

## Chapter 57

### The Total Mass-Energy of the Universe

**The demand for general covariance puts the conservation of mass-energy in a special category. Gravitational energy must exist but is a terribly slippery fish and cannot be pinned down. Consequently the question of the total universal mass-energy appears irresolvable unless there is somewhere ‘outside’ the universe in which to stand to do the measurement. Nevertheless there is a perennial temptation to regard the total mass-energy as zero, since this makes creation *ex nihilo* consistent with the principle of conservation. This Chapter also presents a Newtonian cosmology, the poor man’s approach to the Friedmann equation and the expansion of the universe.**

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#### 1. Is there such a thing as the “Total Mass-Energy of the Universe”?

Venture into this area only with the greatest circumspection. There are major technical barriers to a satisfactory conclusion. Some would say that this is an understatement and that actually it makes no sense to talk of the total mass-energy of an arbitrary universe/spacetime.

This latter point of view can be argued as follows. Relativists know that mass-energy is not a scalar quantity. For a localised system, say a particle, the mass-energy is just the time component of a momentum 4-vector,  $P^\mu$ . Consequently its magnitude is relative to the chosen coordinate system. There is nothing deep or subtle about this. It merely means that the observed kinetic energy of a particle depends upon the relative state of motion between the observer and the particle. So, with respect to what coordinate system would you like the total mass-energy of the universe calculated? There is no satisfactory answer (in general). This objection has even greater force when applied to momentum. If we can evaluate a total  $P^0$  then presumably we can evaluate a total  $P^i$ . But how disconcerting would it be to be told that the total momentum of the universe were  $X$  kgm/s? With respect to what? The reason for our different reactions in the two cases is that our intuition is non-relativistically conditioned to regard mass-energy as a scalar. But it is not.

One response to this issue might be to say, “Ah, clearly the total  $P^\mu$  of the universe must be zero, because this is the only case in which all linear transformations would leave its components invariant and hence produce a unique answer”. I do not propose this as a serious argument. However, if it were, it invites you to consider the universe as an isolated system embedded in a greater super-universe. This illustrates the source of the problem. In order to weigh the universe you must have somewhere outside it on which to stand [paraphrasing Misner, Thorne & Wheeler (1973)]. From this perspective we see that we cannot (in general) deduce the total mass-energy of a universe by integration of quantities within the spacetime of the universe itself. Calculation of the total mass-energy of the universe would seem to require some theory which addresses multiple universes in some larger spacetime.

Well then, if the problem is intractable, what more can we have to say? Firstly, it is important to examine the nature of gravitational energy in more detail, since it has a different status from other forms of energy. Secondly, it is illuminating to consider a Newtonian cosmological model. This has the advantage that our relativistic scruples

are by-passed and a definite result for the total energy is obtained. Whilst this is obviously of dubious relevance to the true relativistic case, one may hope that it provides faithful guidance, if only on the grounds that the Newtonian model correctly reproduces the Friedmann equation. Finally, it is also worth reviewing very briefly what other theories have a bearing on the issue of the total mass-energy (such as inflation theory, for example). However, the source of the driving fascination of the issue of the total mass-energy is the creation of the universe.

The most obvious problem with the creation of the universe in the Big Bang is that we appear to get something from nothing. Before the Big Bang there was nothing, but afterwards we have the whole universe in embryo. Ostensibly this is the ultimate violation of the conservation of mass-energy. An elegant resolution of this problem is provided if the total mass-energy of the universe is actually zero.

Relativists will tell us that this is a mistaken way of looking at the matter. Time itself, they will say, was created in the Big Bang. So there was no “before” and we should not worry about the conservation issue. But does this not strike you as sophistry? Perhaps it is sound physics, but it leaves one feeling less than satisfied. The reason is that it removes the instant of creation from the reach of enquiry by placing it outside the jurisdiction of causality. It seems that if we wish to bring the creation event itself into the scope of science then we must be super-observers standing outside of normal spacetime. Is there then a super-causality in terms of which we can ask “how and why did the Big Bang occur”? Would this super-causality respect the conservation of mass-energy in some sense? We have no grounds for making any such claim. But the centrality of the conservation laws in physics is such that, if we are to continue science into this super-observer realm outside of normal spacetime, our inclination will be to take the conservation laws along with us. The great appeal of the zero mass-energy hypothesis is that it makes the creation of the universe *ex nihilo* consistent with existing scientific principle. But just how credible is it?

## **2. How Could the Total Mass-Energy be Zero?**

The contents of the universe with which we are familiar all contribute positive mass-energy. This is true of the ordinary matter of which the stars, planets, etc., are composed. It is also true of the electromagnetic radiation and neutrinos which pervade the whole of space. It is also true of the dark matter which is postulated in order to explain the gravitational behaviour of galaxies, both their initial formation and their observed rotation. The attractive gravity of this postulated dark matter is its key attribute, and hence it necessarily has positive mass and hence positive mass-energy.

So how can the total mass-energy of the universe possibly be zero if its contents all have positive, non-zero, mass-energy? The answer is the gravitational potential energy, which so far we have not considered. It is clear that the sign of the absolute gravitational potential is negative. Gravity is attractive. Kinetic energy increases as a pair of gravitating bodies move closer together, so their mutual gravitational potential energy must decrease. But their potential energy is zero when infinitely distant. So, at a finite distance their gravitational potential must decrease below zero, i.e., it must be negative.

Can the negative gravitational potential energy possibly have a large enough magnitude to cancel with the positive mass-energy of the whole contents of the universe? Our intuition recoils at the idea. On the scale of our planet it is clearly false. The gravitational potential energy of the Earth is obviously of paltry proportions

compared to its mass-energy. Remarkably, however, on the scale of the whole universe the answer is that the gravitational energy *could* be of the right order of magnitude, as we shall see below. But, for the general relativistic spacetimes which are believed to describe our universe, the calculation of either gravitational energy or total energy is fraught with difficulties. These difficulties are described in the next section. The section after that will make plausible the suggestion that the total energy *could* be zero by means of a well defined calculation using a Newtonian cosmology. The final section discusses briefly some other reasons for believing the total energy might be zero.

### 3. Local and Global Expressions of Conservation in GR

The approach taken here is rather simplistic. For a more rigorous treatment of energy in general relativity see R.M.Wald (1984) or J.Stewart (1996). Our intention is only to convey the nature of the difficulty in defining a unique total universal mass-energy. Let us start by reviewing how conservation laws are formulated mathematically in a spacetime continuum.

Consider a flat spacetime and some 4-vector  $J^\mu \equiv (J^0, \vec{J})$  representing the density and flux of some conserved quantity. The local expression of conservation is the vanishing of the divergence of this 4-vector density, which in flat spacetime and in Lorentzian (Cartesian) coordinates is  $J^\mu{}_{,\mu} = 0$ . That this local differential condition corresponds to global conservation can be seen by integration of the divergence over a prismatic spacetime region defined by a spatial volume  $V$  extruded between two time slices  $t_1$  and  $t_2$ . We thus find, by Gauss's theorem,

$$\int J^\mu{}_{,\mu} d^4x = 0 = \iiint_{\partial V} J^\mu dS_\mu = \int_{t_1}^{t_2} \iint_{\delta V} J^i dA_i dt + \int_{t_2} J^0 dV - \int_{t_1} J^0 dV \quad (3.1)$$

In (3.1),  $dS_\mu$  is an element of the 3-surface which forms the boundary of the 4-dimensional volume integration in the first expression. This closed 3-surface comprises three parts:-

- (i) The curved surface defined by the boundary,  $\delta V$ , of the spatial volume projected along the time axis over the interval  $[t_1, t_2]$ . This is the curved surface of the prism.
- (ii) The 'bottom end cap' of the prism defined by the time slice  $t_1$  and the spatial volume  $V$ ;
- (iii) The 'top end cap' of the prism defined by the time slice  $t_2$  and the spatial volume  $V$ .

If the physical system in question is assumed to be confined within the spatial volume  $V$  so that  $J^\mu = 0$  on the boundary of  $V$  then the first term on the RHS of (3.1) is zero and it becomes,

$$Q = \int_{t_1} J^0 dV = \int_{t_2} J^0 dV \quad (3.2)$$

The quantity  $Q$ , defined by the spatial integral of  $J^0$  and hence equal to the amount of 'stuff' within  $V$ , is conserved since its value is the same whatever time is used to

carry out the integration. More generally, if  $J^\mu$  does not vanish on the boundary of the chosen spatial region,  $V$ , the interpretation of  $\bar{J}$  as the flux of ‘stuff’ out of unit area per unit time means that (3.1) remains consistent with the conservation of ‘stuff’, i.e., it gives,

$$\int_{t_2} J^0 dV = \int_{t_1} J^0 dV - \int_{t_1}^{\int_{t_2}} \oint_{\partial V} J^i dA_i dt \quad (3.3)$$

and (3.3) identifies the amount of ‘stuff’ within region  $V$  at time  $t_2$  with amount at time  $t_1$  less the amount which has flowed out through its boundary in the period  $[t_1, t_2]$ , i.e., that ‘stuff’ is again conserved.

What happens to this argument if spacetime is curved? The local conservation condition, i.e., the vanishing of the ordinary divergence,  $J^\mu{}_{,\mu} = 0$ , becomes the vanishing of the covariant divergence,  $J^\mu{}_{;\mu} = 0$ . The scalar of 4-volume is  $\sqrt{|g|} \cdot d^4x$  so that  $\int \sqrt{|g|} \cdot J^\mu{}_{;\mu} d^4x$  is a scalar with respect to arbitrary coordinate transformations. But we have an algebraic identity,

$$\sqrt{|g|} \cdot J^\mu{}_{;\mu} \equiv \left( \sqrt{|g|} \cdot J^\mu \right)_{;\mu} \quad (3.4)$$

Consequently, because the RHS of (3.4) is algebraically of the same form as an ordinary divergence, the argument based on (3.1) proceeds exactly as before except with  $J^\mu$  replaced by  $\sqrt{|g|} \cdot J^\mu$ . So (3.3) becomes,

$$\int_{t_2} \sqrt{|g|} \cdot J^0 dV = \int_{t_1} \sqrt{|g|} \cdot J^0 dV - \int_{t_1}^{\int_{t_2}} \oint_{\partial V} \sqrt{|g|} \cdot J^i dA_i dt \quad (3.5)$$

and this implies conservation of ‘stuff’ in the same way as before.

It appears, then, that the formulation of conservation is quite straightforward in curved spacetime. And so it is for most interpretations of the ‘stuff’ in question. For example, the above formulation applies to the conservation of electric charge, formulated in terms of the 4-vector of current density,  $J^\mu$ . However, when mass-energy is considered we encounter a difficulty.

Why is the conservation of mass-energy different? The reason is that mass-energy is just one component of the 4-vector of energy-momentum,  $P^\nu$ . All four components of this 4-vector are subject to their own conservation law, as required by Noether’s Theorem corresponding to translational invariance in the four spacetime directions. So, to each component of  $P^\nu$  there corresponds a conserved flux density,  $J^\mu$ , and being tensorial in both indices what we end up with is  $T^{\nu\mu}$ , the stress-energy tensor. What is special about the conservation of energy and momentum is that they are the specific quantities which are conserved by virtue of translational symmetry in the spacetime in question. The conserved quantities are intimately linked to the geometry and we are dealing with the tensor  $T^{\nu\mu}$  not merely a single flux density  $J^\mu$ .

Let us consider firstly the conservation of energy/momentum in flat spacetime. Its local formulation is again the vanishing of the ordinary divergence  $T^{\nu\mu}{}_{;\mu} = 0$ . The interpretation of this follows exactly as before, because, in writing the equivalent of

Equ.(3.1), the extra index  $\nu$  simply goes along for a free ride. So, for example, if we assume that the system is localised and that  $V$  is chosen large enough so that  $T^{\nu\mu} = 0$  on the boundary of  $V$ , then the equivalent of (3.2) is,

$$P^\nu = \int_{t_1} T^{\nu 0} dV = \int_{t_2} T^{\nu 0} dV \quad (3.6)$$

Note that (3.6) implicitly requires Lorentzian (Cartesian) coordinates, because otherwise we are not justified in writing the ordinary divergence as  $T^{\nu\mu}_{;\mu} = 0$ .

Equ.(3.6) shows that  $T^{\nu 0}$  can be interpreted as the density of a set of four conserved quantities labelled by  $\nu$ , and the fact that  $P^\nu$  is the momentum 4-vector follows from the derivation of  $T^{\nu\mu}$  from Noether's Theorem. Moreover, even if  $T^{\nu\mu}$  does not vanish on the boundary of  $V$ , the equivalent of (3.3), i.e.,

$$\int_{t_2} T^{\nu 0} dV = \int_{t_1} T^{\nu 0} dV - \int_{t_1} \oint_{\partial V} T^{\nu i} dA_i dt \quad (3.7)$$

can be interpreted in terms of 4-momentum flowing out of  $\partial V$  with a flux density in direction  $i$  of  $T^{\nu i}$ .

So far, so good: but now the problems start. Consider how we may formulate the conservation of energy-momentum in curved spacetime. The ordinary divergence again generalises to the covariant divergence so that the local expression of conservation must be  $T^{\nu\mu}_{;\mu} = 0$ . In fact this equation is forced by the Einstein field equations,  $G^{\nu\mu} = -\frac{8\pi G}{c^4} \cdot T^{\nu\mu}$ , since the vanishing of the covariant divergence of the

Einstein tensor is an algebraic identity,  $G^{\nu\mu}_{;\mu} \equiv 0$ .

However, we cannot simply insert an extra index in Equ.(3.4) and proceed as before. The resulting expression is simply wrong, i.e.,

$$\sqrt{|g|} \cdot T^{\nu\mu}_{;\mu} \neq \left( \sqrt{|g|} \cdot T^{\nu\mu} \right)_{;\mu} \quad (3.8)$$

Instead, the nearest equivalent expression is of the form,

$$\sqrt{|g|} \cdot T^{\nu\mu}_{;\mu} \equiv \left( \sqrt{|g|} \cdot T^{\nu\mu} \right)_{;\mu} + \left( \sqrt{|g|} \cdot \mathfrak{T}^{\nu\mu} \right)_{;\mu} \quad (3.9)$$

where  $\mathfrak{T}^{\nu\mu}$  is a non-linear expression dependent only upon the metric tensor and its first derivatives. Moreover  $\mathfrak{T}^{\nu\mu}$  is not unique since any  $\mathfrak{N}^{\nu\mu}$  may be added to it provided that  $\left( \sqrt{|g|} \cdot \mathfrak{N}^{\nu\mu} \right)_{;\mu} = 0$ . Different authors favour different expressions for  $\mathfrak{T}^{\nu\mu}$ , see, for example, Misner, Thorne & Wheeler (1973), Equ.(20.22), or Adler, Bazin & Schiffer (1975), Eqs.(11.40, 11.54).

Now, using (3.8), we can proceed as before. Firstly let us assume that the supports of  $T^{\nu\mu}$  and  $\mathfrak{T}^{\nu\mu}$  are both contained within  $V$  and hence that  $T^{\nu\mu}$  and  $\mathfrak{T}^{\nu\mu}$  vanish on its boundary. Then the equivalent of (3.6) is,

$$P^\nu = \int_{t_2} \sqrt{|g|} \cdot (T^{\nu 0} + \mathfrak{T}^{\nu 0}) dV = \int_{t_1} \sqrt{|g|} \cdot (T^{\nu 0} + \mathfrak{T}^{\nu 0}) dV \quad (3.10)$$

Note that it is implicit that Lorentzian (Cartesian) coordinates are used to evaluate (3.10), since only then are the terms on the RHS of (3.9) equal to ordinary flat space divergences – and also for another reason discussed below.

In (3.10) we again have a conserved 4-momentum. However, in addition to the term in  $T^{\nu 0}$ , which represents the stress-energy of the matter and fields comprising the physical system, there is also a contribution from  $\mathfrak{T}^{\nu 0}$  which depends only upon the geometry. Since  $\mathfrak{T}^{\nu 0}$  appears to play the role of a 4-momentum density and depends only upon the spacetime geometry, it is natural to interpret  $\mathfrak{T}^{\nu\mu}$  as the stress-energy tensor of the gravitational field itself – natural, but subtly wrong.

In the case of curved spacetime, (3.10) tells us that the conservation of energy and momentum will not hold for the matter and fields alone, but only if gravity is included as part of the source of energy and momentum. This is perfectly proper and just as it should be. Consequently everything looks rosy – apart from three problems, at least two of which are serious.

The first we have noted already. The algebraic expression for the object  $\mathfrak{T}^{\nu\mu}$  is not unique. This does not necessarily matter very much since an alternative expression will differ from  $\mathfrak{T}^{\nu\mu}$  by a term with  $\left(\sqrt{|g|} \cdot \mathfrak{S}^{\nu\mu}\right)_{,\mu} = 0$  so that (3.9) still holds. Any such alternative expression for  $\mathfrak{T}^{\nu\mu}$  will, upon integration, give the same 4-momentum as (3.10).

The second problem is more fundamental. The object  $\mathfrak{T}^{\nu\mu}$  is not a tensor. There is no generally covariant (i.e., tensorial) quantity  $\mathfrak{T}^{\nu\mu}$  for which (3.9) holds. Whilst  $\mathfrak{T}^{\nu\mu}$  is as near as we can get to a stress-energy tensor for gravity, actually there is no such thing. The best we can do is the stress-energy pseudo-tensor,  $\mathfrak{T}^{\nu\mu}$ , sometimes called an “energy-momentum complex”. This is a more serious problem as regards how (3.10) is to be understood.

In deriving (3.10) from (3.9) we have made the replacement  $dx_1 dx_2 dx_3 = dV$ . But we are only justified in interpreting  $dV$  as a spatial volume element if these coordinates are Cartesian. This is possible only because in this special case we have assumed the spacetime to be asymptotically flat. Equ.(3.10) must be calculated using Lorentzian (spatial Cartesian) coordinates. For a true tensor equation, the coordinate system employed would not matter. The specific coordinate system is required precisely because (3.10) is *not* a tensorial equation. Only by specifying the coordinate system does its value become unique. This is a serious problem because, in general, the spacetime will *not* be asymptotically flat. Without an asymptotically flat region we are left without a definition of the total mass-energy.

The most stark illustration of the non-tensorial nature of  $\mathfrak{T}^{\nu\mu}$  is to recall that the coordinate system can always be chosen to be locally Lorentzian, so that the non-Euclidean nature of the geometry is not locally apparent. This means that  $\mathfrak{T}^{\nu\mu}$  can be transformed to zero at any point by appropriate choice of the coordinate system. If it were a tensor, this would be sufficient to imply that it was identically zero in all coordinate systems. Hence, any attempt to pin down the gravitational energy density at a point is frustrated – gravitational energy is not localisable in GR. Actually this is rather fortunate (or perhaps we should say “unavoidable”) since the gravitational

energy is negative. It would be hard to understand the mass supposedly equivalent to an absolutely negative energy if this were also localisable.

The third problem is again fundamental. Even had a tensorial total stress-energy tensor existed, expressions like  $\int \sqrt{|g|} \cdot T^{\nu 0} d^3x$  are essentially meaningless in curved spacetime. This is because the integrand is vector-valued and the direct addition of vectors at different spacetime positions is, in general, meaningless. Only in the case of flat spacetime with Cartesian coordinates does this become meaningful as a consequence of the fact that the unit vectors are the same everywhere.

Despite all these technical difficulties in the general case, the total mass-energy of a universe which is asymptotically flat can be uniquely and sensibly defined. The asymptotically flat region effectively forms the platform on which we can stand to perform the weighing of the finite region containing mass and curvature. That this can be done is virtually obvious on physical grounds. Standing in the asymptotically flat zone, outside the region of matter fields and curvature from which the mass-energy originates, one need only measure how the gravitational field is reducing. Sufficiently far from the sources this must approximate the far field of the Schwarzschild or Kerr solutions. Fitting to the metric of those solutions provides the total gravitating mass.

Finally, though, we are left with the truly intractable case: curved spacetimes with no asymptotically flat region. There is now no region within which we can find a Lorentzian (Cartesian) coordinate system: nowhere we can stand to do the weighing. There is then no definition of what we could mean by the total mass-energy.

The spacetimes normally assumed to be appropriate for our universe, the FLRW spacetimes, do not have asymptotically flat regions when  $k \neq 0$ . The case of  $k = 0$  is globally flat, but being infinite and homogeneous there is still ‘nowhere to stand to do the weighing’. Hence the total mass of the FLRW spacetimes (or their mass per unit volume for the infinite universes) is undefined. Consequently most physicists regard the question of total universal mass-energy as meaningless.

However a few have not been discouraged and have attempted to calculate the total mass-energy of certain solutions to the Einstein equations, most often by evaluating the gravitational pseudo-tensor in coordinates of their choice. The result has most often been reported as zero, but not always. For example, the total energy of certain FLRW and Bianchi spacetimes have been claimed to be zero by Rosen (1994), Cooperstock (1994), Johr et al (1995), Banerjee and Sen (1997), Xulu (2000) and Radinschi (2001). However, in view of the above remarks it is not clear that these results have any significance. And evaluation in other coordinate systems can result in non-zero energies, e.g., Garecki (2006, 2007).

Moreover, the same methodologies that led to zero total cosmological energy can also lead to the conclusion that gravitational waves carry zero energy, e.g., Rosen and Virbhadra (1993). If true this would mean that gravity waves will never be detected. But the ‘sticky bead’ argument [see, for example, Bondi (1957)] is a convincing argument that, on the contrary, gravitational waves do carry energy – and hence that calculations such as that of Rosen and Virbhadra (1993) are misleading. For a rigorous demonstration that gravitational waves radiate positive energy to infinity see Wald (1984). But there is now also observational evidence. Whilst gravitational waves have not yet been detected, there is at least one instance of good indirect evidence.

The pulsation period of the binary pulsar PSR B1913+16 has been observed to decay by 76.5 microseconds per year, a change of  $\sim 0.13\%$ . This decay rate agrees very well with GR calculations of the rate at which gravity waves are expected to radiate away energy due to the mutual orbit of the two neutron stars. The agreement is better than 0.2% which is strongly suggestive of the reality of energy-carrying gravitational waves. The discovery of this binary pulsar and its implications won the 1993 Nobel Prize in physics for Joseph Taylor and Russell Hulse.

#### 4. Total Energy in a Newtonian Cosmology

In this section we attempt to make plausible the idea that the total energy of the universe could be zero (if only we could define it) by consideration of a Newtonian cosmology in which total energy is well defined.

When cosmologists talk of a Newtonian cosmology they generally do not mean the cosmology in which Newton believed. Most people before Hubble, including Newton, believed that the universe was essentially static and eternal. Of course Newton was well aware of the conflict between the universe being both static and also subject to universal gravitational attraction. His view was that this conflict was resolved by virtue of the universe being infinite, so that the forces on every material element (i.e., the stars) were balanced due to being equal in all directions. (Was he troubled by the obvious instability of such an arrangement? Why did he not consider the possibility of a rotating universe, which would be consistent with a static radial mass distribution whilst also being finite? The required rotation rate between a pair of solar mass stars at a spacing of 4 lyrs would be a mere 0.17 minutes of arc per millennium, and hence would not have been apparent. Not then, anyway. Today it would probably be detectable via an anisotropy of the cosmic microwave background, with a cylindrical symmetry).

##### 4.1 The Newtonian Cosmology Model

A Newtonian cosmology is generally understood to mean a particular model of the universe based on Newtonian gravity. The universe is considered to be a finite sphere of uniform density embedded within an infinite Euclidean space. This is allowed in Newtonian theory because there is no consistency condition between the mass distribution and the spacetime geometry. Unlike Newton's own cosmology, however, this Newtonian cosmology provides a model of an expanding universe. The tangential velocities are assumed negligible. It is a simple matter to derive the equation governing the radial velocity. Remarkably this equation is identical, in the absence of dark energy and pressure, to the Friedmann equation which results from relativistic cosmology. For this reason it is not wholly inappropriate to consider the Newtonian model. In the context of the problem of the total energy, the advantage of considering the Newtonian model is that its gravitational energy is well defined and simple to calculate.

The equation of motion of the Newtonian model is derived as follows. Consider a small mass element,  $m$ , at a radius  $r$  from the centre of the sphere. The gravity inside a uniform spherical shell is zero, and hence only the matter at radii less than  $r$  contributes to the motion of  $m$ . Moreover, the gravity due to the matter at radii less than  $r$  is the same as if all the mass were concentrated at the centre (because the mass distribution remains spherically symmetrical at all times). Hence, the purely radial motion obeys,



$$m\ddot{r} = -\frac{Gm}{r^2} \cdot \frac{4\pi}{3} \rho r^3 \Rightarrow \ddot{r} = -\frac{4\pi}{3} G\rho r \quad (4.1)$$

But the assumption that the density is always uniform plus the conservation of mass can be written,

$$\rho(t)r^3(t) = \rho_0 r_0^3 = \text{constant} \quad (4.2)$$

where the subscript  $_0$  denotes some arbitrary reference time and  $r$  is the radial coordinate of any mass particle whose position at the reference time is  $r_0$ . Now, since  $\dot{r} \equiv \dot{r}dr/dr$  we can integrate (4.2) explicitly to give,

$$\int \dot{r}dr = -\frac{4\pi}{3}G \int \frac{\rho_0 r_0^3}{r^2} dr \Rightarrow \frac{\dot{r}^2}{2} = \frac{4\pi}{3}G \frac{\rho_0 r_0^3}{r} + \text{constant} \quad (4.3)$$

where the constant of integration is constant with respect to time but may (and indeed must) depend upon the reference coordinate,  $r_0$ , of the mass element in question. It is worth noting a blunder which it would be easy to make when carrying out the integration over  $r$  on the RHS of (4.3). Since the density is uniform, i.e., independent of  $r$ , we might be tempted to conclude that the RHS was instead, modulo the constant,

$$-\frac{4\pi}{3}G\rho \int r dr = -\frac{2\pi}{3}G\rho r^2 = -\frac{2\pi}{3}G \frac{\rho_0 r_0^3}{r} \quad (4.4)$$

a result which is of the wrong sign as well as differing by a factor of 2. The error results from forgetting that the integration should track the path of the given mass element as it evolves in time,  $r(t)$ , as opposed from an integration over the sphere at a given time, which is the quantity evaluated in (4.4). The actual integration we need is really a surrogate for a time integral, i.e.,  $\int \dots dr \equiv \int \dots \dot{r} dt$ . Since the density,  $\rho$ , is time dependent, it cannot be taken outside of the integral, as was done in (4.4). Instead the integrand must be re-expressed as in (4.3) because  $\rho_0 r_0^3$  is time independent, by (4.2), thus permitting the integral to be carried out.

We labour this point because it has an important bearing on the integration constant in (4.3). Let us write it in terms of a dimensionless  $k$  as,

$$\dot{r}^2 = \frac{8\pi}{3}G \frac{\rho_0 r_0^3}{r} - kc^2 = \frac{8\pi}{3}G\rho r^2 - kc^2 \quad (4.5)$$

The claim that  $k$  is constant with respect to time, rather than with respect to the reference coordinate,  $r_0$ , is precisely because the integration is really over time. But why have we also claimed that  $k$  must depend upon  $r_0$ ? The reason is that the derivation of (4.5) was based upon the assumption that the sphere remains uniform at all times, i.e., (4.2). The equation of motion, (4.5), must yield this result or it is simply contradictory. But uniform density at all times requires the whole sphere to dilate or contract uniformly, which means that  $r/r_0$  is the same for all points in the sphere.

Hence  $\dot{r}/r_0$  is also the same at all points in the sphere. Dividing (4.5) by  $r_0^2$  we get,

$$\left(\frac{\dot{r}}{r_0}\right)^2 = \frac{8\pi}{3}G\rho \left(\frac{r}{r_0}\right)^2 - \frac{kc^2}{r_0^2} \quad (4.6)$$

The LHS of (4.6), and the first term on the RHS, are thus both independent of  $r_0$ ,

from which it follows that  $\tilde{k} = k / r_0^2$  must be independent of  $r_0$ . So (4.6) is better written as,

$$\left(\frac{\dot{r}}{r_0}\right)^2 = \frac{8\pi}{3} G\rho \left(\frac{r}{r_0}\right)^2 - \tilde{k}c^2 \quad (4.7)$$

Whilst  $k$  is dimensionless and constant in time, it varies across the sphere. In contrast,  $\tilde{k}$  has the dimensions of reciprocal length<sup>2</sup> and is constant both in time and over the sphere.

Note that  $\tilde{k}$  has the dimensions of curvature (the same dimensions as the Einstein tensor,  $G_{\mu\nu}$ ). This is not accidental. Eqs.(4.5-7) are identical to the Friedmann equation, to which the Einstein equations of general relativity reduce for a homogeneous and isotropic spacetime. More accurately, Eqs.(4.5-7) are the particular case when the only source of gravity is a uniform mass density,  $\rho$ , and there is no cosmological constant (dark energy). In GR the parameter  $\tilde{k}$  is the curvature of space.  $\tilde{k} = 0$  corresponds to a flat space,  $\tilde{k} > 0$  to positive curvature, and  $\tilde{k} < 0$  to negative curvature.

Note that if  $\tilde{k} = 0$ , Equ.(4.7) is just  $\dot{r} = Hr$ , i.e., the Hubble relation which says that radial velocity is proportional to radial distance. The equation tells us the value of the Hubble parameter, namely  $H = \sqrt{\frac{8\pi}{3} G\rho}$ .

However, in the Newtonian interpretation,  $\tilde{k}$  is a constant determined by the initial conditions. For example, if the velocity at a given radius is specified at some datum time then  $\tilde{k}$  is determined by (4.7). The velocity at all other radii at this time must be consistent with Equ.(4.7) with this same value of  $\tilde{k}$  if the presumption of uniform density is to remain true as the universe evolves. So the Newtonian model is consistent with a permanently homogeneous universe only if the initial conditions are specially contrived.

Equ.(4.7) can also be written as,

$$\dot{r}^2 = \frac{2GM(r)}{r} - kc^2 \quad (4.8)$$

where  $M(r)$  is the mass up to radius  $r$ , which is independent of time because this mass always remains equal to its starting value, say  $M(r_0)$ . Equ.(4.8) clearly implies that the velocity reduces as the universe expands to a bigger radius – which is just kinetic energy being converted to potential energy. But does the universe expand forever or does it reach a maximum size and collapse? If the latter occurred there would be a time at which the velocity were zero, so that the maximum radius is given by,

$$\dot{r}^2 = \frac{2GM(r)}{r} - kc^2 = 0 \quad (4.9)$$

A finite solution to (4.9) exists only if  $k > 0$ , which is interpreted in GR as a space of positive curvature. The maximum radius attained by a trace mass which was initially at radius  $r_0$  is thus,

$$r_{\max} = \frac{2GM(r_0)}{kc^2} = \frac{8\pi}{3} \cdot \frac{G\rho_0 r_0}{\tilde{k}c^2} \quad (4.10)$$

In particular the maximum radius of the sphere (i.e., the whole matter content of the universe) is,

$$a_{\max} = \frac{2GM}{k_a c^2}, \text{ where } k_a = \tilde{k}a_0^2 \quad (4.11)$$

where  $M$  is the mass of the whole universe (the sphere) and  $k_a = \tilde{k}a_0^2$  is the value of the dimensionless constant which applies at the surface of the sphere.

In the limit that  $k = 0$ , interpreted in GR as flat space, the velocity asymptotically approaches zero as the sphere becomes infinite, i.e.,  $\dot{r} \rightarrow 0$  as  $a \rightarrow a_{\max} \rightarrow \infty$ .

Finally, if  $k < 0$  then (4.8) implies that the outward velocity is always positive. Consequently such a universe expands forever (interpreted in GR as a space of negative curvature).

## 4.2 The Energy of the Newtonian Model

There are several ways to derive the total gravitational potential energy of the Newtonian cosmology model. The answer is,

$$PE = -\frac{3}{5} \cdot \frac{GM^2}{a} \quad (4.12)$$

where  $a$  is the radius of the sphere at the time in question, and  $M$  is the mass of the sphere (which, of course, is a constant in this Newtonian model, since there is no mass-energy equivalence to confuse things). To derive this result recall that the equation for the Newtonian gravitational potential is,

$$\nabla^2 \phi = 4\pi G\rho \quad (4.13)$$

The solution within the uniform spherical mass is,

$$r \leq a: \quad \phi(r) = \frac{GM}{2a} \left( \frac{r^2}{a^2} - 3 \right) \quad (4.14)$$

whereas at radial positions outside the sphere of matter we have,

$$r > a: \quad \phi(r) = -\frac{GM}{r} \quad (4.15)$$

Both (4.14) and (4.15) obey (4.13), in the latter case with  $\rho = 0$ , as is easily confirmed using the spherically symmetric operator  $\nabla^2 \rightarrow \frac{1}{r^2} \partial_r (r^2 \partial_r)$ . In addition the two solutions match at  $r = a$  and (4.15) reduces to zero at large distances from the source. This establishes that (4.14,15) is the unique solution satisfying all the boundary conditions.

The potential energy can now be found using the expression for the field energy density of Newtonian gravity,

$$\xi = -\frac{1}{8\pi G} |\nabla \phi|^2 \quad (4.16)$$

The energy of the gravitational field within the sphere is therefore,

$$PE_{r<a} = -\frac{1}{8\pi G} \int_0^a 4\pi r^2 dr \left( \frac{GM \cdot r}{a^3} \right)^2 = -\frac{GM^2}{2a^6} \int_0^a r^4 dr = -\frac{GM^2}{2a^6} \cdot \frac{a^5}{5} = -\frac{1}{10} \cdot \frac{GM^2}{a} \quad (4.17)$$

Whilst the energy of the gravitational field outside the sphere is,

$$PE_{r>a} = -\frac{1}{8\pi G} \int_0^a 4\pi r^2 dr \left( \frac{GM}{r^2} \right)^2 = -\frac{GM^2}{2} \int_a^\infty \frac{dr}{r^2} = -\frac{GM^2}{2} \left( -\frac{1}{r} \right) \Big|_a^\infty = -\frac{1}{2} \cdot \frac{GM^2}{a} \quad (4.18)$$

Hence, adding (4.17) and (4.18) gives the total energy of the gravitational field to be as claimed in (4.12).

Alternatively, we can appeal to the gravitational potential energy of a small element of mass  $\delta m$  being  $\phi \delta m$  at a position where the gravitational potential is  $\phi$ . The sphere can then notionally be assembled gradually, for example layer by layer – or by letting the density increase slowly from zero. Either way the same expression for the total gravitational potential energy of the universe results, (4.12).

We expect that the total kinetic plus potential energy of the universe should be conserved. To check this note that the kinetic energy of a spherical shell can be written, with the aid of (4.7),

$$\delta KE = \frac{1}{2} \dot{r}^2 \cdot 4\pi r^2 \delta r \rho = 2\pi \rho r^2 \delta r \left[ \frac{8\pi}{3} G \rho r^2 - \tilde{k} r_0^2 c^2 \right] \quad (4.19)$$

It is essential that (4.19) is expressed with the  $r_0$  dependence of  $k = \tilde{k} r_0^2$  made explicit. This is because we must next integrate over all radii and  $r_0 = \lambda r$  where the parameter  $\lambda$  varies with time but not with radius. Hence integrating (4.19) over the sphere gives,

$$KE = 2\pi \rho \left[ \frac{8\pi}{3} G \rho \frac{a^5}{5} - \tilde{k} \lambda^2 c^2 \frac{a^5}{5} \right] = \frac{3}{5} \cdot \frac{GM^2}{a} - \frac{3}{10} \cdot k_a M c^2 \quad (4.20)$$

where  $k_a = \tilde{k} a_0^2$  is the value of the dimensionless constant which applies at the surface of the sphere. But from (4.12) this becomes,

$$KE + PE = -\frac{3}{10} k_a \cdot M c^2 \quad (4.21)$$

So the total energy of the universe is a constant of the motion, as one would expect. Remember that the  $KE$  term in (4.21) is the non-relativistic kinetic energy.

Within the context of this Newtonian cosmology model we note the following corollaries of (4.21):-

- For  $k = 0$  (a flat universe in GR) the total non-relativistic energy (ignoring the rest mass energy) is zero,  $KE + PE = 0$ ;
- For  $k > 0$  (positive curvature in GR) the total non-relativistic energy is negative,  $KE + PE < 0$ . This corresponds to the universe being a bound state that cannot expand forever. From the energy perspective, expanding to  $R \rightarrow \infty$  would imply  $PE \rightarrow 0$  and this would require that the  $KE$  become negative, which is not possible. Hence if  $k > 0$  the universe has a maximum possible size, as we have

seen already. In GR this is interpreted as a positive curvature implying a closed, finite universe (assuming there is no cosmological constant);

- For  $k < 0$  (negative curvature in GR) the total non-relativistic energy is positive,  $KE + PE > 0$ . Hence the universe is not a bound state and will expand forever: even as  $PE \rightarrow 0$  at large expansions, the kinetic energy is still positive and the universe is still expanding at a non-zero rate. In GR this is interpreted as a negative curvature implying that the universe is open and infinite.
- Finally, we note that if we choose the boundary condition  $k_a = 10/3$  then we have a closed universe in which  $KE + Mc^2 + PE = 0$ . In as far as  $KE + Mc^2$  is a first order approximation to the relativistic mass-energy, this suggests that a relativistic solution might exist in which the total relativistic energy is zero. Of course this is by no means a proof, it is merely suggestive.

In conclusion, the Newtonian cosmology demonstrates that the total energy of the universe could feasibly be zero. In the Newtonian cosmology this depends upon the contingent choice of suitable initial conditions, i.e., a suitable value for  $\tilde{k}$  and a contrived choice of initial velocities. Hence zero total mass-energy is only possible, not necessary.

## 5. Theories and Ideas Related to the Total Energy Question

### 5.1 Constraints Imposed by the Stability of Matter

The consensus at present is that something colloquially called “dark energy” is driving an accelerated cosmic expansion rate. If so, then the energy of the universe is not even conserved. This is because dark energy involves a constant energy per unit volume, so the total energy increases in proportion to the volume as the universe expands. If dark energy exists then even a closed, finite universe with positive curvature,  $k > 0$ , would expand forever, and at an accelerating rate. However, the accelerating universal expansion is not necessarily caused by dark energy. It could, for example, be a temporary acceleration due to the formation of large scale structure, see Wiltshire (2007) or Buchert (2008). In what follows it is assumed that dark energy, or other forms of ‘energy creation’, do not exist.

Suppose the universe expands forever, i.e.,  $k \leq 0$ . The gravitational potential energy will therefore tend to zero as the matter density becomes infinitely diluted. Of course there will initially be gravitationally collapsed regions: planets, stars, black holes, galaxies, clusters of galaxies, etc. These have their own local negative gravitational potential energy. However even at the scale of galaxies or galactic clusters, the gravitational self-energy is negligible compared with the rest mass. In any case, given long enough, even these structures will prove unstable and evaporate, emitting their component particles. Even black holes will disappear eventually due to Hawking radiation. Ultimately, and this may involve a truly prodigiously long wait, the universe will contain only the stable fundamental particles: electrons, protons, neutrinos and photons. The energy contribution of the photons, once they are no longer being created, will tend to zero because universal expansion will red-shift their individual energy to nothing. The same would be true for any other zero rest mass particles which might still be present (gravitons?). Consequently, if electrons, protons and neutrinos all decayed via some sequence ultimately into zero rest mass particles, the associated energy would tend to zero. On the other hand, if electrons or protons or

the electron neutrinos are absolutely stable then the total mass-energy of the universe cannot be zero. But on the enormous timescales in question, who knows if they are?

Nevertheless, this observation suggests that a universe which expands forever (and in which **the total mass-energy is constant**) **is unlikely to have zero total mass-energy**. **The reason is that it would be necessary for the conditions in the early universe to anticipate the ultimate instability of the fundamental particles. But these two things would appear to be independent. On the other hand it might be that particle instability is logically necessary in some future fundamental theory.**

**However I tentatively suggest that universes with  $k \leq 0$  are unlikely to have zero mass-energy (though I do not want to imply that this is certainly the case).** This aligns with the Newtonian model. For  $k < 0$  the Newtonian model implied a permanent positive mass-energy. For  $k = 0$ , whilst the non-relativistic energy of the Newtonian model became zero, this still leaves a balance of energy due to the rest mass of any remaining particles.

This leaves the case of a finite, closed universe with  $k > 0$ . The Newtonian model was suggestive that a zero total mass-energy might be possible, including the rest mass energy, if the initial conditions were suitably contrived. This case places no requirement on particle instability. The energy associated with the contents of the universe, its particles and fields, would always be non-zero and positive. But because the universe would attain a finite maximum size, the gravitational energy would never be zero either. The two could possibly cancel – but there is no strong case at present that this is so, either from observational cosmology or from relativistic theory.

However what this line of argument suggests is that if one is biased towards a belief that the total mass-energy of the universe is zero, then the universe is most likely to be closed and finite with positive curvature,  $k > 0$ , which re-contracts at some time. It also implies that there is no ‘dark energy’. This latter point is, however, contrary to the current consensus.

## **5.2 The Hamiltonian Formulation of GR**

If GR can be expressed in Hamiltonian form it might be thought that this would provide insight into the total energy since this is how the Hamiltonian is usually interpreted. In fact a Hamiltonian formulation *is* possible. And it is found that the Hamiltonian vanishes in a closed universe, at least in one particular formulation, see Wald (1984) Appendix E. However, once again the meaning of this result is obscured by technical difficulties. To derive the Hamiltonian formalism it is necessary to “parameterise” the theory and to introduce an associated constraint. This arises because of the arbitrariness of how the spacetime is foliated into time and space slices, a step which is essential in the Hamiltonian formulation. Ideally the theory can ultimately be “deparameterised” by inversion of the constraint equation. However, this inversion appears not to be possible in GR due to its non-linear nature. But the Hamiltonian can always be made to vanish by suitable “parameterisation”, and hence its vanishing does not necessarily have any significance. To quote Wald (1984), *If general relativity could be “deparameterised”, a notion of total energy in a closed universe could well emerge.* But this has not been done, as far as I am aware.

## **5.3 Quantum Cosmology: The Wheeler-deWitt Equation**

Now we are heading into quantum gravity territory and must expect the waters to be choppy. But do not despair. We shall end this section with an accessible example.

The Wheeler-deWitt (WdW) equation is the Hamiltonian formulation of GR but as a quantum theory (roughly, anyway). Here we give only the crudest indication of the WdW equation. For a more complete treatment of quantum gravity topics see, for example, Rovelli (2005) or Fauser et al (2000).

Whilst the classical GR Hamiltonian is identically zero (at least for closed spacetimes), in quantum theory we convert the Hamiltonian function into a Hamiltonian operator. The Hamiltonian constraint then becomes  $\hat{H}|\psi\rangle = 0$ . The Hamiltonian operator  $\hat{H}$  has a specific representation in terms of the metric and the curvature, though it is not without technical ambiguities. The analogy with the usual Schrodinger equation,  $\hat{H}|\psi\rangle = i\hbar \frac{\partial}{\partial t}|\psi\rangle \rightarrow E|\psi\rangle$ , suggests that this might imply that the energy is zero. But it is by no means clear that such an interpretation is justified. The reason why the time dependence disappears is that the canonical Hamiltonian formulation, including the Schrodinger equation,  $\hat{H}|\psi\rangle = i\hbar \frac{\partial}{\partial t}|\psi\rangle$ , is clearly not generally covariant but written in terms of a preferred time coordinate. A fully covariant Hamiltonian formulation can be devised, for example the ADM formalism [see for exampleourgoulhon (2006)]. It is not terribly surprising that when this is done the gauge specific time coordinate disappears. The same consideration for the spatial degrees of freedom produces the ‘momentum constraints’,  $\hat{P}^i|\psi\rangle = 0$ .

The wave-function in the Wheeler-deWitt equation could represent the whole universe. One proposed quantum state of the universe which satisfies the Wheeler-deWitt equation is that of Hartle and Hawking (1983). This is defined by the path integral,

$$\psi(h_{ij}) = N \int D[g_{\mu\nu}] \exp\{-S[g_{\mu\nu}]/\hbar\} \quad (5.1)$$

where  $N$  is a normalisation constant and  $S$  is the Euclidean Einstein action for a given spacetime with 4-metric  $g_{\mu\nu}$ , i.e.,

$$S[g_{\mu\nu}] = \frac{1}{16\pi G} \int R\sqrt{|g|} d^4x \quad (5.2)$$

In (5.2),  $R$  is the usual Ricci scalar, expressible in terms of the metric and its first and second derivatives, but the integral in (5.2) is carried out in Euclideanised spacetime, for which the time coordinate has been replaced by  $it$ . This ensures that the action,  $S$ , is positive and hence that (5.1) is well defined (or, at least, it stands a chance of being well defined). The path integral, (5.1), is carried out over all spacetime metrics  $g_{\mu\nu}$  which reduce to the specified spatial 3-geometry,  $h_{ij}$ , on their boundary. The function  $|\psi(h_{ij})|^2$  is proposed to be the probability for the spatial 3-geometry,  $h_{ij}$ , arising. There is no time in the Hartle-Hawking model. The physics is supposed to reside entirely in the timeless  $\psi(h_{ij})$ . Although time appears in the action integral, (5.2), it there plays the role simply of parameterising the variables over which the path integral, (5.1), is conducted.

From these heady heights of abstraction we can produce an accessible illustration. Consider for simplicity a universe which is empty except for a constant zero-point

energy density  $\rho_{vac}c^2$ . Putting  $\Lambda = 8\pi G\rho_{vac}$  the corresponding Friedmann equation follows immediately from (4.5),

$$\dot{r}^2 = \frac{1}{3}\Lambda r^2 - kc^2 \quad (5.3)$$

The other Friedmann equation, in the acceleration, follows by differentiating (5.3),

$$\ddot{r} = \frac{1}{3}\Lambda r \quad (5.4)$$

More correctly, and to ensure general covariance, we should specify this zero point energy as possessing a stress-energy tensor proportional to the metric tensor,  $\Lambda g_{\mu\nu}$ , of which  $\rho_{vac}c^2$  is the component  $\Lambda g_{00}$ . The generally covariant problem is then stated in the form of the Einstein field equations as  $G_{\mu\nu} = -\frac{8\pi G}{c^2}\rho_{vac}g_{\mu\nu} = \Lambda g_{\mu\nu}$  *Need to sort out the factors of  $c^2$  and the sign convention.* However, doing the job properly does result in the equations (5.3) and (5.4).

For a homogeneous, isotropic universe with no source term other than this zero point energy an effective Lagrangian can be derived [see Atkatz (1994) or Norbury (1998), from whom the development which follows was largely derived)], namely,

$$L = A\left[-r\dot{r}^2 + kc^2r - \Lambda r^3/3\right] \quad (5.5)$$

where  $A$  is some constant ( $\propto 1/G$ ). Seeking the minimum action,  $\delta\int Ldt = 0$ , with respect to variations in the single dynamic variable,  $r(t)$ , results in the Euler-Lagrange equation,

$$\partial_t\left(\frac{\partial L}{\partial \dot{r}}\right) = \partial_t(-2Ar\dot{r}) = \frac{\partial L}{\partial r} = A\left[-\dot{r}^2 + kc^2 - \Lambda r^2\right] \quad (5.6)$$

which gives,

$$\dot{r}^2 + 2r\ddot{r} = -kc^2 + \Lambda r^2 \quad (5.7)$$

It is readily seen that (5.7) is consistent with the Friedmann equations, (5.3), (5.4).

We can now convert the Lagrangian to a Hamiltonian formalism using the canonical procedure, namely to define,

$$p = \frac{\partial L}{\partial \dot{r}} = -2Ar\dot{r} \quad (5.8)$$

and then to put,

$$\begin{aligned} H &\equiv p\dot{r} - L = A\left[-r\dot{r}^2 - kc^2r + \Lambda r^3/3\right] \\ &= A\left[-\frac{p^2}{4A^2r} - kc^2r + \Lambda r^3/3\right] \end{aligned} \quad (5.9)$$

But the Friedmann Equ.(5.3) shows that the Hamiltonian is identically zero, in illustration of the general finding noted previously for Hamiltonians derived consistently with general covariance. Nevertheless, using the expression in terms of

$p$ , Hamilton's equation  $\dot{p} = -\frac{\partial H}{\partial r}$  is consistent with (5.6,7) and hence with the Friedmann equations.



Finally, the Wheeler-deWitt equation implements this ‘Hamiltonian constraint’ by replacing the algebraically vanishing classical Hamiltonian with the quantum equation  $\hat{H}|\psi\rangle=0$ . The Wheeler-deWitt equation is obtained by following the canonical quantisation procedure in which the momentum variable in the Hamiltonian is replaced by an operator thus,  $p \rightarrow \hat{p} = \frac{\hbar}{i} \partial_r$ . The derivative is with respect to the ‘radial’ coordinate because (5.8) defines  $p$  and  $r$  to be conjugate variables. Making this substitution in (5.9) and multiplying through by  $-r$  gives the WdW equation,

$$\left[ -\frac{\hbar^2}{A^2} \frac{\partial^2}{\partial r^2} + 4kc^2 r^2 - \frac{4\Lambda}{3} r^4 \right] \psi(r) = 0 \quad (5.10)$$

*Need to sort out the constants & units here.* (5.10) looks like a Schrodinger equation for a zero energy particle in a potential,

$$V(r) = 4kc^2 r^2 - \frac{4\Lambda}{3} r^4 \quad (5.11)$$

Forearmed with our knowledge of the behaviour of the Schrodinger equation in more conventional contexts, we should be wary of whether any solution exists for zero energy. This depends upon the sign of the curvature,  $k$ . The potential has a crucially different form according to whether we consider a negatively curved universe,  $k < 0$ , or a positively curved universe,  $k > 0$ , as illustrated by Figure 1. **If  $k < 0$  there is no solution. Is this true?**

But if  $k > 0$  there are solutions corresponding to ingoing or outgoing waves in the ‘physical’ region where the potential,  $V$ , is negative (thus permitting positive kinetic energy). In the ‘barrier’ region the potential is positive, thus requiring negative kinetic energy, and hence a classical particle could not reside there. The solution is analogous to a quantum particle tunnelling through the potential barrier from the origin and into the physical region. The suggested interpretation in this case is the creation of a universe *ex nihilo*. From the complete nothing of a universe with no mass-energy and of size  $r = 0$  (at the origin of Figure 1) a real physical universe can spontaneously form – or so the idea goes. Note that there is a key difference between the Wheeler-deWitt equation (5.10) and ordinary quantum tunnelling: there is no time dimension. Quantum tunnelling takes place in the time domain. The particle, initially trapped near the origin takes a finite and calculable time to tunnel through the barrier. The solution to the WdW equation is timeless. It suggests that there is an ever present possibility of universes popping out of the quantum vacuum.

A couple of features are worth noting. Firstly, the universe pops out from its potential barrier at a finite size – the root of (5.11), i.e.,  $r_0 = \sqrt{\frac{3k}{\Lambda}}$ . Obviously the  $\Lambda$  in question

cannot be the infamous dark energy which is postulated to drive the accelerating expansion of the universe – or the universe would have been created at its present size. **Is this a valid argument, or have I misunderstood something?** Rather what is envisaged here is some ‘inflaton’ field (see next section). Secondly, the fact that a solution only exists for  $k > 0$  suggests that only closed universes can arise. This chimes nicely with the intimations made above that only closed universes can have zero energy.

This development assumed a universe empty except for the zero point energy. But the qualitative results are similar if matter is included. In particular the shape of the potential function remains essentially the same. This can be seen by noting that in place of  $\Lambda$  for ponderable matter we would have  $8\pi G\rho \sim GM/r^3$ , and hence the equivalent of the term  $\Lambda r^4$  in the potential, (5.11), is a term  $\sim GMr$ . Hence we get a modified potential of the form,

$$V(r) = 4kc^2r^2 - \frac{4\Lambda}{3}r^4 - \eta GMr \quad (5.12)$$

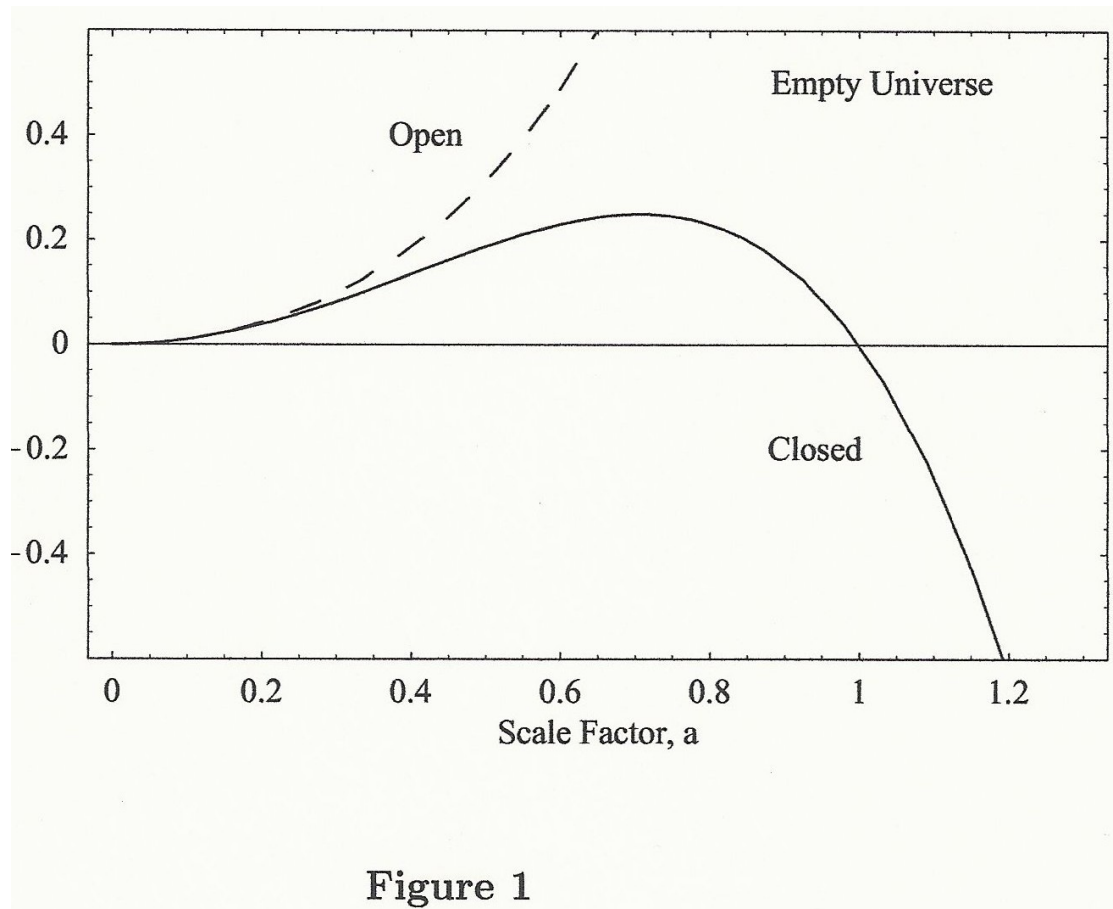
But the extra term linear in  $r$  makes no qualitative difference.

The reader will be aware that all such quantum cosmology ideas are highly speculative, in addition to posing major technical difficulties and interpretational problems. Current thinking has, in any case, moved well beyond the Wheeler-deWitt equation. Our purpose here has only been to illustrate a further context in which the idea of zero total universal energy arises.

#### 5.4 Inflation

*To be added*

**Figure 1** Potential Function (5.11) versus Radius,  $r$  (after Norbury)



**Figure 1**

## References

- R.Adler, M.Bazin and M.Schiffer, "Introduction to General Relativity", McGraw-Hill, 1975.
- D.Atkatz, "Quantum cosmology for pedestrians", 1994 *American J. Phys.* 62, 619
- Banerjee, N. and Sen, S., *Pramana-Journal of Physics*, 49, 609 (1997).
- H.Bondi, Plane Gravitational Waves in General Relativity, *Nature* 179, 1072-1073 (25 May 1957)
- T.Buchert, "Dark Energy from structure: a status report", *Gen.Rel.Grav.*40:467-527, 2008
- F.I.Cooperstock, *General Relativity and Gravitation*, 26, 323-327 (1994).
- B.Fausser, J.Tolksdorf, E.Zeidler, editors, "Quantum Gravity: Mathematical Models and Experimental Bounds", Birkhäuser Verlag, Berlin, 2000.
- J.Garecki, On Energy of the Friedman Universes in Conformally Flat Coordinates, arXiv:0708.2783 (2007)
- J.Garecki, Energy and momentum of the Friedman and more general universes, arXiv:gr-qc/0611056 (2006)
- E.Gourgoulhon, "3+1 Formalism and Numerical Relativity", Lecture notes: Institut Henri Poincare, Laboratoire de l'Univers et de ses Theories, Paris, 22 November 2006
- J.B.Hartle and S.W.Hawking, "Wave function of the universe", *Phys. Rev. D*28 (1983) 2690.
- Johri, V. B., Kalligas, D., Singh, G. P. and Everitt, C. W. F., *General Relativity and Gravitation*, 27, 313 (1995).
- C.W.Misner, K.S.Thorne and J.A.Wheeler, "Gravitation", W.H.Freeman & Co., San Francisco(1973).
- J.W.Norbury, "From Newton's Laws to the Wheeler-DeWitt Equation", *European Journal of Physics*, vol. 19, pg. 143-150 (1998)
- I.Radinschi, The Energy Distribution of the Bianchi Type VI<sub>0</sub> Universe, *Chinese Journal of Physics*, **39** (2001) 231-235.
- N.Rosen, *General Relativity and Gravitation*, 26, 319-321 (1994).
- Rosen, N. and Virbhadra, K. S., *Gen. Relativ. Gravit.* 25, 429 (1993).
- Carlo Rovelli, "Quantum Gravity", Cambridge University Press (2005)
- J.Stewart, "Advanced General Relativity", Cambridge University Press, 1996 (see Chapter 3)
- R.M.Wald, "General Relativity", University of Chicago Press, 1984 (see Chapter 11)
- D.L.Wiltshire, "Exact solution to the averaging problem in cosmology", *Phys.Rev.Lett.*99:251101, 2007
- S.S.Xulu, *Int. J. Theor. Phys.* 39, 1153 (2000).
- S.S.Xulu, "The Energy-Momentum Problem in General Relativity", PhD Thesis Department of Mathematical Sciences, University of Zululand, 18 November 2002, arXiv:hep-th/0308070.

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