Another Exceedingly Simple Climate Change Model

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1. Introduction

The intention is to devise a simple means of examining whether the claims of the mainstream climate modellers are reasonable. The IPCC consensus is that doubling the atmospheric CO_2 concentration from its 2008 value of 385 ppm to 770 ppm would cause an increase in the global average surface temperature of around 2.3°C, with a 90% confidence interval of roughly 1.8°C to 3.3°C - see the 2014 IPCC Synthesis Report, Figure SPM.5(b). My simple model of 2009 implied a global mean temperature increase with respect to 2008 of 1.65°C if CO_2 reaches 800 ppm.

Here I present another model on a different basis. The Maximum Entropy Production Principle (MEPP) is used to derive the latitudinal variation of temperature, from which a global average is derived. This provides rather more information of relevance to climate, namely the equatorial-polar temperature difference. The driving forces underlying global wind patterns include the equatorial-polar temperature difference and the Earth's rotation (manifest as the Coriolis force), as well as the mean temperature and other factors (e.g., ocean behaviour).

2 The Basic Idea: Heat Flows and Entropy Production

The Earth is a heat engine. Due to geometry, the Earth's poles receive less energy per unit area from the Sun than do tropical regions (the variation going as the cosine of latitude). The Earth becomes a heat engine by virtue of the resulting temperature difference between the equator and the poles. Heat therefore flows from the equator to the poles. This heat is transported by winds and ocean currents, i.e., by convection. The result is weather. Weather is the work done by the Earth considered as a heat engine.

The Earth also radiates heat into space, as well as receiving radiant heat from the Sun. The total heat radiated away from the Earth is in balance with that received from the Sun. (Were it not, climate change would be happening on a timescale of hours or days not decades).

How can the Earth's surface temperature be found as a function of latitude? One way is to appeal to the Maximum Entropy Production Principle (MEPP). When an amount, dQ, of heat flows from a temperature T_1 to a lower temperature, $T_2 < T_1$, there is an increase in entropy, E, of at least,

$$dE = \left(\frac{1}{T_2} - \frac{1}{T_1}\right) dQ \qquad (1)$$

The MEPP claims that the latitudinal temperature distribution arranges itself so as to maximise the total global rate of entropy production. The Principle applies only in conditions of dynamic equilibrium, when temperatures are steady and heat flows constant. The MEPP is contentious. Unlike its counterpart in equilibrium thermodynamics, the MEPP cannot be proved rigorously. Indeed, it is unlikely to be exact. Nevertheless, it has been used with good results in a number of settings, not least in modelling the Earth's atmosphere by Paltridge [1]. Since then proponents of the hypothesis have pointed to its apparent success in modelling the atmospheres not only of the Earth (Dyke and Kleidon [2]) but also of Titan and Mars (Lorenz, Lunine and Withers [3]). Other successful applications of the MEPP have included,

- Transitions among multiple steady states of oceanic thermohaline circulation (Shimokawa & Ozawa [4]);
- The behaviour of the Earth's mantle (Lorenz [5]);
- Bacterial photosynthesis (Juretic and Zupanovic [6]);
- The evolution of the molecular motor ATP synthase (Dewar, Juretic and Zupanovic [7]);
- Fluid flow problems (Niven [8]); and,
- Heat transfer in chaotic Rayleigh-Benard convection (Bradford, [9]).

Note that the entropy rate to be maximised is that arising within the Earth system itself. A greater entropy production arises due to the rejection of heat as infrared radiation into space. The latter makes possible the spontaneous evolution of highly organised, complex entities such as plants and animals. The spontaneous evolution of ordered structures would violate the second law of thermodynamics were it not for the export of vast quantities of entropy into space - life's entropic dumping ground. But this is not relevant to the MEPP as applied to the terrestrial system itself. Here we are concerned with temperatures arranging themselves to maximise entropy production within the Earth's fluid systems.

3. Formulation of the Model Equations

3.1 Idealisation of the Incident Radiation

The heat flux from the Sun impinging on the outer layer of the atmosphere is $S_0 = 1367$ W/m^2. However, a smaller heat flux actually impinges on the ground (or sea) due to reflection of some of the radiation. This smaller flux is written $S_{\max} = (1-a)S_0$, where a is the albedo, i.e., the fraction of radiation reflected. The subscript denoting 'maximum' refers to the fact that $S_{\max}$ is the flux hitting the ground only at mid-day at the equator. Measuring time in hours, from a datum of mid-day, the radiation reaching the Earth's surface at time t and latitude θ, is,

During daytime: $$S(\theta,t) = S_{\max}\cos\theta\cos\frac{\pi t}{12} \qquad (2a)$$

During night time: $$S(\theta,t) = 0 \qquad (2b)$$

Using a diurnal average thus results in

$$S(\theta) = \frac{1}{\pi}S_{\max}\cos\theta = \left(\frac{1-a}{\pi}\right)S_0\cos\theta \qquad (3)$$

The factor of $\cos\theta$ results from the curving of the Earth's surface away from the Sun at higher latitudes.

A further simplification, implicit in the above, is the ignoring of seasons, i.e., the approximation that the Earth's axis is perpendicular to the plane of the ecliptic. This is roughly, though not precisely, equivalent to using annually averaged temperatures.

3.2 The Heat Flows

The diurnally and annually averaged temperature of the Earth's surface at latitude θ is denoted T, or as $T(\theta)$ if the angular dependence requires emphasis. More precisely what we mean by T is a time average formed as $\langle T^4 \rangle^{\frac{1}{4}}$. The average radiation emitted from the Earth into space at angle θ is thus $\varepsilon\sigma T(\theta)^4$ per unit area, where σ is Stefan's constant (5.67 x 10^{-8} W/m^2K^4) and ε is the effective emissivity. The term 'effective' here refers to the fact that the term ε accounts for the throttling of the radiative transport through the atmosphere, as well as the actual emissivity of the Earth's surface.

Thus, the effect of increased CO_2 in the atmosphere will be accounted for in this model through its effect in reducing ε, i.e., increasing the impedance to radiative heat flow through the atmosphere.

Denoting the Earth's radius by r, the area of the Earth's surface between latitudes θ and $\theta + \delta\theta$ is $2\pi r^2 \cos\theta \cdot \delta\theta$. The rate of heat loss by radiation from this annular region is thus,

$$2\pi r^2 \varepsilon\sigma T(\theta)^4 \cos\theta \cdot \delta\theta \qquad (4)$$

The radiative heat power into this annular region is $2\pi\left(\frac{1-a}{\pi}\right)S_0 r^2 \cos^2\theta \cdot \delta\theta$

$$= 2(1-a)S_0 r^2 \cos^2\theta \cdot \delta\theta \qquad (5)$$

Finally, we need a model for the heat flow through the Earth's fluid systems (northwards in the northern hemisphere, southwards in the southern hemisphere). This heat flow is driven by the temperature gradient over the Earth's surface, so we will use an extremely simple model,

$$\text{heat flow per unit circumference per second} = -k\frac{dT}{rd\theta} \qquad (6)$$

for some constant k. This unknown constant will be determined by maximising the entropy production rate. The minus sign in (6) is because heat flows from hot to cold. Note the factor of r in the denominator of (6), required because the arc length increment is $ds = rd\theta$. The heat flowing per second across the circle of latitude θ is thus,

$$-2\pi k \cos\theta \frac{dT(\theta)}{d\theta} \qquad (7)$$

from which the r dependence has cancelled. Similarly, the heat flowing per second across the circle of latitude $\theta + \delta\theta$ is,

$$-2\pi k \cos(\theta + \delta\theta)\frac{dT(\theta + \delta\theta)}{d\theta} \qquad (8)$$

The net rate of flow of heat out of the annular region between latitudes θ and $\theta + \delta\theta$ is thus,

$$-2\pi k \frac{d}{d\theta}\left(\cos\theta \cdot \frac{dT}{d\theta}\right) \cdot \delta\theta \qquad (9)$$

In steady state, the heat into the annular region, given by (5), must balance with the sum of the heat losses by radiation, given by (4), and by convection, given by (9). Hence, after cancelling terms, we get the heat balance equation,

$$\left(\frac{1-a}{\pi}\right)S_0\cos\theta=\varepsilon\sigma T^4-\frac{\widetilde{k}}{\cos\theta}\frac{d}{d\theta}\left(\cos\theta\cdot\frac{dT}{d\theta}\right) \tag{10}$$

where $\widetilde{k}=k/r^2$.

<u>Boundary Conditions</u>

If the constants a, S_0, ε and $\widetilde{k}$ are known, equ.(10) can be solved by numerical integration provided that suitable boundary conditions are imposed. We require two boundary conditions because the differential equation is of second order. These conditions are that $\frac{dT}{d\theta}=0$ at the equator ($\theta=0$) and at the poles ($\theta=\pm\pi/2$). The first of these follows from symmetry (there can be no heat flowing across a symmetry plane), whilst the second must be so because there cannot be heat flow into a point.

3.3 The Entropy Production Rate

The heat flowing through the annular region between latitudes θ and $\theta+\delta\theta$ is given by (7). This heat flows from temperature $T(\theta)$ to temperature $T(\theta+\delta\theta)$ and hence the rate of entropy production is,

$$\begin{aligned}&-2\pi k\cos\theta\frac{dT(\theta)}{d\theta}\left(\frac{1}{T(\theta+\delta\theta)}-\frac{1}{T(\theta)}\right)\\&=-2\pi k\cos\theta\frac{dT(\theta)}{d\theta}\cdot\frac{d}{d\theta}\left(\frac{1}{T}\right)\cdot\delta\theta\\&=2\pi k\cos\theta\left(\frac{1}{T}\frac{dT}{d\theta}\right)^2\cdot\delta\theta\end{aligned} \tag{11}$$

Hence, the total rate of entropy production over the whole Earth is

$$\dot{E}=4\pi k\int_0^{\pi/2}\cos\theta\left(\frac{1}{T}\frac{dT}{d\theta}\right)^2\cdot\delta\theta \tag{12}$$

4. Observed Temperature Distribution

Antarctic temperatures tend to be lower than arctic temperatures. However, splitting the difference, the average terrestrial temperature is about -27^{o}C at the poles and about +27^{o}C at the equator. Recall that these are diurnal and annual averages. Estimating the temperature at intermediate latitudes by $T(\theta)=-27+54\cos\theta$ the global average temperature is given by,

$$\langle T\rangle=\int_0^{\pi/2}T\cos\theta\cdot d\theta \tag{13}$$

which evaluates to $54\frac{\pi}{4}-27=15.4$ ^{o}C, which is a reasonable value for the current global average temperature. Note that accuracy in modelling the absolute temperature is not crucial to the issue of faithfully reproducing the sensitivity of temperatures to the CO_2 concentration.

5. Solution Procedure - Uniform Albedo Approximation

Solution of the differential equation, (10), proceeds by re-writing it as,

$$y = \frac{dT}{d\theta} \qquad \text{and} \qquad \frac{dy}{d\theta} = y \tan\theta + \frac{1}{\tilde{k}}\left(\varepsilon\sigma T^4 - \frac{(1-a)}{\pi} S_0 \cos\theta \right) \tag{14}$$

Initially the albedo, a, is assumed to be uniform over the Earth and equal to 0.3.

The solution procedure consists of assuming some initial values for the effective emissivity, ε, and the convective heat transfer constant, $\tilde{k}$.

Starting from the condition $y = 0$ at $\theta = 0$, numerical integration of (14) can be accomplished by simple finite differences in small angular steps provided some equatorial temperature, $T(0)$, is assumed. However, in general this will fail to result in y being zero at the poles. The assumed equatorial temperature, $T(0)$, is therefore iterated until y is zero at the poles. An automated procedure for this can be programmed. This then provides the complete temperature distribution for the assumed a, ε and $\tilde{k}$.

The next step consists of re-running the solution for different $\tilde{k}$ values for the same a and ε. The physically correct solution is given by that $\tilde{k}$ which maximises the entropy production rate, which is found by numerical evaluation of (12).

The global average temperature is found for this solution by numerical evaluation of (13).

Finally, the effective emissivity, ε, is iterated until the correct global average temperature is reproduced, i.e., 15.4^{o}C = 288.6K.

The solution is provided for all latitudes, so in particular the equatorial and polar temperatures are found.

Carrying out this procedure for $a = 0.3$ produces $\varepsilon = 0.6$, $\tilde{k} = 0.42$ and an equatorial temperature of 27.8^{o}C, in good agreement with reality. However, the predicted polar temperature of -15.1^{o}C is too high compared with the observed -27^{o}C. The reason for this is probably that the albedo near the ice caps is far higher than 0.3, more like 0.9, thus depressing temperatures further. This will be explored below. For now we continue with this simple "uniform a" model to see what it predicts for the sensitivity to CO_2 concentration.

6. The Effect of CO_2 Changes on the Effective Emissivity (ε)

6.1 Cross Sections and Rosseland Averaging

The Earth is in heat balance: the rate at which energy is absorbed from the Sun equals the rate at which energy is re-radiated back into space. Were this not the case the planet's temperature would be changing extremely rapidly, i.e., on a timescale of hours or days. If we ignore the variation in the Earth's temperature with latitude and assume a uniform temperature, T_u, then equating the total heat from the Sun with the heat radiated back into space gives,

$$4\pi r^2 \varepsilon \sigma T_u^4 = (1-a)\pi r^2 S_0$$

or,
$$\varepsilon T_u^4 = \frac{(1-a)S_0}{4\sigma} \qquad (15)$$

Using $a = 0.3$ gives $\varepsilon^{\frac{1}{4}} T_u = 255$ K. Thus, if T_u is to agree, at least roughly, with the actual global mean surface temperature of ~288 K, we require an effective emissivity of ε ~0.61. This is extremely close to the emissivity deduced from the simple model of §5 (namely 0.6) which included a latitudinal temperature variation.

The mechanism by which CO_2, and other 'greenhouse gases', can increase terrestrial temperatures lies in impeding the flow of infrared radiation through the atmosphere. Impeding this heat flow causes the effective emissivity to reduce. In as far as the right hand side of (15) is fixed, this inevitably causes an increase in T_u. (We will discuss possible simultaneous changes in the albedo later).

All molecules will tend to absorb or scatter photons to some degree. However, molecules of H_2O, CH_4 and CO_2, for example, have far larger cross sections for infrared photon interactions than O_2 or N_2. Consequently, if any of the former (greenhouse gases) replace the latter (dominant constituents of air), the atmosphere will interfere more with the passage of infrared radiation. The probability that a molecule will scatter or absorb an incident photon is measured by a quantity with the units of area, and hence called a 'cross section', though this is not the geometrical cross section. The number of photons scattered or absorbed by a given molecule per second equals the flux of incident photons times the molecule's cross section.

The cross section varies by many orders of magnitude depending on the frequency of the radiation. Cross sections are large (and hence scattering is very likely) if the frequency of the radiation is close to a molecular resonance. The cross section of CO_2 in the infrared is shown below against wavelength (taken from Freedman and Schwenke, [10]),

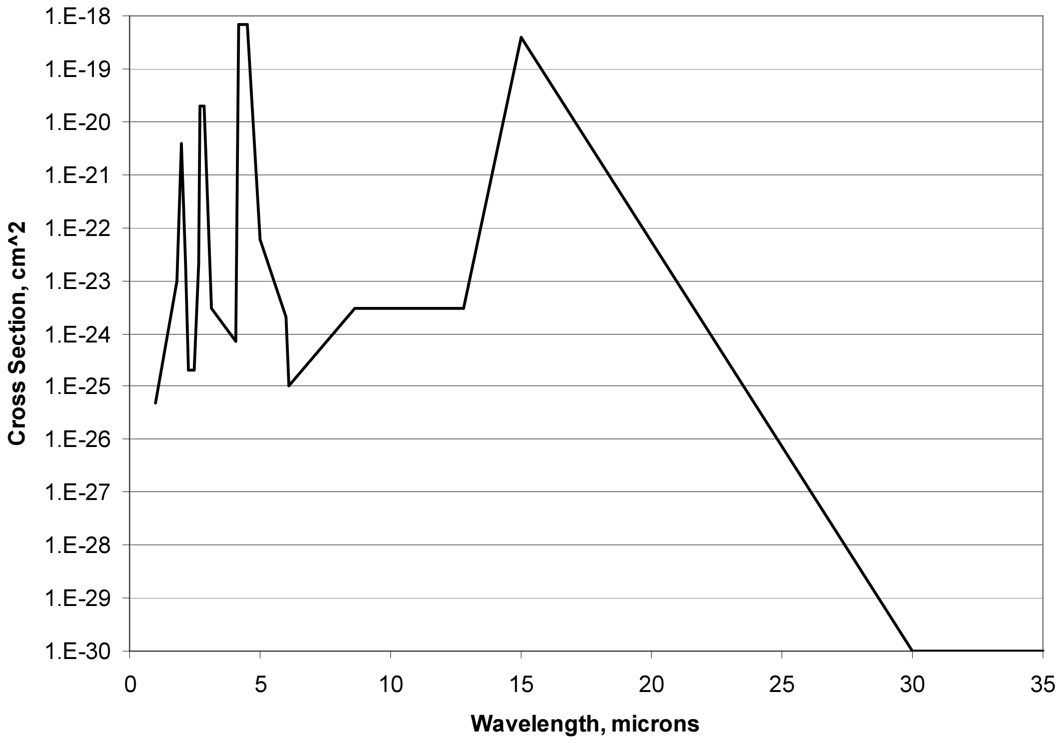


Figure 1: Cross-Section of CO_2 v Wavelength

In the air there are many different molecules, each with their own cross sections, A_i, where the subscript i denotes the chemical species. If the number of molecules of type i per cubic metre is ρ_{Ni}, the total cross section of the gas in a cubic metre of air is $\sum_i \rho_{Ni} A_i$. Hence, if one photon enters the side of a one metre cube, the probability of it being scattered is $\sum_i \rho_{Ni} A_i$. But this is the probability of scattering per 1 metre travelled, so the mean free path of a photon is,

$$\Lambda = \frac{1}{\sum_i \rho_{Ni} A_i} \tag{16}$$

It is convenient to define the average cross section by,

$$\rho_N \langle A \rangle = \sum_i \rho_{Ni} A_i = \frac{1}{\Lambda} \tag{17}$$

where ρ_N is the number density of all molecules. This *would* establish the connection between the cross section(s) and the mean free path if the cross sections were independent of frequency. But the cross sections vary greatly with frequency and so (17) is ambiguous. Clearly some average over frequencies is required, but not a simple average because only a certain range of frequencies occurs in the black body spectrum. Consideration of this issue leads to a particular weighted average cross section, the Rosseland average, defined by,

$$\frac{1}{\langle A\rangle_R}=\frac{\int \frac{1}{\langle A\rangle}\cdot\frac{dB}{dT}\cdot d\omega}{\int \frac{dB}{dT}\cdot d\omega} \tag{18}$$

where $B=\frac{\hbar\omega^3}{\pi^2c^3\left(e^x-1\right)}$ is the energy density of black body radiation, whose temperature derivative is thus,

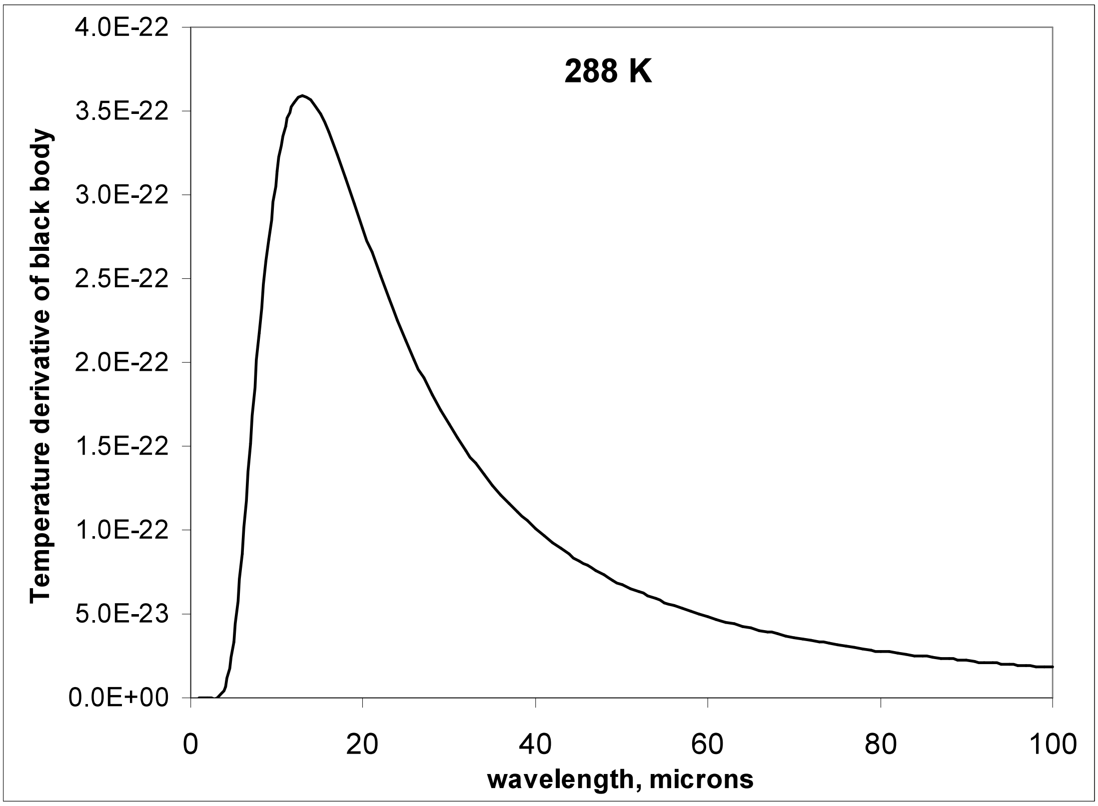
$$\frac{dB}{dT}=\frac{\hbar}{\pi^2T}\left(\frac{\omega}{c}\right)^3\frac{xe^x}{\left(e^x-1\right)^2} \tag{19a}$$

where,

$$x=\frac{\hbar\omega}{kT} \tag{19b}$$

and $\omega=2\pi f$ where f is the frequency, and the wavelength is thus $\lambda=c/f$. Figure 2 illustrates $\frac{dB}{dT}$ for a mean surface temperature of 288 K.

Figure 2: The function dB/dT for 288 K



(17) and (18) therefore give,

$$\Lambda=\frac{\int \frac{1}{\langle A\rangle}\cdot\frac{dB}{dT}\cdot d\omega}{\rho_N\int \frac{dB}{dT}\cdot d\omega} \tag{20}$$

6.2 Relation of the Mean Free Photon Path to Emissivity

The probability that a given photon will survive travelling distance x without being scattered or absorbed is,

$$\text{probability of free travel over distance } x = \exp\left\{-\frac{x}{\Lambda}\right\} \tag{21}$$

At one optical depth (Λ) below the outer extremity of the atmosphere, the effective emissivity is unity. Hence the temperature of this position in the atmosphere is given by (15) with the emissivity replaced by unity,

$$T_1^4 = \frac{(1-a)S_0}{4\sigma} \tag{22}$$

which gives $T_1 = 255$ K for $a = 0.3$. Assuming an average lapse rate in the troposphere of 6.5°C per km, and an average surface temperature of 288 K, this temperature occurs at a height above the ground of $h_1 = 5.08$ km.

The effective emissivity at ground level is the probability that a photon emitted from ground level will survive propagation over the height h_1 without being scattered or absorbed (after which its effective emissivity becomes unity). But this probability is given by (21) with $x = h_1$ so we conclude that,

$$\varepsilon = \exp\left\{-\frac{h_1}{\Lambda}\right\} \tag{23}$$

Using the emissivity we have already established (0.6), and $h_1 = 5.08$ km, equ.(23) gives a mean free photon path of 9.95 km. Hence (20) tells us that,

$$\Lambda = \frac{\int \frac{1}{\langle A \rangle} \cdot \frac{dB}{dT} \cdot d\omega}{\rho_N \int \frac{dB}{dT} \cdot d\omega} = 9.95 \text{ km} \tag{24}$$

Of course the mean free path strictly varies with height, because the temperature and pressure of the atmosphere, and hence both ρ_N and dB/dT, vary with height. For sake of argument we shall use conditions at the mid-height of $h_1/2 = 2.54$ km, i.e., a temperature of 271.5 K and a pressure of 0.717 bara. (The later is based upon a pressure reduction with height of a factor of $\exp\{-h/7.64km\}$, with h in km). The number density of any gas at STP is 2.68 x 10^{25} m^{-3}, and hence the number density of the atmosphere at $h_1/2$ is,

$$\rho_N = (273.15/271.5)*(0.717/1)* 2.68 \times 10^{25} \text{ m}^{-3} = 1.93 \times 10^{25} \text{ m}^{-3} \tag{25}$$

So the density is 72% of that at ground level. Rather than attempt a dead-reckoning reproduction of the mean free path (9.95 km) using the cross sections of all the contributing species, instead we shall use an effective frequency-independent mean cross section, averaged over all species except CO_2, denoted A_0.

If f is the fraction of the molecules in the atmosphere which are CO_2. and if the CO_2 cross-section per molecule is A_{CO_2}, the cross-section averaged over all molecules in the atmosphere is,

$$\langle A \rangle = (1 - f)A_0 + fA_{CO_2} \tag{26}$$

The factor of $1 - f$ is virtually unity and can be absorbed into the definition of A_0. Hence (24) becomes,

$$\Lambda = \frac{\int \frac{1}{(A_0 + fA_{CO_2})} \cdot \frac{dB}{dT} \cdot d\omega}{\rho_N \int \frac{dB}{dT} \cdot d\omega} = 9.95 \text{ km} \tag{27}$$

Taking 2008 as our reference year, when the CO_2 concentration was 385 ppm, i.e., $f = 0.000385$, then (27) holds for a frequency independent A_0 if,

$$A_0 = 3.906 \times 10^{-30} \text{ m}^2 \tag{28}$$

or a Rosseland average cross section of $\langle A \rangle = 1/\rho_N \Lambda = 5.206 \times 10^{-30} \text{m}^2$.

We can now re-evaluate (27) for any desired CO_2 concentration, f, from which we find the fractional change in the mean free path, $\delta\Lambda / \Lambda$, using 385 ppm (i.e., 2008) as datum. The relationship, (23), between Λ and the emissivity then permits the fractional change in emissivity to be found. Taking the derivative of (23) yields,

$$\frac{\delta\varepsilon}{\varepsilon} = -\ln\varepsilon \cdot \frac{\delta\Lambda}{\Lambda} = 0.511 \frac{\delta\Lambda}{\Lambda} \tag{29}$$

However, (29) assumes the height of the layer of unit optical depth (h_1) remains unchanged. But because there is almost the same proportion of CO_2 in the air above h_1 as below it, the position of unit optical depth is bound to rise, increasing h_1. On the other hand, the average temperature gradient between the ground and h_1 must increase as the CO_2 concentration increases, due to the greater impedance to heat flow. Consequently an upper bound to the increase in h_1 is obtained by dividing the average temperature rise obtained assuming (29) by the lapse rate, i.e.,

$$\delta h_1 \le \frac{\delta T_{av}}{6.5} \text{ (km)} \tag{30}$$

(29) is then replaced by,

$$\frac{\delta\varepsilon}{\varepsilon} = -\ln\varepsilon \cdot \frac{\delta\Lambda}{\Lambda} - \frac{\delta h_1}{\Lambda} = 0.511 \frac{\delta\Lambda}{\Lambda} - \frac{\delta h_1}{\Lambda} \tag{31}$$

Note that the change in h_1 increases the magnitude of the change in the emissivity, because $\delta\Lambda < 0$ when $\delta h_1 > 0$, and vice-versa. So, whilst (29) is a lower bound for the emissivity change, an upper bound is,

$$\frac{\delta\varepsilon}{\varepsilon} = 0.511 \frac{\delta\Lambda}{\Lambda} - \frac{\delta T_{av}}{6.5\Lambda} \tag{32}$$

In summary, a postulated change in CO2 concentration is used to find the fractional change in Λ which in turn is used to find the fractional change in emissivity from either (29) or (32). This revised emissivity is then used in the model of §5.

7. Results - Uniform Albedo Approximation

The temperature changes due to CO_2 increases are given in Table 1 (lower bounds) and Table 2 (upper bounds), and in graphical form in Figure 3.

The agreement with the IPCC calculations is very good

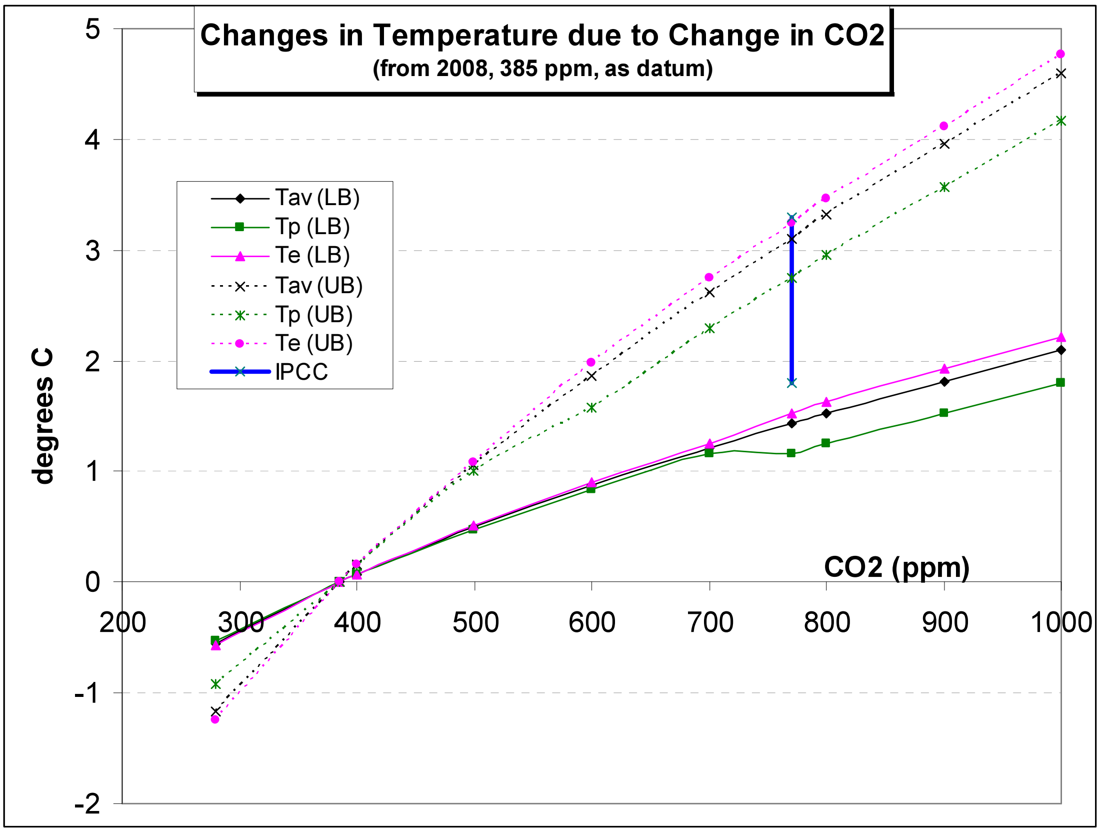
Table 1: Lower Bound Temperature Changes, ^oC (based on equ.29)
changes are with respect to 2008 (385 ppm)
T_e = equatorial temperature; T_p = polar temperature; T_{av} = global average

CO_2 ppm	Λ	LB emissivity	LB del-Te	LB del-Tav	LB del-Tp	LB del(Te-Tp)
280	10.10	0.605	-0.57	-0.55	-0.53	-0.04
385	9.95	0.600	0.00	0.00	0.00	0.00
400	9.93	0.599	0.07	0.07	0.07	0.01
500	9.82	0.596	0.51	0.50	0.47	0.04
600	9.72	0.593	0.90	0.88	0.83	0.06
700	9.63	0.590	1.25	1.22	1.16	0.09
770	9.57	0.588	1.53	1.43	1.16	0.37
800	9.54	0.587	1.63	1.53	1.26	0.37
900	9.47	0.585	1.92	1.82	1.53	0.40
1000	9.40	0.583	2.22	2.10	1.80	0.42

Table 2: Upper Bound Temperature Changes, ^oC (based on equ.32)
changes are with respect to 2008 (385 ppm)
T_e = equatorial temperature; T_p = polar temperature; T_{av} = global average

CO_2 ppm	del_h_1 (km)	LB emissivity	LB del-Te	LB del-Tav	LB del-Tp	LB del(Te-Tp)
280	-0.085	0.610	-1.25	-1.16	-0.91	-0.34
385	0.000	0.600	0.00	0.00	0.00	0.00
400	0.011	0.599	0.16	0.16	0.15	0.01
500	0.076	0.591	1.08	1.06	1.01	0.08
600	0.135	0.585	1.97	1.87	1.58	0.40
700	0.187	0.579	2.75	2.62	2.29	0.46
770	0.220	0.575	3.24	3.10	2.75	0.49
800	0.235	0.573	3.46	3.32	2.96	0.51
900	0.279	0.568	4.12	3.96	3.56	0.56
1000	0.324	0.563	4.77	4.59	4.17	0.60

Figure 3: Uniform Albedo Model Results - Temperature Changes
The range of IPCC results for doubling the 2008 CO_2 concentration (to 770 ppm) is shown as the bold blue line



Future Work

- Include latitude variation in albedo
- Include effect of CO2 changes on albedo

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