Chapter 52

Does a Uniformly Accelerating Charge Radiate?

Yes (I think) – but not everyone can see it (perhaps).

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A point charge undergoes constant acceleration wrt its instantaneous rest frame. It has done so for the indefinite past and will do so for the indefinite future. You might think the question of the electromagnetic radiation it emits would be simple book work, presenting no problems of principle. You would be wrong. This problem is notorious.

Part of the difficulty lies in a paradox. We can look at the charge from two different frames of reference. In the first, an inertial frame, we have a Minkowski spacetime in which some applied force is causing our point charge to accelerate uniformly at a rate g (wrt its instantaneous rest frame). According to our usual understanding of electromagnetism this will cause electromagnetic radiation to be emitted. On the other hand we can adopt a frame of reference which is comoving with the charge. In this frame we see a uniform gravitational field, with the charge being supported against gravity by the externally applied force. The charge is, of course, stationary in this frame. The relativistic principle of equivalence requires that we regard both frames of reference as equally valid. But, in the comoving frame, why should an apparently stationary charge emit radiation? Or does a charge held stationary in a uniform gravitational field truly emit radiation?

I spent a long time worrying about this question. Only later did I realise that a great many others had done so too. A roll-call of excellent physicists in the first half of the twentieth century fell victim to its snares. Even now it is not entirely clear that the matter is settled, though I think that a consensus has emerged. Gratifyingly this does involve a true resolution of the paradox (if you believe it).

Constant acceleration wrt a fixed inertial frame is not possible, of course. It would soon lead to the velocity of light being exceeded. The type of motion that is envisaged here is constant acceleration, g, wrt the instantaneous rest frame of the charge. This is so-called hyperbolic motion. Assuming that motion takes place along the *z*-axis, the motion is defined by,

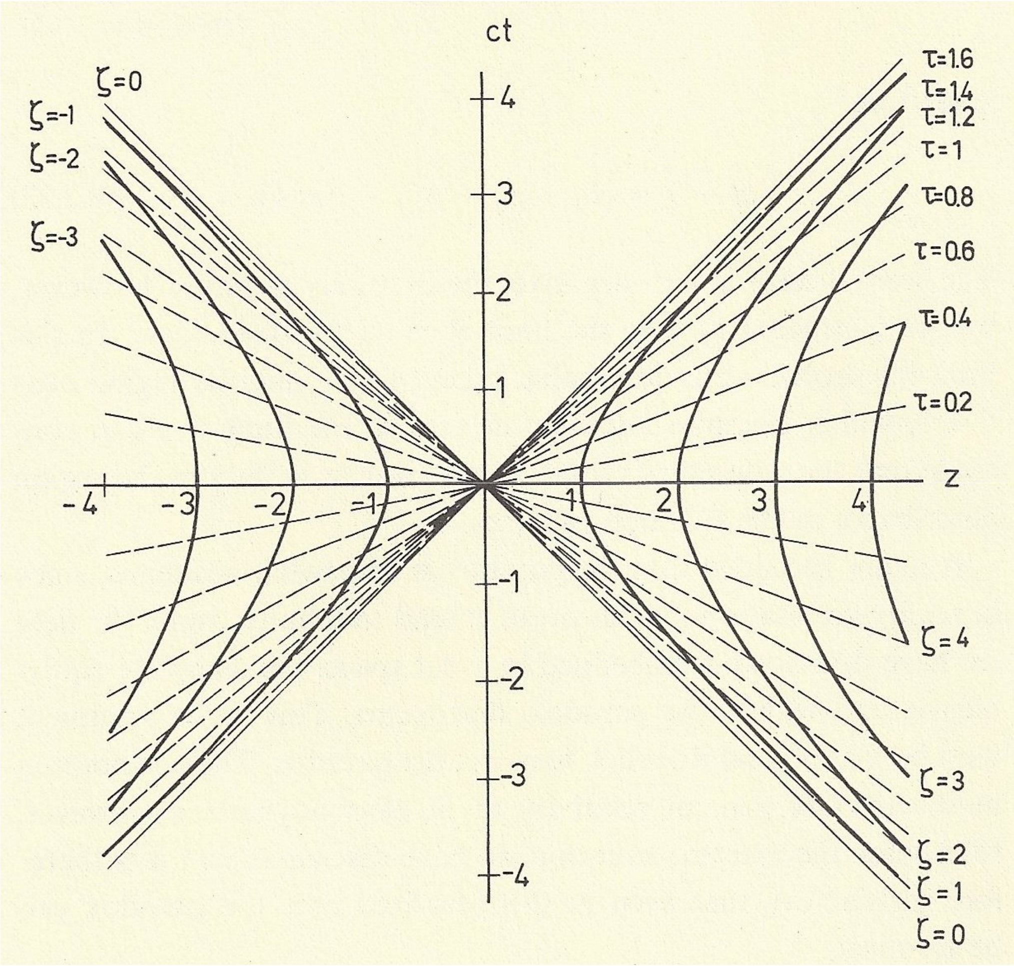
$$z^2 = \alpha^2 + (ct)^2 \tag{1}$$

where $\alpha = c^2 / g$. This defines hyperbolae with branches in the regions $z \geq \alpha$ and $z \leq -\alpha$, such as those shown in Figure 1. We shall take our charge to be travelling on the former, on the positive *z*-axis. The asymptotes are $z = \pm ct$. At early times the charge is approaching the origin at a speed arbitrarily close to the speed of light, and similarly moves away at arbitrarily close to the speed of light at sufficiently late times. The velocity, v, and acceleration, a, as seen from the inertial frame are readily derived from (1) and are,

$$v = \frac{c^2 t}{\sqrt{\alpha^2 + (ct)^2}}; \qquad a = \frac{g}{\gamma_v^3}; \qquad \gamma_v = \frac{1}{\sqrt{1 - v^2 / c^2}} \tag{2}$$

So the velocity has the asymptotic behaviour described above and the acceleration in the inertial frame is vanishingly small at early and late times but approximately g near $t = 0$.

Figure 1 The hyperbolae of constant acceleration



Before reviewing the history of this issue, it is worth recalling why we are inclined to believe that accelerating charges radiate. The electromagnetic field due to a charge in arbitrary motion is usually formulated in terms of the retarded potentials. The derivation can be found in standard electromagnetism texts, such as Duffin (1968), Jackson (1975) or Bleaney and Bleaney (1965). The full field contains terms independent of acceleration which fall off with distance as $1/r^2$. However the acceleration dependent terms in the fields fall off as $1/r$. Specifically they are,

$$\overline{E}_{acc} = \frac{e}{4\pi\varepsilon_0 c^2 \left(r - \overline{r}\cdot\overline{v}/c\right)^3}\,\overline{r}\times\left[\left(\overline{r} - r\overline{v}/c\right)\times\overline{a}\right] \tag{3}$$

$$\overline{B}_{acc} = \frac{\overline{r}\times E'_{acc}}{rc} \tag{4}$$

The notation $_{acc}$ denotes that only the acceleration dependent terms are displayed. Equs.(3,4) are more tricky than they at first appear. This is because all the quantities $\overline{r}, \overline{v} = \dot{\overline{r}}, \overline{a} = \dot{\overline{v}}$ must be understood as retarded. That is, in evaluating the field at a given point and time, you must first find the point (and time) on the charge's trajectory such that an influence propagating at the speed of light reaches your required field point at the desired time. Thus, $\bar{r}$ is the vector connecting your field point to the position where the charge ***was*** such that a signal emitted from it reaches your field point at the desired time. More succinctly put, $\bar{r}$ is the path of a light beam connecting the emission event with your observation event. The velocity and acceleration, $\bar{v}$ and $\bar{a}$, also relate to this retarded time and position.

The energy flux is given by the Poynting vector, $\bar{N} = \vec{E} \times \bar{B} / \mu_0$. The form of (3,4) implies that there will generally be a non-zero acceleration dependent term in the Poynting vector which falls off as $1/r^2$ when the acceleration is non-zero. Such a term gives a non-zero total power flux through a closed surface of arbitrarily large size, $\oiint_{r\to\infty} \bar{N} \cdot d\bar{S} \neq 0$. In contrast, the non-acceleration dependent terms in the Poynting vector fall off as $1/r^3$ or $1/r^4$ and hence will produce no such energy flux at infinity. Hence the acceleration dependent terms alone, (3,4), are associated with the production of radiation, i.e., the propagation of energy to infinity.

In the general case, actually carrying out an integral like $\oiint_{r\to\infty} \bar{N} \cdot d\bar{S}$ by insertion of (3,4) is made problematical by the fact that the fields must be evaluated at the retarded position and time. The Poynting vector at each point on the surface of integration is therefore determined by a different point on the trajectory of the charge, in general. In suitable circumstances approximations may render the integration tractable. For example, if the charge moves only slowly, so the distinction between retarded and current positions can be neglected, or perhaps the charge merely oscillates and does not change its position much as a consequence. In these situations the total radiated power can be evaluated to be,

$$P = \frac{e^2 a^2}{6\pi\varepsilon_0 c^3} \qquad (5)$$

This is the Larmor formula, the standard electromagnetic received wisdom. Equ.(5) is the expected relationship between the acceleration of a charge and the resulting radiated power. But note that the approximations required to derive (5) simply from (3,4) are not applicable to hyperbolic motion. With this background we turn to the history of the problem of hyperbolic motion.

The story starts with Born (1909) who purported to have derived the solution for the fields. The Born solution is given by Equs.(6a-d) below if the delta function terms are ignored. However, his solution will not do, as pointed out by Milner (1921). To understand why, refer to Figure 1. Our source is travelling on a hyperbola in the positive z region. Signals travelling along the forward light cone from any point on this trajectory can only reach points above the line $z = -ct$. Points (strictly, events) below this line, for $ct < -z$, must have zero fields because there is no retarded point on the charge's trajectory which connect with them via a null line (i.e., a light ray). For example, there is zero field at the origin for $t < 0$ because no signal from the charge can have reached there yet. But the Born solution is non-zero for all z, t, and hence is not appropriate. Milner suggested that the Born solution actually describes two charges, the original charge and one of opposite sign travelling on the other branch of the hyperbola at negative z. But this will not do either, as remarked by

Bondi and Gold (1955), because this situation would require zero field in the time-like region within the backward light cone[1], i.e., the region $t < 0, |ct| > |z|$.

This problem with the Born solution did not prevent Pauli (1920,1958) from using it as the basis of his discussion of the problem, concluding that eternal hyperbolic motion does ***not*** result in the emission of radiation. In his text book, von Laue (1919) also came to this conclusion. The arguments advanced in favour of this conclusion were,

- The Born field is symmetrical between past and future;
- At $t = 0$, with the charge instantaneously at rest, the magnetic field vanishes, and hence so does the Poynting vector. Since changing to a different inertial frame, by a Lorentz transformation, can reduce any point on the trajectory to rest, the field simply moves with the charge;

The first of these is actually just the result of the *flaw* in the Born solution, and should not be true of the actual situation. Past/future symmetry is broken by causality and the physical relevance of the retarded potentials only, not the advanced potentials.

As for the second, I struggle to see that it is an argument at all. In any one inertial frame it only establishes that the energy flux is instantaneously zero. But the same can be said of an oscillating dipole, at two instants in every cycle, which nevertheless indubitably radiates. The observation that, for any given time, an inertial frame can be found such that the instantaneous radiated energy flux is zero does not mean that the radiated energy flux is zero at all times in a fixed inertial frame.

We should be suspicious of an argument based on the fields at $t = 0$. In our inertial frame we need to evaluate the integral of the Poynting vector over a large closed surface. It must be large to capture just the radiation field. But, as noted above, the value of the Poynting vector on this surface relates to retarded times and positions on the charge's trajectory. Evaluation of the integral at $t = 0$ therefore involves the acceleration, a, as it was at times roughly r / c earlier, where r is the size of the surface of integration. Since r must be large, so must this time be significantly in the past – and by (2) this means that the acceleration, $a = g / \gamma_v^3$, as seen by the inertial observer will be very small, since the charge's speed at this time will be nearly *c*. But the bulk of the radiation, as seen by this inertial observer, will occur when the acceleration is greatest, i.e., when the *retarded* time is close to zero. So the energy flux through our large surface will be greatest at times around r / c.

Consequently, Pauli's arguments for zero radiation appear to be invalid. One may speculate, however, on the extent to which the general relativistic perspective may have influenced his judgment. Did the equivalence principle lead him to believe that he knew what the right answer was? If so, I sympathise. Until recently I also thought that zero radiation was the correct result. And so did many others, including Feynman. Papers continue to be published expounding this view, for example Mariwalla and Hari Dass (2002). The first to claim that, on the contrary, hyperbolic motion *does* result in radiation appears to have been Drukey (1949).

[1] I suspect the Born solution actually applies to the original charge plus one of opposite charge travelling on the other branch of the hyperbola *but travelling backwards in time and creating fields via the advanced potentials*. The reversal of causality implicit in the latter would prevent this from normally being considered as a physical solution.

Then Bondi and Gold (1955) rectified the problem with the Born solution. This cannot be done merely by insisting by *fiat* that the field in the region $ct < -z$ be zero. What would result fails to satisfy Maxwell's equations on the boundary $ct = -z$. However, Bondi and Gold showed that a solution is obtained which is valid everywhere if a singular term $\propto \delta(x+ct)$ is added to the Born solution, in addition to requiring that the field in the region $ct < -z$ be zero. However, Bondi and Gold appear then to have fallen for Pauli's false arguments. Whilst they claimed that there was radiation present in their amended solution, they appear to associate this only with the delta-function.

The turning point probably came mostly as a result of the paper by Fulton and Rohrlich (1960), shortly after supported by Lanz (1962) and later buttressed by Rohrlich (1990,1999,2000). Fulton and Rohrlich (1960) was perhaps the first source to emphatically claim that not only was radiation emitted, but it was finite and constant in time in the instantaneous rest frame. Moreover, in the instantaneous rest frame, the usual Larmor formula, (5), applies with the instantaneous acceleration being *g*. Contrary to Bondi and Gold (1955) who associated radiation only with the singular delta function, $\delta(x+ct)$, Fulton and Rohrlich, and Lanz, claimed ordinary, continuous radiation emission. But there were two problems with this: the first was what it might imply for the equivalence principle, throwing us back into the paradox, and the second was the issue of the radiation reaction.

Normally a charge which is accelerating due to the action of some imposed force would experience a back-reaction from its radiation emission, increasing the work required from the externally agency to maintain the acceleration. This is the source of energy carried away as radiation. A study of the history of radiation reaction calculations in hyperbolic motion would be a long digression in its own right. Suffice it to say that repeated calculations seemed to show that the radiation reaction was zero. This, of course, meshed nicely with claims that there was no radiation, whilst it left those who claimed that there *was* radiation with a problem. From where did the energy come?

Feynman threw his weight behind the "no radiation" camp. In his "Lectures on Gravitation", Feynman (1999)[2], he says "we have inherited a prejudice that an accelerating charge should radiate", and then he goes on to argue that the usual Larmor formula, (5), "has led us astray" because it applies only to cyclic or bounded motions. He then derives an expression implicating the third time derivative of position to radiation, concluding that constant acceleration would therefore not result in radiation. Again one wonders to what extent he convinced himself of this in order to save the equivalence principle. The requirement for a non-vanishing third derivative is what I had myself thought to be the correct resolution of the paradox until recently.

Following Fulton and Rohrlich (1960) the view that the radiation was real and 'ordinary' was on the ascendancy, being supported by many papers thereafter, such as Cohn (1978), Parrott (1997,2002), Gupta and Padmanabhan (1998), Harpaz and Soker (1998), and Eriksen and Gron (2000a,b,c,2002,2004). It is appropriate now to take a look at the fields. The following is the Born solution, augmented by the delta-function term advised by Bondi and Gold, but expressed in cylindrical polars, as used by Fulton and Rohrlich,

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[2] Note that the lectures on which the book "Feynman (1999)" was based were delivered in 1962-63.

$$4\pi\varepsilon_0 E_\rho = \frac{8e\alpha^2 z\rho}{\xi^3} - \frac{2e\rho}{\alpha^2+\rho^2}\delta(z+ct) \qquad E_\phi = 0 \tag{6a}$$

$$4\pi\varepsilon_0 E_z = -\frac{4e\alpha^2\left[\alpha^2+(ct)^2+\rho^2-z^2\right]}{\xi^3} \tag{6b}$$

$$4\pi\varepsilon_0 B_\phi = -\frac{8e\alpha^2 ct\rho}{\xi^3} + \frac{2e\rho}{\alpha^2+\rho^2}\delta(z+ct) \qquad B_\rho = B_z = 0 \tag{6c}$$

$$\xi = \left\{4\alpha^2\rho^2 + \left[\alpha^2+(ct)^2-\rho^2-z^2\right]^2\right\}^{1/2} \tag{6d}$$

$$\overline{E} = \overline{B} = 0 \text{ for } ct < -z \tag{6e}$$

Here $x = \rho\cos\phi$, $y = \rho\sin\phi$ and the particle is moving along the z-axis, in accord with (1). In (6a-d) the coordinates ρ, ϕ, z are ordinary coordinates, not retarded coordinates, unlike (3,4). Although we have included Bondi and Gold's delta-function in (6a) and (6c), it will play no part in the subsequent discussion. The radiation of interest, it will be seen, derives from the non-singular terms. The relevance of the delta functions is that they permit us to assert (6e) which gets rid of the unphysical fields in the causally unconnected region.

The confusion over whether or not the above solution contains a radiation field can now be understood in terms of two distinct limiting conditions. Firstly consider a fixed time, t. To discover whether we have a radiation field we need to look at the form of the fields at spatial infinity, i.e., for $\rho, z \to \infty$. Putting $r = \sqrt{\rho^2+z^2}$, (6d) gives $\xi \to r^2$ and hence (6a,b,c) give the limiting behaviour of the fields to be,

$$E_\rho \to\sim \frac{\alpha^2}{r^4} \qquad E_z \to\sim \frac{\alpha^2}{r^4} \qquad B_\phi \to\sim \frac{\alpha^2 ct}{r^5} \tag{7}$$

Hence the Poynting vector falls off at large distances extremely rapidly, as $1/r^9$, and there is apparently no radiation. However, a little thought tells us that we should not be surprised at this result. We are evaluating the fields at a very large distance but for a fixed, finite time. Such fields originate from the source at very early retarded times, when the source was still approaching at nearly the speed of light and hence with vanishingly small acceleration, a, as seen by our inertial observer [recall (2) with very large γ_v]. But little radiation would be expected when the acceleration is very small.

The greatest accelerations are seen by the inertial observer at around $t \sim 0$. But any radiation emitted at $t \sim 0$ will reach a distance surface at radius r only a time r/c later. This suggests that we should evaluate the fields on our large sphere at a time which is commensurate with its size, i.e., for $r^2 = \rho^2 + z^2 = (ct)^2$. The difference is that the limiting condition now also involves $ct \to \infty$, corresponding to *retarded* times around zero. This has a profound effect since (6d) now gives,

$$\xi = \left\{4\alpha^2\rho^2 + \alpha^4\right\}^{1/2} \to 2\alpha\rho \tag{8}$$

and the fields are of order,

$$E_\rho \to\sim \frac{1}{\alpha r} \qquad E_z \to\sim \frac{1}{\alpha r} \qquad B_\phi \to\sim \frac{1}{\alpha r} \tag{9}$$

So the Poynting vector is now of order $\frac{1}{\alpha^2 r^2}$ and hence is a radiation field which integrates to a finite, non-zero value over a limiting large sphere. Moreover, the result is proportional to $\alpha^{-2} \propto g^2$, as expected. Of course this establishes only that there is energy flux through the surface at one particular time. The task is completed by integrating the energy flux over time. This is bound to be finite because we know that we get radiation only from retarded times around $t \sim 0$. The tolerance on this will be a timescale governed by $\alpha \sim g^{-1}$ and hence we expect that the total radiated energy will integrate to something proportional to $g^2 \times g^{-1} = g$. Carrying out the space and time integrals yields the result,

Total energy radiated = ??????? (10)

But if there really ***is*** radiation, what is the solution to the radiation reaction problem and how is the equivalence principle paradox resolved?

The view put forward by Peierls (1979), and also by Harpaz and Soker (1998) in a different form, is that the source of the energy which is radiated is the charge's field itself. Normally the field of a charge is not available to do work. If a charge is initially in a state of constant velocity, and also ends in a state of constant velocity, whatever energy is within the charge's field is clearly the same before and after any intervening complex motion. So no energy has been extracted from the field to do work or cause radiation. However, the situation is different when the initial state is constant acceleration with incoming velocity (nearly) $-c$, and the final state is also accelerating but with outgoing velocity c. The relevance of retarded, but not advanced, potentials breaks the symmetry and we have no reason to suppose that the charge's field is the same, and hence has the same energy content, in the initial and final states. With the energy balance problem solved (perhaps) the radiation reaction is free to be zero, it is no longer required.

The resolution of the paradox with the equivalence principle seems first to have been published by Peierls (1979), quoting his source as the anticipated work of Boulware, which was subsequently published, Boulware (1980). More recently de Almeida and Saa (2006) have provided a nice, accessible exposition of Boulware's thesis. The solution is elegant and I am inclined to believe it. The key is the realization that the comoving observer does not have access to the whole of the (z,t) plane, in contrast to the inertial observer, who does. The comoving observer is aware only of the region $-z \le ct \le +z$.

The spacetime of the comoving observer is that of a Rindler coordinate system, as shown in Figure 1. These are hyperbolae like the trajectory of our charge, (1), except that α now plays the part of a spatial coordinate, taking values in the range $[0,+\infty]$ for $z > 0$ and values in the range $[-\infty,0]$ for $z < 0$. The time coordinate is, of course, the proper time for a comoving observer, so the complete specification of the Rindler coordinates (ς,τ) is,

$$z = \alpha \cosh\frac{c\tau}{\alpha} \qquad ct = \alpha \sinh\frac{c\tau}{\alpha} \tag{11}$$

The importance of this limitation on what the Rindler observer can see is that the radiation all goes into the region he *cannot* see, the region $ct > |z|$. This neatly solves the problem of the inertial observer seeing radiation whilst the comoving observer does not, whilst saving the equivalence principle.

Peierls (1979) has presented a very simple argument which shows why the radiation is observed only in the region $ct > |z|$, which the comoving observer cannot see. It is essentially the same issue as was raised in deriving the limiting forms (8,9). The identification of radiation requires us to consider a spatially large surface, $r \to \infty$. But this means we must also consider commensurately late times $ct \sim r$ such that the retarded times correspond to times near $t \sim 0$ when the radiation is emitted. So this means that $ct = \sqrt{z^2 + \rho^2} > |z|$ and hence is not within the spacetime observable by the comoving Rindler observer.

Peierls's version of the argument is worth stating also. Suppose the radiation is emitted at time t_0 from point z_0. When the signal has travelled a distance R in a direction at angle θ to the z-axis we will have $z = z_0 + R\cos\theta$ and $t = t_0 + R/c$. Hence,

$$ct - z = (ct_0 - z_0) + R(1 - \cos\theta) \tag{12}$$

The first term on the RHS of (12) is necessarily negative, since this defines the region wherein the charge lies (Figure 1). The second term on the RHS of (12) is positive except exactly on $\theta = 0$. Since, in identifying radiation, we need to consider $R \to \infty$ it follows that (12) is positive where radiation can be identified. But this means that the region of radiation is $ct > z$, which the comoving observer cannot see.

A paraphrasing of de Almeida and Saa (2006) makes a fitting conclusion, "*A free-falling charge will radiate according to an observer at rest, because in a constant gravitational field, any particle should move with uniform acceleration. However, an observer falling freely with the charge would observe it at rest and no radiation at all. If the equivalence principle is assumed to be valid, we would conclude that a charged particle at rest on a table should radiate, because for free-falling inertial observers the particle is accelerating. To explain this puzzle we need to recognize that the concept of radiation has no absolute meaning and depends both on the radiation field and the state of motion of the observer. This dependence is the main conclusion of a celebrated and long debate, exhaustively presented in the recent series of papers by Eriksen and Grøn (2000a,b,c,2002,2004). We can conclude that comoving observers have no access to the radiation field of a uniformly accelerated charge. The concept of a horizon emerges naturally in this context. The electromagnetic field generated by a uniformly accelerated charge is observed by a comoving observer as a purely electrostatic field.*"

However, it would be wrong to end on a note which sounds definitive. There are still dissenting voices and some aspects of the near-consensus which now exists may prove unreliable. In this respect the very thorough review of Lyle (2008) is most noteworthy. In particular, whilst Lyle is content with the existence of radiation in the inertial frame, he demurs from the Boulware- Eriksen-Grøn resolution of the equivalence principle paradox.

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