Lower Bound Collapse Solution for Thick Half-Pipe under P + M Loading
Exact Integration of Step-Function Stress Distribution

Let *a*,*b* be the inner and outer radii, and *y* the perpendicular distance from the base diameter.

The postulated stress distribution is: for $y < \lambda$, $\sigma = \sigma_y$; for $y > \lambda$, $\sigma = -\sigma_y$. This produces moments in the desired sense, but these are defined as negative.

The centroid is $y_c = \frac{1}{A}\int y dA = \frac{4}{3\pi}\cdot\frac{b^3 - a^3}{b^2 - a^2}$.

The moment, *M*, is defined wrt the centroid, i.e., $M = \int \sigma(y - y_c)dA$.

The exact interaction diagram is expressed parametrically in terms of λ by,

$$\frac{M}{M_0} = \frac{\tilde{M}}{M_0} - \frac{P}{P_0}$$

where,

$$\frac{P}{P_0} = 1 + \frac{4}{\pi(b^2 - a^2)}\left[a^2\theta_a - b^2\theta_b + \lambda^2(\tan\theta_b - \tan\theta_a)\right]$$

$$\frac{\tilde{M}}{M_0} = 1 + \frac{2}{b^3 - a^3}\left[a^3\sin\theta_a - b^3\sin\theta_b + \lambda^3(\tan\theta_b - \tan\theta_a)\right]$$

and,

$$\theta_b = \cos^{-1}\left(\frac{\lambda}{b}\right)$$

$$\theta_a = \cos^{-1}\left(\frac{\lambda}{a}\right) \text{ if } \lambda \le a \text{, else } \theta_a = 0$$

$$P_0 = \frac{\pi}{2}(b^2 - a^2)\sigma_y$$

$$M_0 = \frac{2}{3}(b^3 - a^3)\sigma_y$$

Note that M_0 is ***not*** the collapse moment at *P* = 0, but merely a convenient normalising quantity.

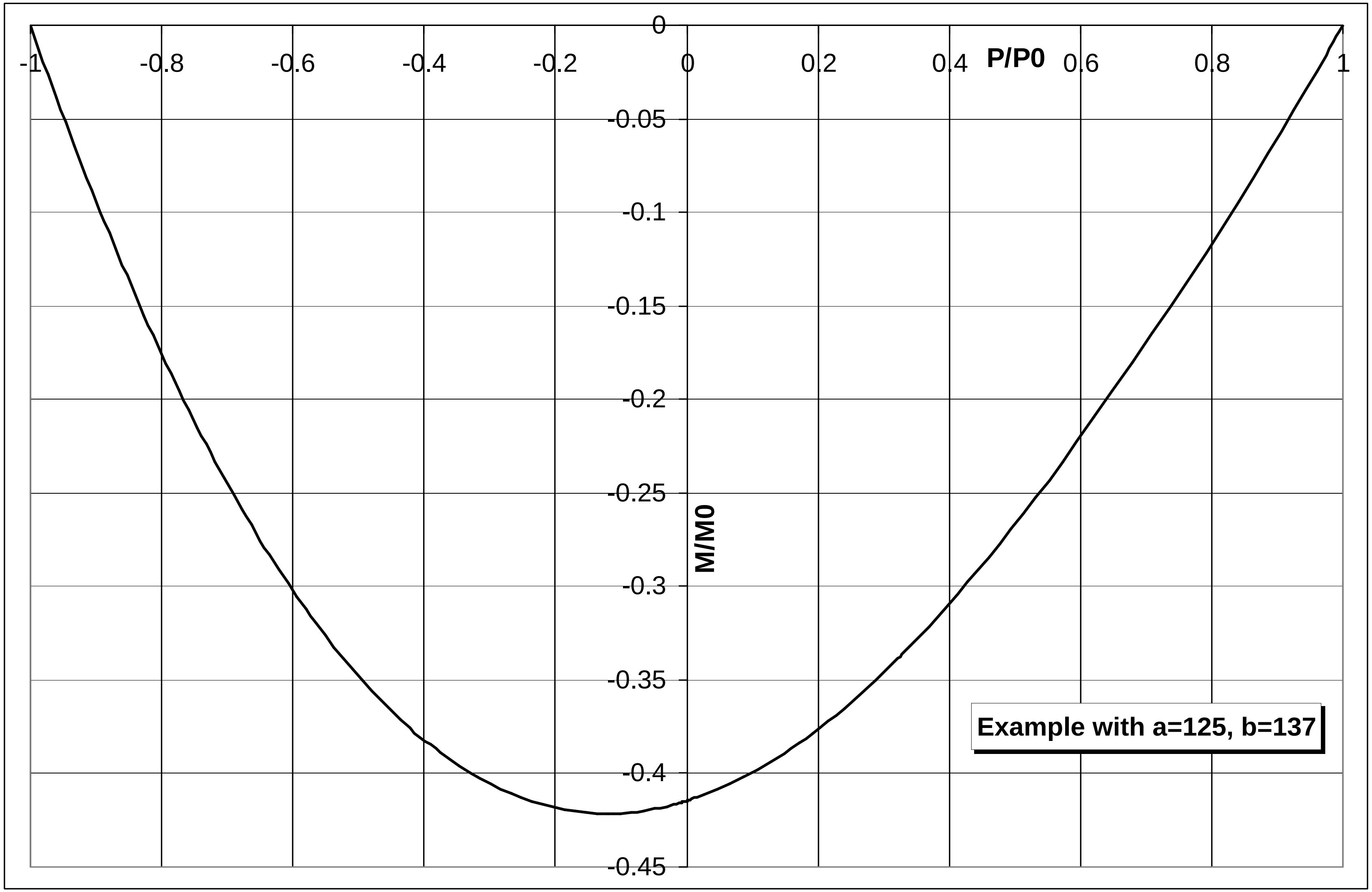
The parameter λ is varied from 0 to *b*, corresponding to $P/P_0 = -1$ and $P/P_0 = +1$ respectively.

Example for a = 125, b = 137 – See attached plot of interaction surface

Collapse moment for *P* = 0 is *M* = -0.4147M_0. This compares with the approximate solution using a step function of θ which gives $M = -(\sqrt{2} - 1)M_0 = -0.4142M_0$. The correction due to finite thickness is extremely small.

The maximum collapse moment is -0.4214M_0 and occurs for P = -0.119P_0.

$\tilde{M}$ is the moment about *y* = 0 and is zero for the case of interest (GH96), giving collapse at $\tilde{M} = 0$, *M* = -0.3341M_0 and **$P = +0.3341P_0$**.



0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
-0.4
-0.45
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
P/P0
M/M0
Example with a=125, b=137