Is it right to insert C(t) into a ccg law derived from C* controlled tests?

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Experimental ccg laws are derived from data in the C* controlled regime (earlier data being specifically rejected by the test analysis procedure). However, in plant applications, if C(t) > C* then C(t) is inserted into the ccg law derived on the basis of C* control. Is this correct?

This question is examined below from the perspective of continuum damage mechanics (CDM), assuming standard ductility exhaustion. The creep deformation equation is assumed to be,

Primary creep: $$\dot{\varepsilon}^c = \tilde{B} t^m \sigma^{n_1} \tag{1}$$

Secondary creep: $$\dot{\varepsilon}^c = B\sigma^n \tag{2}$$

C* control, considered in §1 below, applies for times after the redistribution time, t_R.

In the case of C(t) control, considered in §2 below, elastic-creep conditions will be assumed and also times short compared with the redistribution time, $t << t_R$.

1. Recap - Derivation of the $\dot{a} - C^*$ relation in C* control

The crack is assumed to have already incubated and be growing.

Steady creep conditions are assumed to have been achieved so that the crack tip fields are controlled by $C*$ [as opposed from *C(t)*].

Let r_c be the size of the creeping zone ahead of a pre-existing crack tip. Imagine a material element which starts a distance r_c from the crack tip, and lies on the line along which the crack will grow. Since r_c is, by definition, the boundary of the creep zone from the initial crack tip position, the material element initially has zero creep strain (zero damage). The distance of this material element from the crack tip, *r*, will reduce as the crack grows towards it. So we write $r(t)$. The creep strain rate in this material element can therefore be written $\dot{\varepsilon}_c(r(t))$, where the notation emphasises that the strain rate will increase as the material element becomes nearer to the crack tip. The ductility exhaustion failure criterion is thus,

$$\int_0^{t_c} \dot{\bar{\varepsilon}}_c(r(t))dt = \varepsilon_f \tag{3}$$

where t_c is the time for the crack tip to grow the distance r_c, and $\bar{\varepsilon}_c$ denotes the Mises creep strain. In (3), the creep ductility, ε_f, is that relevant to the state of stress, and the strain rate, prevailing within the creep zone. In truth this will vary with position in the creep zone and hence ε_f should really appear within the integral, i.e., the ductility exhaustion failure criterion is strictly,

$$\int_0^{t_c} \frac{\dot{\bar{\varepsilon}}_c(r(t))}{\varepsilon_f(r(t))} dt = 1 \tag{4}$$

This refinement is ignored here for simplicity and we shall use (3). The instantaneous crack growth rate is,

$$\dot{a} = -\frac{dr}{dt} \tag{5}$$

So we can change the integration variable in (3) from t to r to give,

$$-\int_{r_c}^{0} \frac{\dot{\overline{\varepsilon}}_c(r)}{\dot{a}} \cdot dr = \varepsilon_f \tag{6}$$

It is now assumed that secondary creep, equ.(2), prevails. This is consistent with the assumption of C* control, though it may not necessarily always be true in practice. Hence the explicit HRR expression for the strain rate is,

$$\dot{\varepsilon}_{ij}^{c}(t) = B\left[\frac{C^*}{BI_n r}\right]^{\frac{n}{n+1}} \hat{\varepsilon}_{ij}(\theta, n) \tag{7}$$

Substituting (7) into (6) and carrying out the integral gives,

$$\dot{a}\varepsilon_f = \int_0^{r_c} \dot{\overline{\varepsilon}}_c dr = B\left[\frac{C^*}{BI_n}\right]^{\frac{n}{n+1}} \tilde{\overline{\varepsilon}}(\theta, n) \int_0^{r_c} \frac{dr}{r^{\frac{n}{n+1}}} = \frac{(n+1)B^{\frac{1}{n+1}}}{I_n^{\frac{n}{n+1}}} C^{*\frac{n}{n+1}} \cdot \tilde{\overline{\varepsilon}}(\theta, n) \cdot r_c^{\frac{1}{n+1}} \tag{8}$$

where the dimensionless HRR function for the Mises strain, $\tilde{\overline{\varepsilon}}_c(\theta, n)$, is evaluated at the angle, θ, relevant to growth, probably $\theta = 0$. Note that the crack growth rate, $\dot{a}$, has implicitly been assumed constant and taken out of the integral. This approximation assumes sufficiently small amounts of growth that C*, and hence $\dot{a}$, do not vary significantly over the period of interest.

Equ.(8) is a growth law of the form $\dot{a} = A.C^{*q}$ and we can see that,

$$q = \frac{n}{n+1} \tag{9}$$

$$A = \frac{(n+1)B^{\frac{1}{n+1}}}{I_n^{\frac{n}{n+1}}} \cdot \frac{\tilde{\overline{\varepsilon}}(\theta, n)}{\varepsilon_f} \cdot r_c^{\frac{1}{n+1}} \tag{10}$$

If primary creep prevails, despite C* control, then in the above derivation the constant B is replaced by the time dependent $\tilde{B}t^m$, which cannot strictly be taken out of the integral. However, in view of the weak dependence of A on B it may be an adequate approximation in primary creep to replace n by n_1 and B by $\tilde{B}t^m$ in equs.(9,10).

2. Derivation of $\dot{a} - C(t)$ relation in C(t) control

Define Q as,

Primary creep: $$Q = \frac{n_1}{n_1 + 1} \tag{11a}$$

Secondary creep: $$Q = q = \frac{n}{n+1} \tag{11b}$$

The following derivation is carried out for primary creep, but can be reduced to the secondary creep case easily later. The creeping HRR field gives the creep strain rate near the crack tip as,

$$\dot{\varepsilon}^c(t,r) = \left(\tilde{B}t^m\right)^{1-Q}\left(\frac{C(t)}{I_{n_1}r}\right)^Q \tilde{\varepsilon}(\theta,n_1) \tag{12}$$

Consider a point initially at a distance r_c from the crack tip. The crack grows to this point in time t_c. Ductility exhaustion therefore requires,

$$\varepsilon_f = \int_0^{t_c} \dot{\varepsilon}^c(t,r(t))\cdot dt \tag{13}$$

Because the primary creep rate has an explicit time dependence, it is not possible to change the integration variable to r, as was done in §1. In (13), $r(t)$ is an unknown function required to have initial and final values $r(0)=r_c$ and $r(t_c)=0$.

Elastic-creep is assumed with $t << t_R$ so that $C(t)$ can be written in terms of the LEFM SIF as,

$$C(t) = \frac{K^2}{(n_1+1)Et} \tag{14}$$

Substitution of (14) into (12) and the result into (13) gives,

$$\varepsilon_f = \int_0^{t_c}\left(\tilde{B}t^m\right)^{1-Q}\left(\frac{K^2}{I_{n_1}(n_1+1)Ert}\right)^Q \tilde{\varepsilon}(\theta,n_1)\cdot dt = D\int_0^{t_c}\frac{t^{m(1-Q)-Q}}{r(t)^Q}dt \tag{15}$$

where,
$$D = \tilde{B}^{1-Q}\left(\frac{K^2}{I_{n_1}(n_1+1)E}\right)^Q \tilde{\varepsilon}(\theta,n_1) \tag{16}$$

In taking the SIF, K, out of the integral it has implicitly been assumed that the time periods considered are sufficiently short that the crack growth is small compared with the ligament, so that the SIF is not significantly affected by the change in crack length. Failing that, K may be interpreted as some average SIF.

Equ.(15) is actually an integral equation in the unknown function $r(t)$. It is simply solved by guessing the solution to be of the form,

$$r = r_c\left(1-\tau^\alpha\right) \quad \text{where,} \quad \tau = \frac{t}{t_c} \tag{17}$$

Substitution of (17) into (15) gives,

$$\varepsilon_f = \Im D\frac{t_c^{(1-Q)(1+m)}}{r_c^Q} \quad \text{where,} \quad \Im = \int_0^1 \frac{\tau^{m(1-Q)-Q}}{\left(1-\tau^\alpha\right)^Q}d\tau \tag{18}$$

But r_c is the crack growth, Δa, which occurs in time $t = t_c$, so we write,

$$\Delta a^Q = \frac{\Im D}{\varepsilon_f}t^{(1-Q)(1+m)} \tag{19}$$

i.e., $$\Delta a = r_c - r = r_c \left(\frac{t}{t_c}\right)^{\alpha} = \left(\frac{\Im D}{\varepsilon_f}\right)^{1/Q} t^{(1-Q)(1+m)/Q} \qquad (20)$$

Hence (17) is indeed a solution of (15) and we can identify the index α to be,

$$\alpha = \frac{(1-Q)(1+m)}{Q} \qquad (21)$$

Because $-1 < m \le 0$ and $0.5 < Q < 1$ it follows that $0 < \alpha < 1$. Differentiating (20) gives the instantaneous crack growth rate,

$$\dot{a} = \alpha \left(\frac{\Im D}{\varepsilon_f}\right)^{1/Q} t^{\alpha - 1} \qquad (22)$$

We can now substitute $C(t)$ in place of the explicit time dependence using (14), giving,

$$\dot{a} = \tilde{A} \cdot C(t)^{\tilde{q}} \qquad (23)$$

where, $$\tilde{q} = 1 - \alpha = 1 - \frac{(1-Q)(1+m)}{Q} \qquad (24)$$

and, $$\tilde{A} = \frac{\alpha}{I_{n_1}} \left(\frac{\Im \tilde{\tilde{\varepsilon}}}{\varepsilon_f}\right)^{\frac{1}{Q}} \tilde{B}^{\frac{1}{n_1}} \left(\frac{K^2}{(n_1+1)E}\right)^{\alpha} \qquad (25)$$

3. Comparison of ccg derived from C* and C(t) based CDM approaches

The algebraic form of the ccg expression based on C*, equs.(9,10), is very different from that based on C(t), equs.(23-25). The former includes a dependence on r_c whereas the latter does not, and the latter includes a dependence on K whereas the former does not. To compare the two predictions it is therefore simplest to consider some numerical examples typical of plant. These are based on the RCC-MR deformation law for 316 at 550°C which gives the parameters,

Primary creep: $n_1 = 4.18$, $\tilde{B} = C_1 C_2 / 100 = 1.25 \times 10^{-14}$

Secondary creep: $n = 8.2$, $B = 5.29 \times 10^{-26}$

The HRR parameters used will assume plane stress. This is because the use of plane strain in equs.(9,10) is known to be problematical due to both the creep strain and the Spindler fraction being almost vanishingly small on $\theta = 0$ (see http://rickbradford.co.uk/T73S03TutorialNotes42.pdf). Plane stress, on the other hand, produces results which are in good agreement with experimental ccg laws. Hence, the HRR parameter $\tilde{\tilde{\varepsilon}}$ on $\theta = 0$ is taken as 0.952, whilst the parameter I_n is taken as 3.59 for primary creep (n ~ 4) and 3.09 in secondary creep (n ~ 8). The Spindler fraction is 0.46, whilst the uniaxial ductility is assumed to be 10.7% (BE) or 2.6% (LB).

The C* based equations, (9,10), will assume secondary creep. The creep zone size assumed will be 1 cm, which produces results in reasonable agreement with experiment.

The C(t) based equations, (23-25), will be illustrated for both primary and secondary creep. The results will be given for elastic SIFs of 10 and 30 MPa√m (and Young's modulus 160 GPa).

Numerical evaluation of the integral defining $\Im$, (18), requires care because the integrand is singular at both $\tau = 0$ and $\tau = 1$. For the record the evaluation here gave,

For $Q = 0.8913$, $m = 0$: $\Im = 80.09$ (secondary creep)

For $Q = 0.8069$, $m = C_2 - 1 = -0.579$: $\Im = 55.54$ (primary creep)

Since the index, $\tilde{q}$, in the C(t) based equation, (23), differs from q, comparison is best carried out for example numerical values of C(t). For this purpose values of 10^{-7} and 10^{-8} MPa.m/hr are chosen, since these are characteristic of plan t applications.

Results

Table 1 gives the results assuming lower bound ductility (multiaxial value 1.2%). Table 2 are the corresponding results for best estimate ductility (multiaxial value 4.92%).

Table 1

C* based ccg				
q	0.891	0.891	0.891	0.891
A	0.291	0.291	0.291	0.291
ccg at C*=10^-7 (mm/yr)	1.467	1.467	1.467	1.467
ccg at C*=10^-8 (mm/yr)	0.188	0.188	0.188	0.188
C(t) based ccg				
	secondary creep		primary creep	
q-tilda	0.878	0.878	0.899	0.899
A-tilda	0.188	0.246	0.176	0.219
K	10	30	10	30
ccg at C(t)=10^-7 (mm/yr)	1.176	1.537	0.781	0.974
ccg at C(t)=10^-8 (mm/yr)	0.156	0.204	0.098	0.123
ratio of C(t) to C* based ccg 10^-7	0.80	1.05	0.53	0.66
ratio of C(t) to C* based ccg, 10^-8	0.83	1.08	0.52	0.65

Table 2

C* based ccg				
q	0.891	0.891	0.891	0.891
A	0.071	0.071	0.071	0.071
ccg at C*=10^-7 (mm/yr)	0.357	0.357	0.357	0.357
ccg at C*=10^-8 (mm/yr)	0.046	0.046	0.046	0.046
C(t) based ccg				
	secondary creep		primary creep	
q-tilda	0.878	0.878	0.899	0.899
A-tilda	0.038	0.050	0.030	0.038
K	10	30	10	30
ccg at 10^-7	0.240	0.314	0.135	0.169
ccg at 10^-8	0.032	0.042	0.017	0.021
ratio of C(t) to C* based ccg 10^-7	0.67	0.88	0.38	0.47
ratio of C(t) to C* based ccg, 10^-8	0.70	0.91	0.37	0.46

Conclusions

- Calculations based on CDM imply that the use of C(t) in a ccg law derived under C* control is conservative, but not overly conservative.
- In the derived laws $\dot{a} = AC^{*q}$ and $\dot{a} = \tilde{A}C(t)^{\tilde{q}}$, the indices are almost the same, $\tilde{q} \approx q$, whilst the $C(t)$ law has a slightly smaller coefficient, $\tilde{A} < A$, though the difference is not great compared with the scatter in ccg laws.
- The upper bound growth, based on lower bound ductility, is up to a factor of 2 faster if the C* based ccg law is used, i.e., CDM with a C(t) controlled crack tip field indicates slower growth.
- The best estimate growth is up to a factor of 3 faster if the C* based ccg law is used.
- However, if secondary creep prevails near the crack tip, the C(t) and C* based estimates of growth are closer, differing by not more than 30%.
- Hence, current practice in plant assessments appears to be justified.