enough to balance the rate of nuclear heat generated. At the centre of the star there is a maximum power density consistent with purely radiative heat transfer, i.e., $\varepsilon_v < \varepsilon_v^{max} = 4\pi c G\rho / \kappa$, where κ is the opacity (the subscript $_v$ denotes power per unit volume). Since the power density depends upon the square of the proton density, this limit on power density results in an upper limit on proton density. It evaluates to about 2.6 kg/m^3 for our example. This is based on the diproton reaction rate from equation (6) together with the reaction sequence given in the Appendix. Of significance is the fact that at 10^6 K, and a density of 2.6 kg/m^3, the opacity of pure hydrogen is only beginning to rise above the lower bound provided by Thompson scattering (namely ~0.3 m^2/kg, compared with the Thompson opacity of 0.034 m^2/kg).

Satisfying hydrostatic equilibrium and heat balance everywhere within the star would determine the unique central density for a given central temperature (if any stable solution exists). In the absence of a complete stellar model, however, we have instead derived the range within which the central density must lie, namely between 0.015 kg/m^3 and 2.6 kg/m^3.

An estimate for the star's lifetime is provided by the reaction time based on equation (6), noting that this is the slowest reaction in the subsequent sequence, as demonstrated in the Appendix. Using the above limiting densities suggests lives between 200 Myrs and 30 Byrs. This encompasses the biophilic range, permitting biological evolution the order of billions of years to carry out its work.

The mass of our star may be estimated from $kT_c \approx 0.24 G M_p M^{2/3} \rho_c^{1/3}$, which is an approximate rendering of the virial condition (i.e. being gravitationally bound), except that average quantities have been replaced by their central values. This implies masses of between ~3 and ~50 solar masses (the latter being, by construction, of the lower bound density). These fall within the usual stellar range and hence seem achievable, e.g. there is no obvious objection as regards the availability of cooling mechanisms during star formation.

Estimation of the surface temperature is more contentious. To do so we have assumed that one quarter of the star's mass is involved in nuclear reactions at the central rate. This results in luminosities of 300 to 8,000 times solar luminosity. The radius is estimated using $R \approx 2.5(M/\rho_c)^{1/3}$, which suggests sizes 50 to 650 times solar size. Finally, the preceding results imply a surface temperature between 1,400K and 7,600K. Thus, the surface temperature is quite uncertain, but is not obviously inconsistent with planetary life based on conventional biochemistry. The point here is that biophilic stars require surface temperatures consistent with photons of an energy compatible with driving photosynthesis, or some comparable biochemistry.

Of course we have not definitively established that such stars could exist. To do so would require explicit demonstration that hydrostatic equilibrium and heat balance were respected at all points in the star, i.e. a complete stellar model. More problematical still, in our present state of knowledge, would be the requirement to demonstrate that such stars could actually form. Fortunately this is not where the burden of proof lies. We have seen that the elementary stability constraints can be consistent with a star which is sufficiently long lived, sufficiently luminous, and has a suitable range of surface temperatures, to mimic the actual conditions of our universe to some approximation. In view of this, the burden of proof lies with any contention that diproton stability is anthropically catastrophic. It would appear not to be.

It is rather remarkable that a reaction which is so many orders of magnitude faster than the usual weak pp capture reaction can result in a stable star simply by reducing the temperature and density. It is reasonable to ask whether this trick could be repeated for an even faster reaction. The limit may be that at still lower temperatures the opacity will start to climb steeply (Kramer's opacity $\propto 1/T^{7/2}$). This will severely restrict the power density which can be balanced by purely radiative heat transfer. It appears that our example is close to this limit.

It is amusing to speculate how physicists in an alternative universe, containing stars exclusively like that of our example, might view their situation. They might point to three remarkable ‘coincidences’. The first would be that the Thompson lower bound opacity is attained at a temperature *just* low enough to support the required stellar heat transport. The second would be the good fortune that identical particles were involved in the first stellar reaction, thus suppressing the reaction rate due to the exclusion principle, and hence creating stars of sufficient longevity to support biological evolution. The third piece of luck, undeniable surely, would be that the strong nuclear force was *just* strong enough to bind the diproton – without which the first stellar nuclear reaction would not be possible, and hence there would be no stars and no chemical elements!

5. Conclusions

Increases in g_s sufficient to bind the diproton do not lead to significant production of diprotons during BBN. This has been demonstrated by direct evaluation of the pp capture rate for increases in g_s of up to 40%, corresponding to diprotons being roughly twice as stable as deuterons are in our universe (i.e. a binding energy of ~4MeV).

Under solar conditions, the stability of diprotons would lead to a reaction pathway for converting protons to deuterons perhaps ~10 orders of magnitude faster than the usual weak capture reaction. This would prevent stars of the familiar hot, dense type from occurring in the universe. Nevertheless, elementary stellar stability requirements can be met by lower density stars, with lower central temperatures, when the diproton is stable. Such a universe therefore appears compatible with stars whose lifetimes are of the order of billions of years, and whose luminosities and surface temperatures are appropriate for the nurturing of biological life based on conventional molecular chemistry.

The above observations challenge the contention that the strong nuclear force has a fine-tuned anthropic upper bound requiring the diproton to be unbound.

However, this does not preclude there being other mechanisms which might impose an anthropic upper bound to the strong nuclear force. One possibility is the well-known ‘Hoyle’ resonance energies which promote the production of carbon and oxygen in stars. Even very small changes in g_s would presumably play havoc with these very delicate nuclear balances. We also note that increases in g_s rather greater than 40% would lead to deuterium being stable before 1 second. At such times the leptonic reactions which interconvert protons and neutrons were still active. The neutrons would thus escape into the sanctuary of helium-4 whilst the neutron:proton ratio was still determined by thermal equilibrium. If g_s were increased by a factor of ~4, so that deuterium was stable at about a millisecond, the primordial universe would contain only ~3% hydrogen. It is not clear if this is anthropically deselected, but if so it is a far weaker tuning of g_s than is usually envisaged.

Appendix - Stellar “ppI” reaction sequence with a stable diproton

The possibilities for the reaction sequence, analogous to the usual ppI sequence, are listed in Table 3. Reactions involving nuclei with Z > 2, analogous to the ppII/ppIII sequences, have been ignored for simplicity, as have reactions involving neutrons as a reactant. The reaction rates given below have been taken from either Hoffman (2002) or Smith (1988), with the exception of [1a] which is derived above as equation (6). No correction has been made for the increased strength of the nuclear force as regards the rates of reactions after [1a]. For the electromagnetic reactions, this is reasonable. The justification for the other reactions is that reaction [1a] will be found to be the rate determining step. Hence, faster subsequent reactions will not cause a major change to the scenario outlined below.

Table 3. Reactions and rates analogous to the "ppI" sequence for a stable diproton

Label	Reaction	Rate at 10^6 K $s^{-1}(mole/cm^3)^{-1}$	Reference
[1a]	$p + p \rightarrow {}^2_2He + \gamma$	3.5 x 10^{-14}	herein
[1b]	${}^2_2He \rightarrow {}^2_1D + e^+ + \nu_e$	*Assumed fast*	-
[2]	$p + D \rightarrow {}^3_2He + \gamma$	1.64 x 10^{-11}	Smith (1988)
[3]	${}^3_2He + {}^3_2He \rightarrow {}^4_2He + 2{}^1_1p$	1.46 x 10^{-41}	Hoffman (2002)
[4]	${}^2_1D + {}^2_1D \rightarrow {}^4_2He + \gamma$	7.85 x 10^{-16}	Hoffman (2002)
[5]	${}^2_1D + {}^2_1D \rightarrow {}^3_1H + {}^1_1p$	6.72 x 10^{-9}	Hoffman (2002)
[6]	${}^2_1D + {}^2_1D \rightarrow {}^3_2He + {}^1_0n$	6.33 x 10^{-9}	Hoffman (2002)
[7]	${}^3_1H + {}^2_1D \rightarrow {}^4_2He + {}^1_0n$	1.88 x 10^{-7}	Hoffman (2002)
[8]	${}^3_1H + {}^1_1p \rightarrow {}^4_2He + \gamma$	3.56 x 10^{-11}	Smith (1988)
[9]	${}^3_2He + {}^2_1D \rightarrow {}^4_2He + {}^1_1p$	3.84 x 10^{-19}	Hoffman (2002)
[10]	${}^3_2He + {}^3_1H \rightarrow {}^4_2He + {}^2_1D$	1.42 x 10^{-22}	Hoffman (2002)
[11]	${}^3_2He + {}^3_1H \rightarrow {}^4_2He + {}^1_0n + {}^1_1p$	2.0 x 10^{-22}	Hoffman (2002)

The usual ppI sequence involves reactions [2] and [3]. At the low temperature considered, the reactions involving reactant nuclei with double charges are strongly suppressed by the Coulomb barrier. In particular, reaction [3] is not active and so the ppI sequence in our alternative universe must follow a different path. Reactions [4], [9], [10] and [11] are also too slow to contribute significantly. The dominant reaction sequences are thus,

[1a]→[1b] → [2] + [6] (^{3}He production)

[1a]→[1b] → [5] → [7] + [8] (^{4}He production)

Note that helium-3 is not burnt at this temperature. The above rates are consistent with the timescale of the hydrogen burning phase being determined by reaction [1a]. Equilibrium deuteron and tritium densities are around 10^{-3} and 3 x 10^{-5} of the proton density respectively. The end product of the hydrogen burning phase is a mixture of roughly 75% helium-3 and 25% helium-4. Such a star would exhibit a distinct helium-3 burning phase after the hydrogen phase (following additional gravitational collapse to raise the temperature sufficiently to activate reaction [3]). Only after exhaustion of the helium-3 would the usual helium-4 burning phase occur, following further collapse of the core.

The power density suggested in the main text is an upper bound based on full conversion to helium-4 in the hydrogen burning phase. The heat released per helium nucleus equals, to a good enough approximation, the helium-4 binding energy less twice the neutron/proton mass difference. The binding energy is increased significantly in our hypothetical universe. Based on the energy levels of a square potential well, we estimate the binding energy to be,

$$\frac{B}{B_{actual}} = \left(\frac{g_s / g_s^{actual} - 0.85}{1.0 - 0.85} \right)^2$$

because a reduction in g_s to $\sim 0.85 g_s^{actual}$ results in the deuteron being unbound (i.e. B = 0). This results in helium-4 binding energy estimates of 154, 255 and 380 MeV respectively, for g_s increased by x1.2, x1.3 and x1.4. The energy release is thus quite prodigious by normal standards. If the binding energy were estimated assuming scaling according to $\left(g_s / g_s^{actual}\right)^4$ we would get 57, 81 and 109 MeV respectively. Hence, we have employed a rather generous upper bound power density since this strengthens the arguments of the main text.

References

Agrawal, V., Barr, S. M., Donoghue, J. F., Seckel, D. 1998a, *Anthropic considerations in multiple-domain theories and the scale of electroweak symmetry breaking, Phys.Rev.Lett*,. **80,** 1822.

Agrawal, V., Barr, S. M., Donoghue, J. F., Seckel, D. 1998b, *The anthropic principle and the mass scale of the standard model, Phys.Rev*., D **57,** 5480.

Barrow, J. D., Tipler, F. J. 1986, *The Anthropic Cosmological Principle*, Oxford University Press.

Blatt, J. M., Weisskopf, V. F. 1952, *Theoretical Nuclear Physics*, John Wiley.

Carr, B. J., Rees, M. J. 1979, *The anthropic principle and the structure of the physical world, Nature,* **278**, 605.

Carter, B. 1974, *Confrontation of cosmological theories with observation: Proc. IAU Symp.63*, 291, ed M S Longair, Dordrecht, Reidel.

Davies, P. C. W. 1982, *The Accidental Universe*, Cambridge University Press.

Davies, P. C. W. 2004, *Multiverse cosmological models, Mod.Phys.Lett.*, A **19,** 727.

Davies, P. C. W. 1972, *Time variation of the coupling constants*, *J.Phys*. A, **5,** 1296.

Dine, M. 2003, *Is there a string theory landscape: some cautionary remarks*, Preprint hep-th/0402101.

Dyson, F. J. 1971, *Energy in the universe*, *Sci.Am.*, **225,** 51.

Evans, R. D. 1955, *The Atomic Nucleus*, McGraw-Hill.

Guth, A. H. 2007, *Eternal inflation and its implications*, *J.Phys.*A*: Math.Theor*., **40,** 6811.

Harnik, R., Kribs, G. D., Perez, G. 2006, *A universe without weak interactions*, *Phys.Rev*. D, **74,** 035006.

Hoffman, R. 2002, *Strong and Electromagnetic Reaction Rates 2002*, Lawrence Livermore National Laboratory, Nuclear Theory & Modelling Group (http://www-phys.llnl.gov/Research/RRSN/semr/light_reacs.html).

Hogan, C. J., 2000, *Why the universe is just so*, *Rev.Mod.Phys.*, **72,** 1149.

Hogan, C. J. 2006, *Nuclear astrophysics of worlds in the string landscape*, *Phys.Rev*. D, **74,** 123514.

Linde, A. 1994, *Sci. Am*., **271,** 32.

Linde, A. 2007a, *Sinks in the landscape, Boltzmann brains, and the cosmological constant problem*, *J.Cosmol.Astropart.Phys.*, JCAP **0701**, 022.

Linde, A. 2007b, *Inflationary cosmology*, Preprint hep-th/0705.0164.

Martel, H., Shapiro, P. R., Weinberg, S. 1998, *Likely values of the cosmological constant, Astrophys.J.*, **492,** 29.

Rees, M. J. 1999, *Just Six Numbers*, Weidenfeld & Nicolson, London.

Smith, M. 1988, *Caughlan & Fowler 1988 Thermonuclear Reaction Rates*, Oak Ridge National Laboratory (http://www.phy.ornl.gov/astrophysics/data/cf88/directory.html).

Susskind, L. 2003, *The anthropic landscape of string theory*, Preprint hep-th/0302219.

Tegmark, M., Aguirre, A., Rees, M. J., Wilczek, F. 2006, *Dimensionless constants, cosmology and other dark matters*, *Phys.Rev.* D, **73,** 023505.

Weinberg, S., 2007, *Living in the multiverse,* in *Universe or Multiverse,* ed B Carr, Cambridge University Press, and Preprint hep-th/0511037.