Chapter 46

The Standard Model of Particle Physics and QCD

The standard model could hardly be left out, though whether it is a "choice cut" or a bit of a mess is another matter. For a long time progress in physics was synonymous with increasing elegance and simplicity. The full electroweak-QCD Lagrangian, not to mention extracting predictions from it, is not exactly simple.

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1. A Little Early History: 1879 - 1936

The first sub-atomic particle to be discovered was, of course, the electron. The ready production of cathode rays in vacuum tubes made the discovery simple (easy for me to say). That the cathode rays were, in fact, negatively charged particles of matter was first proposed by Crookes (1879), he of Crookes' Radiometer fame. The dominant prevailing view at the time was that the cathode rays were "ethereal disturbances", in other words something akin to light. The discovery of the electron is normally attributed to J.J.Thomson (1884). He showed that cathode rays are deflected both by electric and magnetic fields, and thereby deduced the speed and charge:mass ratio of the particles comprising the rays. The latter was about two thousand times the charge:mass ratio of the hydrogen ion, the lightest known particle at the time. Assuming the cathode ray particles had the same magnitude of charge as the hydrogen ion would then imply the electron, as we now call it, would be roughly two thousand times lighter than the hydrogen ion (or proton, as we now call it). However, recall that the quantisation of charge was not demonstrated by Millikan until 1909, so the reasoning was not so straightforward at the time.

The discovery of the proton is not normally marked by a specific experiment. Rather it is the discovery of the atomic nucleus which is associated with a specific experiment, namely Rutherford's gold foil alpha particle scattering experiment. If you thought that laboratory leaders getting the accolade for discoveries made by a research student was a modern phenomenon, think again. It was Rutherford's idea to measure the deflections of alpha particles passing through thin gold foils. This was made possible by the experimental technique for detecting single alpha particles using scintillation of zinc sulphide coatings, a method developed by himself and his assistant Hans Geiger (he of Geiger Counter fame). Early experiments confirmed the expected small angle of deflection. Geiger had to find an occupation for a research student, Ernest Marsden, and had the idea of setting him the task of looking for any alpha particles scattered through large angles. The unexpected result that, in fact, some were, was explained by Rutherford on the basis of the concentration of virtually all the mass of the gold atoms in a very tiny fraction of the atomic volume, i.e., the nucleus. This work was published in 1911 and was seminal. For the first time, the basic sub-structure of atoms had been identified: the 'solar system' model of the central nucleus and the orbiting electrons. This model was essential to prepare the ground for quantum mechanics, the latter arising in the context of explaining quantitatively the atomic spectra (amongst other things). That the charge of the nucleus was due to positively charged particles, protons, was a concept which was then natural and gained widespread acceptance over the decade following 1911. By 1911 Rutherford already had the Nobel prize - not in physics but in chemistry, for his earlier work which established that radioactivity consisted of a transmutation of the chemical elements. It is hard now for us to appreciate just how revolutionary that concept, too, was at the time. Transmutation had a dirty name in proper science due to its association with alchemy. Despite being the founder of much of sub-atomic physics, Rutherford would never receive the Nobel prize for physics.

What was discovered first: the neutron or anti-matter? It was the neutron - but only just. Chadwick's paper announcing the neutron was published in February 1932. Anderson's paper on the positron was published in February 1933, though Anderson's cloud chamber observations are often dated at August 1932.

The story of the discovery of positrons is one of the great romances of physics. Between 1911 and 1928, quantum mechanics was developed by Bohr, Heisenberg, Schrodinger and Pauli in a form capable of predicting the basics of the atomic spectra. In 1928 Dirac published his relativistic quantum equation which considerably improved the prediction of the hydrogen spectrum. The negative energy solutions bothered him, and in 1929 he published his interpretation in terms of 'holes' and speculated that these 'holes' might be protons. The idea was untenable as he later accepted, and in 1931 proposed that 'anti-electrons' must exist. The prediction was just in time since the positron was discovered in 1932 - thus making Dirac's prediction arguably the most stunning prediction based on pure theory in the whole of science to-date. The positrons were detected using cloud chambers exposed to cosmic rays by Carl Anderson. With the benefit of hindsight, several different workers had actually seen positron tracks in cloud chambers up to three years earlier, but none had followed up the observations. The experimental detection of the positron, despite its extremely short lifetime before being annihilated, has a great advantage over the experimental detection of the neutron: it is charged whereas the neutron is not. Virtually all interactions with matter which lead to a particle being detectable are based on its electromagnetic interaction - a thing conspicuous by its absence in the case of neutral particles. For example, cloud chamber tracks depend upon the ionisation caused by the passage of a charge particle. Similarly, the scintillation of a chemical coating depends upon an interaction with atomic electrons due to the charge of the incident particle. Neutral particles are the devil to detect.

So how did Chadwick discover the neutron? It had to be indirect. The opportunity arose because, in fact, other workers had already produced beams of neutrons, though they did not know what they were. In 1930 the German physicists Bothe and Becker bombarded the light metal beryllium with alpha particles, and noticed that a very penetrating radiation was emitted. This radiation was non-ionising (i.e., neutral). At the time the known forms of radiation were either electromagnetic (hence gamma rays if energetic enough) or alpha or beta rays. Both the latter were known to be charged and hence Bothe and Becker assumed their neutral radiation must be gamma rays.

In 1932 Irène and Frédéric Joliot-Curie investigated this radiation in France. They let the radiation hit a block of paraffin wax, and found it caused the wax to emit protons. They measured the speeds of these protons (5.3 MeV). Rutherford, in whose Cavendish lab Chadwick was working, realised that such energetic protons could not result from gamma rays unless the gamma rays had unrealistically high energies. (The point here is that when a light particle strikes a far heavier target particle, the latter can only attain a small fraction of the energy from the light particle - which tends to just bounce off with little loss of energy). On the other hand, if the neutral 'rays' in question were composed of neutral particles with a mass roughly equal to that of protons, then the energy of the Joliot-Curie's spalled protons could be easily explained. The key to Chadwick's conclusive demonstration that this was the correct explanation lay in his use of beryllium, a very light nucleus, as the target - which still produced neutrons. Chadwick studied the interaction of the neutral 'rays' with hydrogen and nitrogen and was able to deduce the mass of the neutrons to within about 0.2% of the currently accepted value.

And what about our little friend, the neutrino? This was first postulated by Pauli as a means of making beta decay of nuclei consistent with the conservation laws. It is an example of how things seem easy in retrospect. These days we naturally think of beta decay as being the decay of the neutron, $n \to p + e + \overline{\nu}_e$ and hence the postulate that the neutrino exists does not seem a particularly remarkable feat of intellect. But Pauli made this suggestion in 1930, two years before Chadwick's discovery of the neutron! It was the unstable nucleus that Pauli had on the LHS of this reaction, and the daughter nucleus in place of the proton on the RHS. After the discovery of the neutron, it was Enrico Fermi in 1934 who first proposed the reaction in the form $n \to p + e + \overline{\nu}_e$ and provided a consistent theory which united the Chadwick neutron, the neutron-proton model of the nucleus, the Dirac electron-positron theory of quantum electrodynamics and the Pauli neutrino. This was what we would now refer to as the four-line vertex model of the weak interactions, a theory which was still taught (and still useful) in the 1970s when I was a student. Fermi's 1934 paper is notable for being both ahead of its time and also being rejected by Nature for being "too remote from reality". All the best people have papers rejected - at least, that's the thought with which I comfort myself.

Prior to 1936 the only known particles were the constituents of ordinary matter, the electron, the proton and the neutron, plus the positron (the neutrino had not yet been experimentally detected, being only a postulate at that time). This was to change in 1936, the year that one might take as the start of particle physics.

Since 1911 the basic puzzle in sub-nuclear physics was what held together the highly concentrated positive charge in the nucleus? Why did the nucleus not blow itself apart under the action of its own Coulomb repulsion? By 1934 a primitive version of quantum electrodynamics existed based on Dirac's theory. This had already given rise to the view that the electromagnetic interaction between charged particles could be considered as mediated by an exchanged photon, the quantum of the 'force field', in this case the electromagnetic force. This led Yukawa in 1934 to postulate that the mysterious force which held the nucleus together was also mediated by a scalar particle, a spin zero boson. To explain the very short range of the nuclear force this intermediate boson was postulated to have non-zero rest mass, unlike the photon. The term "meson" was introduced for such particles, derived from the Greek for "intermediate". It is elementary to observe that the Klein-Gordon equation, which by then was known to be the relevant quantum equation for a scalar particle, has a solution of the form e^{-mr}/r, and hence decays exponentially with distance for non-zero particle mass, m. Moreover, $1/m$ should set the size scale of the nucleus. Since the nucleus is around a couple of fermi (10^{-15} m) in size, this suggests a meson of mass around 100 MeV or so.

Again it was cloud chamber cosmic ray studies by Carl Anderson and Seth Neddermeyer that discovered the muon in 1936. Track curvatures allowed the mass to be determined, ~106 MeV. This was nicely in agreement with Yukawa's prediction so it was natural to suppose that the newly discovered particle was Yukawa's meson. For this reason the muon was initially referred to as the mu meson, a nomenclature that was still used by old timers as late the 1970s, causing immense confusion to students - because, of course, the muon is not Yukawa's particle after all - and it is not even a meson. The muon is actually a lepton and plays no part in ordinary matter. In this sense it is something entirely novel and unexpected. ("Who ordered that?" exclaimed physicist Isidor Rabi). Yukawa's particle is actually the pi meson - to be discovered a little later...

2. The Exploding Particle Zoo: 1947 - 2013

The period from 1937-1946 was taken up with approach to, and execution of, the second world war. This was a period of considerable increase in the knowledge of nuclear physics. This included the development of the nuclear reactor and hence controlled nuclear reactions. It also included the development of the so-called "atom bomb" - which should, of course, really be called the "nuclear bomb" - and hence uncontrolled nuclear reactions. However, pure research was largely on hold due to the war effort.

It was again cosmic rays which provided the source of the next particle to be discovered. This time it was a photographic technique rather than cloud chambers which was the crucial experimental step forward. And this time it really was Yukawa's meson that was found, of mass ~140 MeV. Now called the pi meson, it was found in both its positively and negatively charged forms in cosmic rays by Cecil Powell and co-workers at Bristol university in 1947, for which Powell received the Nobel prize in 1950 - the year after Yukawa got the Nobel for his prediction of it. The neutral pion was definitively identified in 1950 via its decay products, a pair of photons. By this time cyclotrons were being used, and neutral pions were seen both in cosmic rays and from accelerators.

Also from cosmic ray studies came the discovery of the kaons (K mesons) of mass ~494 MeV by the British physicists Rochester and Butler, 1947, and the lambda baryon, Λ^0, by the Australian physicists Hopper and Biswas in 1950. Rabi would have had cause to again ask who ordered those? These particles could be found from cosmic ray studies using photographic plates only because their lifetime was sufficiently long (0.1 ns or longer). This fact would ultimately be linked to their "strangeness", though this would be understood only later.

Any further particle physics became possible only with the advent of the particle accelerator, starting with the cyclotron. A cyclotron is a type of particle accelerator in which charged particles accelerate outwards from the centre along a spiral path. The particles are held to a spiral trajectory by a static magnetic field and accelerated by a rapidly varying (radio frequency) electric field. If the particles become fast enough that relativistic effects become important, the beam becomes out of phase with the oscillating electric field, and cannot receive any additional acceleration. The classical cyclotron is therefore only capable of accelerating particles up to a few percent of the speed of light. This is extremely limiting and cyclotrons quickly gave way to synchrotrons and linear accelerators.

A synchrotron is a particular type of cyclic particle accelerator, descended from the cyclotron, in which the guiding magnetic field (bending the particles into a closed path) is time-dependent, being synchronized to a particle beam of increasing kinetic energy. The synchrotron is one of the first accelerator concepts to enable the construction of large-scale facilities, since bending, beam focusing and acceleration can be separated into different components. Currently the large hadron collider (LHC) at CERN in Switzerland is the largest and most powerful synchrotron yet built, being 17 miles in circumference. It consists of two parallel beam pipes, each housing a proton beam circulating in opposite directions. The beams are made to collide at four different points around the circumference, there being different types of detectors at each intersection point. At full power the LHC will deliver colliding proton beams each at 7 TeV, hence a total energy of 14 TeV. So far (as of June'14) the LHC has run at 2 x 4 = 8 TeV. This was sufficient to find the Higgs boson.

The largest linear accelerator to-date is at the Standford Linear Accelerator Centre (SLAC), in California, and is 2-miles long. It can accelerate electrons and positrons to an energy of 50 GeV each, hence a total centre-of-mass energy of 100 GeV for colliding beams. It is not coincidence that this energy is sufficient to produce Z bosons.

In the 1950s the particle zoo proliferated madly. By 1960 or so it was clear that this zoo of particles desperately required systematising. This was achieved by Murray Gell-Mann and George Zweig in 1964 with the quark model - at that time using just three quarks. The quark model was soon itself rationalised in terms of SU(3) flavour symmetry. Later that provisional idea would give way to the full 'standard model' and its corresponding field theory, including SU(3) colour symmetric quantum chromodynamics (QCD) as a part. From 1950 onwards it would be tedious, and pointless, to itemise each new particle discovery. However, a few stand out as particularly significant and are discussed very briefly below. Some of the rest of the zoo are illustrated in the context of the quark model and the standard model in §4.

- In 1955 the antiproton was experimentally confirmed in accelerator work by Berkeley physicists Emilio Segrè and Owen Chamberlain, for which they were awarded the 1959 Nobel Prize in Physics. This was the first antimatter particle other than the positron to be found and confirmed the expectation that all particles would have antiparticles. Since 1955 anti-protons have been commonly detected in cosmic ray studies.
- The $\overline{\nu}_e$ (anti-electron-neutrino) was finally detected experimentally in 1956 by Cowan and Reines, the first of the neutrinos to be detected. The basis of the experiment is inverse beta decay, $\overline{\nu}_e + p \to n + e^+$ using a reactor as the source of (anti)neutrinos. The positron quickly finds an electron, and they annihilate each other. The two resulting gamma rays (γ) are detectable. The neutron can be detected by its capture on an appropriate nucleus, releasing a gamma ray. The coincidence of both events - positron annihilation and neutron capture - gives a unique signature of an antineutrino interaction. Note how indirect this is. It is standard procedure in particle physics now to regard decay products or secondary reactions to be a sufficient signature of the existence of the sought particle. There is little choice when the particle in question is neutral.
- In 1962 the muon neutrino, ν_μ, was detected and shown to be distinct from the electron neutrino by Lederman, Schwartz and Steinberger.
- Perhaps the final bit of glory from cosmic ray studies was the discovery of a charged Ξ baryon by the Manchester group in 1952. The neutral Ξ particle was detected at Lawrence Berkeley Laboratory in 1959. It was also observed as a daughter product from the decay of the omega baryon (Ω−) observed at Brookhaven National Laboratory in 1964.
- The discovery of the Ω^- in 1964 was a great triumph for the quark model since it was found only after its existence, mass, charge and decay products had been predicted by Gell-Mann in 1962 and independently by Ne'eman.
- In the late 1960s and 1970s, so-called deep inelastic scattering experiments (electrons scattered off protons) confirmed the particle-like sub-structure of the protons, consistent with the expectations from the quark model. This can be regarded as roughly analogous to Rutherford's identification of the nucleus from the alpha

particle scattering experiments. When I was doing my PhD work in the mid 1970s, the analysis of these deep inelastic experiments was a popular research area.

- In 1974 charmonium was discovered. This is the famous J/ψ particle which consists of a charm and an anti-charm quark. It got named twice because it was discovered simultaneously at both SLAC and Brookhaven. It was a milestone in particle physics because until then just three quarks (u, d, s) were sufficient to explain all known particles. The J/ψ was the first clear indication of the existence of a fourth quark, as had been predicted by Bjorken and Glashow in 1964. I recall this discovery clearly when I was an undergraduate. It made a big splash.
- In experiments between 1974 and 1977 the team led by Martin Perl at SLAC discovered the tau lepton. The tau and anti-tau pairs were produced in electron-positron annihilation experiments, $e^- + e^+ \to \tau^- + \tau^+$. The tau's were identified via their decay signature to an electron, a positron, a muon or an anti-muon, plus a pair of neutrinos, e.g., $\tau^- \to e^- + \overline{\nu}_e + \nu_\tau$, etc. This was important in terms of its theoretical implications because the existence of a third charged lepton implied, at least to some theoreticians, that a third generation of quarks also remained to be found.
- In 1977 the Upsilon meson (Υ) was found at Fermilab. The quark structure of Υ is $b\overline{b}$ where b is the fifth, "bottom or "beauty", quark, hence implying that there was indeed a third quark generation.
- In 1983 the W and Z bosons were finally detected at CERN. These are the intermediate vector bosons which mediate the weak nuclear force. Their existence is key to the U(1) x SU(2) gauge theory of the combined electroweak interaction, part of the standard model. Hence confirmation of their existence was an important validation of the model. One of the early predictions of the model was the existence of neutral weak currents, mediated by the neutral Z. When I was doing my PhD in the mid 1970s the experimental confirmation of neutral weak currents was a popular research topic.
- In 1995 the t quark was identified at Fermilab - the sixth and (as far as we know) the last of the quarks, known as "top" or "truth" according to taste.
- In 2000 the tau neutrino was first observed directly, again at Fermilab, the third and last of the neutrino flavours. Note, though, that one of the outstanding questions is whether neutrinos are distinct from their anti-particles (i.e., Dirac fermions) or identical to their anti-particles (Majorana fermions). The matter is not yet settled. The latest null result in the search for neutrinoless double beta decay suggests they are Dirac fermions, with distinct anti-particles. Watch this space.
- Last, but not least, a particle was discovered in 2012/13 at the LHC in CERN which physicists are sufficiently confident is the Higgs boson that Peter Higgs and François Englert won the Nobel prize in 2013. This was the final piece in the standard model jigsaw. There are, however, a host of unanswered questions as yet about this particle, at a mass of ~126 GeV.

3. Glossary

Boson	A particle with integral spin. Named after the Indian physicist Satyendra Nath Bose, which indicates the pronunciation. Please don't pronounce boson as bosun, which is a ship's officer.
Fermion	A particle with half-integral spin.
Meson	Originally a particle which mediates an interaction between other particles, from the Greek for "intermediate". These days a meson is defined as the bound state of a quark and an anti-quark. A meson is always a boson. The old fashioned term "mu meson" refers to the muon - which is a lepton not a meson! Moreover, the term "meson" is no longer used to refer to the force carrying particles of the fundamental interactions (i.e., the gluons and the intermediate vector bosons, W and Z, are not mesons).
Baryon	Orginally meaning "heavy particles", the lightest being the nucleons, the term baryon now means a particle composed of three quarks. The baryons are therefore necessarily fermions.
Lepton	Originally meaning "light particle", which is appropriate for the electron, the muon and the neutrinos, the leptons are now defined as the fundamental fermions which do not feel the strong nuclear force (i.e., they do not couple to gluons). This is another way of saying that they are neither composed of quarks nor are one of the 'force carrying' particles. The neutrinos 'feel' only the weak nuclear force, whereas the charged leptons feel both the weak force and the electromagnetic force. They cannot now be regarded as light since the tau has mass 1777 MeV, nearly double that of the nucleons. The leptons are fermions.
Hadron	A hadron is any particle which feels the strong nuclear force, though it is normally used in a sense which excludes the quarks and counts only the composite particles. Hence "hadron" generally means a meson or a baryon, though more exotic hadrons may exist.
Quark	One of the constituent particles of which hadrons are composed
Gluon	The carriers of the strong force which bind quarks together. There are eight distinct gluons differing in their colour state.
Flavour	Flavour is defined by the combined specification of isospin (or charge), strangeness, charm, beauty and truth. Hence the six quarks (and six antiquarks) of the same colour are distinguished by their flavour.
Hyperon	A hyperon is any baryon containing one or more strange quarks, but no charm, bottom, or top quark. The Λ^0 was the first discovered hyperon, though its status in this respect was not initially known.
Generation	Pairs of quarks and leptons are grouped into "generations", corresponding to the columns in the table of the fundamental particles, see Fig.5. Thus u,d,e,ν_e is generation 1; c,s,μ,ν_μ is generation 2; t,b,τ,ν_τ is generation 3.
Intermediate vector bosons	This usually refers to the W and Z particles, which are the force-carriers of the weak interaction.
Scalar particle	A scalar particle is simply a particle with zero spin
Vector particle	A vector particle is simply a particle with spin 1.

4. The Eightfold Way - Flavour SU(3) - 1964

By the early 1960s there were so many known hadrons that it became a matter of urgency to find some means of classifying them and explaining why some types of particle occurred and some did not. This was achieved by Murray Gell-Mann and George Zweig in 1964 with the quark model - at that time using just three quarks. There were some elementary precursors to this model.

The first was the concept of isospin (more properly called isobaric spin). This has nothing to do with spin as angular momentum. It is merely a dimensionless label. The idea is that some pairs, triples, etc., of particles can usefully be regarded as different charge states of the same underlying particle. The first example is the nucleon, whose neutral charge state is the neutron and whose positive charge state is the proton. Another example is the neutral and positively charged kaons, K^0, K^+, and another is the three pions π^-, π^0, π^+. Different isospin states of 'the same' particle invariably have closely similar mass, though not identical. By analogy with real spin, isospin was thought of as an example of SU(2) symmetry, with the differing charge states of 'the same' particle corresponding to the different elements in a representation of the group, each particle type labelling a different irreducible representation. Hence the nucleon and the kaon are manifestations of the 2-dimensional (spinor) representation of SU(2), the pion a 3-dimensional (vector) representation, etc.

The second concept which provided a degree of order to the multitude of particle behaviours was "strangeness". Strangeness was introduced to explain the fact that certain particles, such as the kaons and certain hyperons, were created easily in particle collisions, yet decayed much more slowly than expected for their large masses and large production cross sections. Noting that collisions seemed to always produce pairs of these particles, it was postulated that a new conserved quantity, dubbed "strangeness", was preserved during their creation (the pairs consisting of one strange and one anti-strange particle), but not conserved in their decay. In our modern understanding, strangeness is conserved during the strong and the electromagnetic interactions, but not during the weak interactions. Consequently, the lightest particles containing a strange quark cannot decay by the strong interaction, and must instead decay via the much slower weak interaction, explaining the anomalously long lifetime of strange particles (which might mean a whopping 0.1 to 10 ns).

Gell-Mann and Zweig found that the particles could be assembled into groups of eight or ten in a 2D pattern which was parameterised by the isospin (or charge) and the strangeness. The set of these multiplets are analogous to the periodic table of the elements. The lowest mass multiplets are shown in Figures 1-4.

Fig.1: Meson Nonet, Spin 0

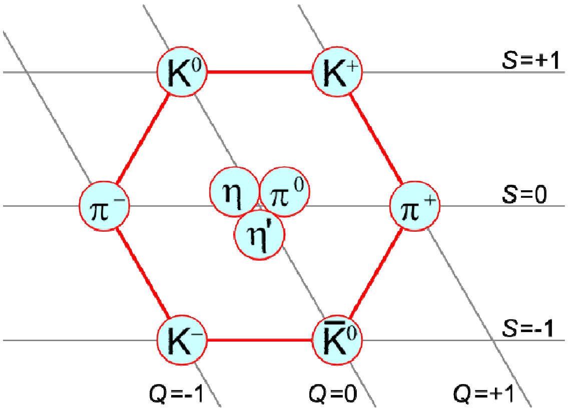


Fig.2: Meson Nonet, Spin 1

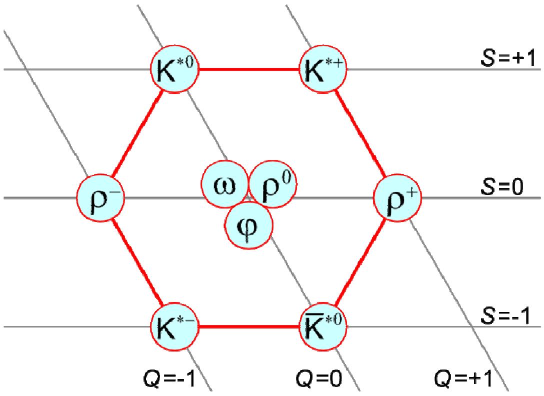


Fig.3: Baryon Octet, Spin 1/2

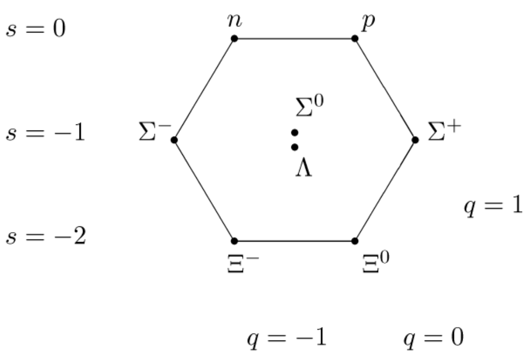
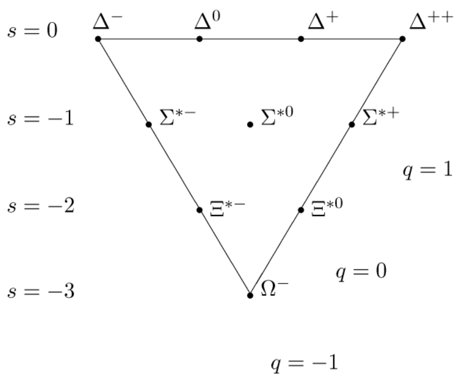


Fig.4: Baryon Decuplet, Spin 3/2



The nonets of mesons are to be considered as octets plus a singleton particle. The particles can thus be systematised as singlets, octets and decuplets.

Just as isopin (charge) was associated with SU(2), so the larger classification was suggested to be associated with an SU(3) symmetry group. Each multiplet was considered to be a manifestation of an irreducible representation of SU(3). The different particles within a multiplet transform into each other under the action of one of the elements of that representation of SU(3). If the SU(3) symmetry were exact, the masses of all the particles in a multiplet would be the same. They are not. At this point it is convenient to introduce the quark structure of the hadrons - and so we might as well introduce all the elementary particles of the standard model.

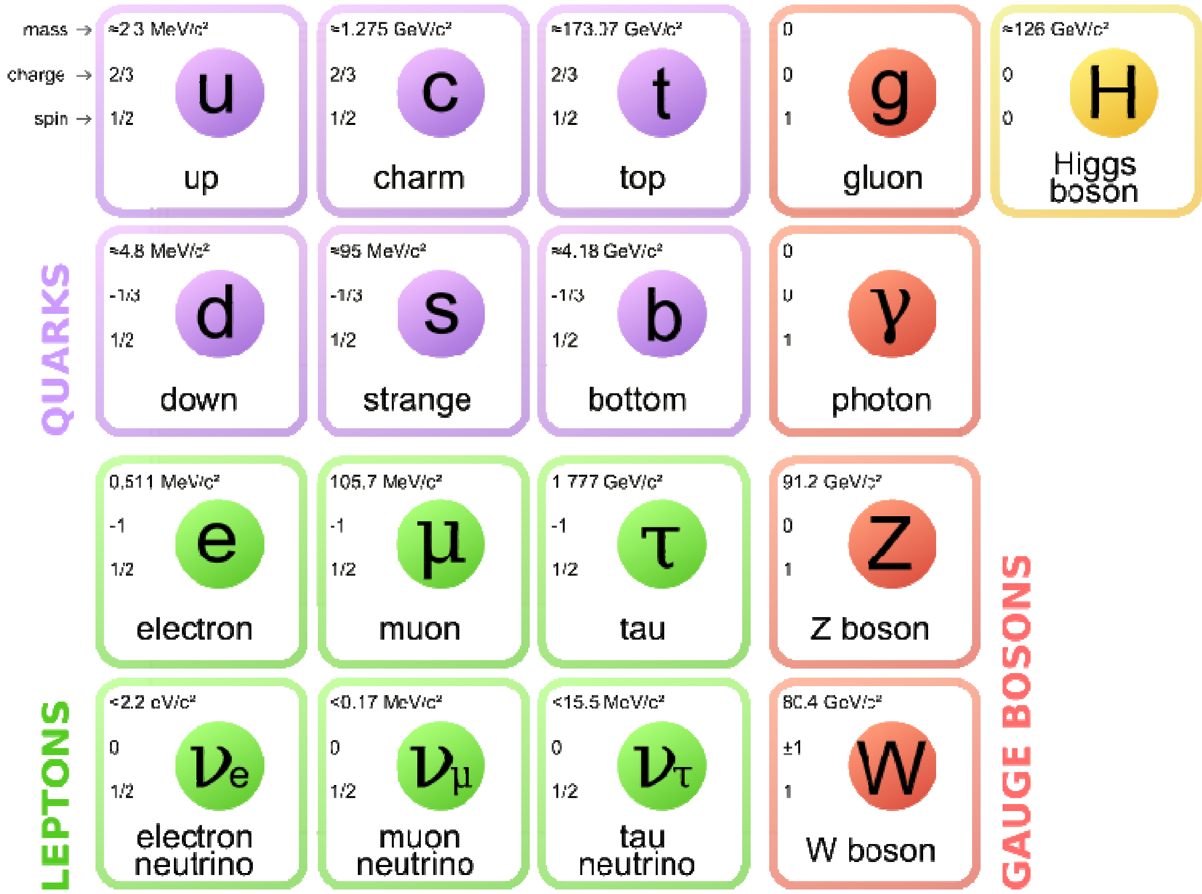
5. The Standard Model

In 1964 only three quarks were known: u, d and s. The full standard model as it stands at 2014 has six quarks (Fig.5), arranged as three pairs, each pair corresponding to a pair of leptons so that the first three columns in Fig.5 are considered to be three "generations" of particle. Generation 1 are the lowest mass and were found first, generation 2 the next in terms of mass and were found second, and generation 3 are the highest mass and were found last. I shall refer to "top" and "bottom" as interchangeable with "truth" and "beauty". Salient features are,

- The quarks and leptons are all spin 1/2 fermions;
- The gauge bosons, which are the force-carrying particles, are all spin 1;
- The Higgs is the only spin 0 (scalar) fundamental particle;

- The Higgs, the photon, the gluons, the neutrinos and Z are all uncharged, and hence do not interact electromagnetically.
- Quark charges of 2/3 or -1/3 mean that the mesons ($q\bar{q}$) and the baryons (qqq) all have integral charge (including neutral).

Figure 5: The Fundamental Particles of the Standard Model



Note that the convention for the flavour quantum numbers is that their sign is the same as that of their charge. Hence,

- Quark c has charm +1
- Quark t has truth +1
- Quark s has strangeness -1
- Quark b has beauty -1

Other important features are,

- All the quarks may be considered as carrying a baryon number of 1/3, whilst all the other particles have baryon number 0. Hence mesons ($q\bar{q}$) have baryon number zero, whilst the baryons (qqq) all have baryon number 1.
- The Higgs, the photon, the gluons and Z equal their own anti-particle, and W^+ and W^- are an anti-particle pair.
- In contrast, the quarks and the leptons are all distinct from their anti-particles (though this has not been established for the neutrinos).

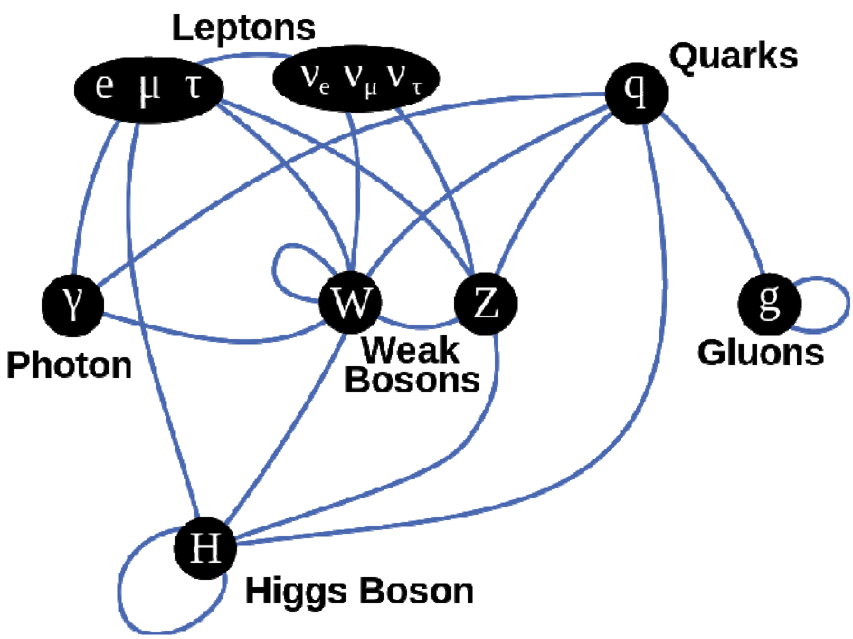
We will see shortly that the quarks come in three "colours" and the gluons in eight "colours", hence, counting anti-particles as well, the total number of fundamental particles is calculated in Table 1, below.

Table 1: The Number of Fundamental Particles in the Standard Model

	Types	Generations	Antiparticle	Colours	Total	Spin
Quarks	2	3	Pair	3	36	1/2
Leptons	2	3	Pair	None	12	1/2
Gluons	1	1	Own	8	8	1
W	1	1	Pair	None	2	1
Z	1	1	Own	None	1	1
Photon	1	1	Own	None	1	1
Higgs	1	1	Own	None	1	0
		Total			**61**	**-**

Which particles experience, (i) the electromagnetic, (ii) the weak force, and, (ii) the strong force? This is synonymous with asking which particles couple to (i) the photon, (ii) the W or Z, and, (iii) the gluons. The answer to (i) is simply "the charged particles". The answer in every case is shown diagrammatically in Fig.6.

Figure 6: Which Particles Interact Via the e/m, Weak and Strong Forces?



Notable features of Fig.6 are,

- All, and only, those particles with non-zero rest mass couple to the Higgs (because it is the Higgs mechanism which generates rest mass);
- Only the quarks, and the gluons themselves, feel the strong force;
- All particles except the gluons feel the weak force;
- The Higgs, the gluons and the W couple to themselves.

6. The Quark Content and Masses of the Low Mass Hadrons

The quark model explains the mesons as having the structure $q\bar{q}$ and the baryons as having the structure qqq. Here we make explicit the quark structure of the hadrons shown in Figs.1-4, for which only the first three quarks, u, d, s, are required. We also take a quick look at the mass systematics of these hadrons.

Table 2: Quark Structure and Masses (MeV) of Low Mass Hadrons

Pseudo-Scalar Meson Octet			
$\pi^\pm,\pi^0$ $u\bar{d}, d\bar{u}, (u\bar{u}-d\bar{d})/\sqrt{2}$ 140, 135	η $c_1(u\bar{u}-d\bar{d})+c_2 s\bar{s}$ 548	$K^\pm, K^0, \bar{K}^0$ $u\bar{s}.s\bar{u}, d\bar{s}, s\bar{d}$ 494, 498	
Vector Meson Octet			
$\rho^\pm,\rho^0$ $u\bar{d}, d\bar{u}, (u\bar{u}-d\bar{d})/\sqrt{2}$ 776	ω $c_1(u\bar{u}-d\bar{d})+c_2 s\bar{s}$ 783	$K^{*\pm}, K^{*0}, \bar{K}^{*0}$ $u\bar{s}.s\bar{u}, d\bar{s}, s\bar{d}$ 892	
Spin 1/2 Baryon Octet			
p,n uud, udd 938.3, 939.6	$\Sigma^+,\Sigma^0,\Sigma^-$ uus, uds, dds 1189,1193,1197	Λ uds 1116	Ξ^0,Ξ^- uss, dss 1315,1321
Spin 3/2 Baryon Decuplet			
$\Delta^{++},\Delta^+,\Delta^0,\Delta^-$ uuu, uud, udd, ddd 1232	$\Sigma^{*+},\Sigma^{*0},\Sigma^{*-}$ uus, uds, dds 1383,1384,1387	Ξ^{*0},Ξ^{*-} uss, dss 1532,1535	Ω^- sss 1672

Hence, the different multiplets have rather different mass ranges.

The different charge states of the same particle may differ by up to ~8 MeV, and generally less, a relatively small difference.

The masses of the different strangeness states within the same multiplet can be 'explained' very roughly by the higher mass of the strange quark (~95 MeV) and the number of strange quarks in the hadron. (The pseudo-scalar octet does not fit this very well, but more sophisticated considerations do better).

The absolute masses of the hadrons bears no relation to the sum of the rest masses of the two or three component quarks because the u and d quarks are so light (~2 MeV and ~5 MeV respectively). Hence, only ~1% of the mass of the nucleons is accounted for by the rest mass of their valence u and d quarks. This 'problem' was not known when the quark model was first proposed. The bulk of the hadronic mass, it turns out, is due to the sea of virtual quarks and gluons which bind the valence quarks. But this required the development of quantum chromodynamics (QCD) before it was appreciated.

Historically, Gell-Mann predicted the existence of the Ω^- in 1962 on the basis of the quark model. It was a major triumph that the particle was then detected in 1964.

Note that flavour SU(3) explains the existence of multiplets which are singlets, octets and decuplets, because the irreducible representations into which combinations of three types of quarks fall are given symbolically by, Mesons ($q\bar{q}$): $3 \otimes 3 = 8 \oplus 1$

Baryons (qqq): $3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$

7. Colour - The Strong Force 'Charge'

The alert reader will have spotted a problem with the quark model as represented by Table 2, specifically in respect of the spin 3/2 baryons. The particles Δ^{++}, Δ^{-} and Ω^{-} have quark structure uuu, ddd and sss respectively. But being spin 3/2 and composed of spin 1/2 quarks, the three quarks must be in a symmetric spin state and a symmetrical spatial state (of zero orbital angular momentum). But how can we have fermions in a symmetric state? The postulate which was made to avoid this problem was that there was another, hidden, quantum number in terms of which the three quarks differed. This new quantum number must take at least three values in order for all three quarks to differ. This was the origin of the concept of colour. In QCD, §8, it has been identified with the 'charge' of the strong force.

Colour is, in fact, the label of the quantum states within an irreducible representation of the SU(3) gauge theory which is at the heart of the strong interaction. Thus, the quarks transform as the fundamental, 3 dimensional, irreducible representation of the colour SU(3) group, whereas the gluons transforms as the adjoint, 8 dimensional, representation. In terms of the three fundamental quark colour states, r,g,b, the eight possible gluon states are given in Table 3.

Table 3: The Gluon Colour States

$$\begin{array}{ll}
(r\bar{b}+b\bar{r})/\sqrt{2} & -i(r\bar{b}-b\bar{r})/\sqrt{2} \\
(r\bar{g}+g\bar{r})/\sqrt{2} & -i(r\bar{g}-g\bar{r})/\sqrt{2} \\
(b\bar{g}+g\bar{b})/\sqrt{2} & -i(b\bar{g}-g\bar{b})/\sqrt{2} \\
(r\bar{r}-b\bar{b})/\sqrt{2} & (r\bar{r}+b\bar{b}-2g\bar{g})/\sqrt{6}.
\end{array}$$

8. Quantum Chromodynamics (QCD) and the Complete Lagrangian

This will not be a lecture on quantum field theory. It is assumed that the reader can 'read' a field theory Lagrangian and interpret it in terms of what interaction vertices will occur in the associated Feynman diagrams. Discussion of the Higgs mechanism, by which the elementary particles acquire mass, will be deferred to Chapter 48, here http://rickbradford.co.uk/ChoiceCutsCh48.pdf. The complete Lagrangian will be given here, including all the interactions, the electromagnetic, weak and strong forces. Hence, all, and only, the interactions depicted in Fig.6 can be identified as terms in the Lagrangian. Technically, the Lagrangian is that of a non-abelian gauge field theory with spontaneously broken $U(1) \otimes SU(2) \otimes SU(3)$ symmetry. Roughly speaking the three sub-groups correspond respectively to the electromagnetic interaction, $U(1)$, the weak force, $SU(2)$, and the strong force, $SU(3)$.

The Lagrangian is given in its post-symmetry-breaking form, so that particle masses are explicitly present. In this form the Lagrangian is *not* symmetric under $U(1) \otimes SU(2) \otimes SU(3)$. Moreover, it would not be a renormalisable theory as presented here either, since the Higgs field is omitted. In Chapter 48, http://rickbradford.co.uk/ChoiceCutsCh48.pdf, we will see that strict

$U(1)\otimes SU(2)\otimes SU(3)$ symmetry applies when the masses are replaced by zero. In this form, with zero masses, the theory becomes renormalisable. How do we have our cake and eat it too? How can we take benefit from the renormalisability of the massless theory, whilst also ending up with a theory which is consistent with the known non-zero masses? The answer lies in the Higgs mechanism in which a scalar field, the Higgs field, has a spontaneously broken symmetry making the renormalisable massless theory equivalent to a theory with masses, the masses of the quarks, leptons, W and Z being generated by the interaction with the Higgs field. More of this in Chapter 48, http://rickbradford.co.uk/ChoiceCutsCh48.pdf,

8.1 The QCD Lagrangian

The QCD part of the Lagrangian of the standard model describes the quarks, the gluons and their interaction, the strong force. It is,

$$L_{QCD} = L_g + L_{qg} \tag{8.1}$$

where L_g is the free gluon Lagrangian and L_{qg} includes both the free quark Lagrangian and the interaction terms between the gluons and the quarks, i.e., the strong interaction. Hence,

$$L_g = -\frac{1}{4}F^{(a)}_{\mu\nu}F^{(a)\mu\nu} \quad \text{where, } F^{(a)}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f_{abc} A^b_\mu A^c_\nu \tag{8.2}$$

where μ,ν represent the four spacetime indices, as usual, and a,b,c represent the eight types of gluon, i.e., the eight gluon colours as listed in Table 3. The A^a_μ are thus the eight gluon vector 'fields' (analogous to the electromagnetic 4-potential). The f_{abc} are the structure constants of the $SU(3)$ Lie algebra, i.e., they define the commutators between the generators, $\lambda^a/2$, of the Lie algebra,

$$\left[\frac{\lambda^a}{2},\frac{\lambda^b}{2}\right] = if_{abc}\frac{\lambda^c}{2} \tag{8.3}$$

In the Gell-Mann representation the generators are chosen to be as closely analogous to the Pauli matrices as possible, namely,

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

The structure constants are completely anti-symmetric in all three indices and the only non-zero cases are those given below, or obtained therefrom by permutation,

$$f^{123} = 1$$
$$f^{147} = -f^{156} = f^{246} = f^{257} = f^{345} = -f^{367} = \frac{1}{2} \qquad (8.4)$$
$$f^{458} = f^{678} = \frac{\sqrt{3}}{2},$$

The quark-gluon part of the Lagrangian is,

$$L = i\overline{\psi}_q^i \left(\hat{D\!\!\!/}_{ij} - m_q \delta_{ij} \right) \psi_q^j \qquad \text{where} \quad \hat{D\!\!\!/}_{ij} \equiv \gamma^\mu \left(\hat{D}_\mu \right)_{ij} \qquad (8.5)$$

and, $$\left(\hat{D}_\mu \right)_{ij} = \delta_{ij} \partial_\mu + i \frac{g_s}{2} \lambda_{ij}^a A_\mu^a \qquad (8.6)$$

where summation over i, j and over q, and of course over μ, is implied in (8.5), and summation over a is implied in (8.6). i, j run over the three quark colours, q runs over the six quark flavours, and μ of course runs over the four spacetime indices, whilst a runs over the eight gluon colours.

Note that there is only one contingent coupling constant, g_s, defining the strong force (i.e., the quark-gluon coupling and also the gluon-gluon coupling). The differing strength of the strong interaction between different coloured gluons and different colours and flavours of quark is defined in terms of the (known) SU(3) structure constants and Gell-Mann matrices.

However, the quark masses m_q also need to be determined experimentally, i.e., they are not determined by the theory. The Higgs mechanism provides an alternative perspective on these masses, as arising from a Yukawa coupling between the Higgs field and the initially massless quark field. However, there is a one-to-one relationship between (unknown) Yukawa coupling constants and the resulting masses, so this does not reduce the number of unknown parameters. This is a greatly disappointing feature of the standard model, in my opinion.

8.2 The Electroweak Lagrangian

To improve transparency the Lagrangian below is simplified, not strictly correct. Some of the liberties I have taken are explained in the notes which follow. The Lagrangian describing the leptons, and the intermediate vector bosons W and Z, and their interactions via the electromagnetic and weak forces is,

$$L_{em-weak} = L_{WZ} + L_L + L_{qew} \qquad (8.7)$$

where L_{WZ} is the free Lagrangian for the intermediate vector bosons, W and Z; L_L is the free Lagrangian for the leptons plus the electroweak interaction between the leptons and the intermediate bosons; and, L_{qew} is the electroweak interaction Lagrangian between the quarks and the photon and the W and Z.

To formulate L_L it is convenient to define,

$$\Psi_L = \begin{pmatrix} \psi_{\nu_L} \\ \psi_L \end{pmatrix} \qquad (8.8a)$$

and also the matrix, $$M_L = \begin{pmatrix} m_L & 0 \\ 0 & m_{\nu_L} \end{pmatrix} \tag{8.8b}$$

so that we can write,

$$L_L = \overline{\Psi}_L \left(\hat{D\!\!\!/}_L - iM_L \right) \Psi_L \tag{8.8c}$$

where L is summed over the three lepton generations, and hence (8.8c) accounts for the three charged leptons and the three neutrinos. The covariant derivative is,

$$D\!\!\!/_L = \gamma^\mu \left\{ \partial_\mu + i\frac{g}{2}\sigma_j W_\mu^j - i\frac{g'}{2}B_\mu \right\} \tag{8.9}$$

where σ_j are the three Pauli matrices, which of course are the generators of SU(2). In (8.8c) it is understood that the 2x2 matrix (8.8b) and the 2x2 Pauli matrices σ_j act on the upper and lower Dirac spinors of (8.8a). On the other hand, the Dirac matrices γ^μ act on the Dirac 4-spinors in the usual manner. (In other words, $\gamma^\mu \sigma_j$ is to be understood as a direct product).

The W_μ^j are the three 4-vector fields, analogous to the electromagnetic 4-potential, which are the gauge fields arising from SU(2) symmetry. The single 4-vector field B_μ arises as the gauge field of the U(1) symmetry and hence might appear to be the electromagnetic field. However, things are not quite that simple because B_μ and W_μ^3 mix, so that the physical electromagnetic field is a linear combination thereof (the mixing angle being the Weinberg angle). Specifically,

$$Z_\mu = cW_\mu^3 - sB_\mu \qquad \text{and} \qquad A_\mu^{em} = sW_\mu^3 + cB_\mu \tag{8.10}$$

where A_μ^{em} is the physical electromagnetic 4-potential, and $c = \cos\theta_W$, $s = \sin\theta_W$ where the Weinberg angle is determined experimentally to be $\sin^2\theta_W = 0.231$.

The usual quantum of electrical charge is $e = g\sin\theta_W = g'\cos\theta_W$. Since $\alpha = e^2/4\pi = 1/137$ this gives $e = 0.30$, $g = 0.63$, $g' = 0.34$. Defining a weak coupling by $g_w = g/2\sqrt{2} = 0.22$ the "weak fine structure constant" is $\alpha_w = g_w^2/4\pi = 0.0038 = 1/262$ and hence of a similar order to the electromagnetic fine structure constant, α. The reason why the weak force is weak is not because the coupling constant is small, it is because the masses of the W and Z are large. The old-fashioned, low-energy weak coupling, the Fermi constant, is given by $G_F = \frac{\sqrt{2}\cdot\alpha_w}{m_W^2}$ where $m_W = 80.425$ GeV is the W mass. Hence the Fermi constant is found to be $G_F \approx 1.1\times10^{-5}\,\text{GeV}^{-2} \approx 10^{-5}/m_n^2$, in agreement with the usual expression. (Of course this is not a prediction. In reality the knowledge of the Fermi constant, e.g., from the muon lifetime, determines the Weinberg angle, the reverse of the manner in which I have presented it here).

The quanta of the $W_\mu^{1,2}$ fields are the charged intermediate bosons $W^\pm$, whereas the quanta of the Z_μ field is the neutral Z boson, the conveyer of the neutral weak current. (NB: There is no neutral W particle).

As confessed below, (8.8c) is not really quite right. Certain terms must be dropped to ensure that the neutral neutrinos do not couple to the physical electromagnetic potential. Also, the W_μ^j fields couple only to the left-handed lepton fields, so the right-handed terms need to be dropped. I have fudged these things just to make the essentials stand out more clearly. The liberties I have taken in (8.8c) include,

- The weak interaction does not respect CP conservation. CP violation was first observed in the transformations of the neutral kaons. Later it was observed in B mesons and D mesons. CP violation is easily incorporated into the electroweak theory without harming its $U(1)\otimes SU(2)$ symmetry. The way to do this had already been formulated before the advent of gauge theories. The key is to recognise that the weak interaction is chiral, i.e., left-handed and right-handed spin do not couple identically. To be correct, (8.7) and (8.8) should be reformulated to reflect the different treatment of the left and right handed parts. Essentially this consists of dropping the coupling between the right-handed lepton currents and the W^μ field. In old terminology, the weak interaction is a "V-A" coupling, i.e., the coupling term is neither via a vector current, $\overline{\psi}_L\gamma^\mu\psi_L$, nor an axial vector current, $\overline{\psi}_L\gamma^\mu\gamma^5\psi_L$, but via the difference of the two, $\overline{\psi}_L\gamma^\mu(1-\gamma^5)\psi_L$, a 'left handed' current. Note that the strong force appears to be CP conserving, though why it should be is unclear. CP violation could coexist with the SU(3) symmetry, so the vanishing of CP violating terms has been described as a fine tuning issue. The issue of whether CP violation occurs elsewhere in the complete Lagrangian is of central cosmological importance in addressing why the universe is all matter and no antimatter.
- Whilst the charged lepton masses arise from the spontaneous symmetry breaking of the $U(1)\otimes SU(2)$ symmetry via the Higgs mechanism, this is not true of the neutrino masses which I have included in (8.8). Non-zero neutrino masses are currently problematical in the standard model for precisely this reason - they are not generated by the Higgs mechanism.

The Lagrangian for the free W and Z fields follows from the requirement that B_μ be invariant under U(1), and W_μ^j be invariant under SU(2), from which we get,

$$L_{WZ} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}G_{j\mu\nu}G_j^{\mu\nu} \qquad (8.11)$$

where, $$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \qquad (8.12)$$

and, $$G_{\mu\nu}^j = \partial_\mu W_\nu^j - \partial_\nu W_\mu^j + g\varepsilon_{jmk}W_\mu^m W_\nu^k \qquad (8.13)$$

(8.13) is a special case of the general form of a non-abelian gauge field, (8.2), for the particular case of SU(2) for which the structure constants are just the alternating tensor, ε_{ijk}.

The electroweak interactions of the quarks are given by the Lagrangian,

$$L_{qew} = \overline{\psi}_q\lambda^\mu\left(\pm\frac{g}{2}W_\mu^3 + \frac{g'}{2}B_\mu\right)\psi_q \qquad (8.14)$$

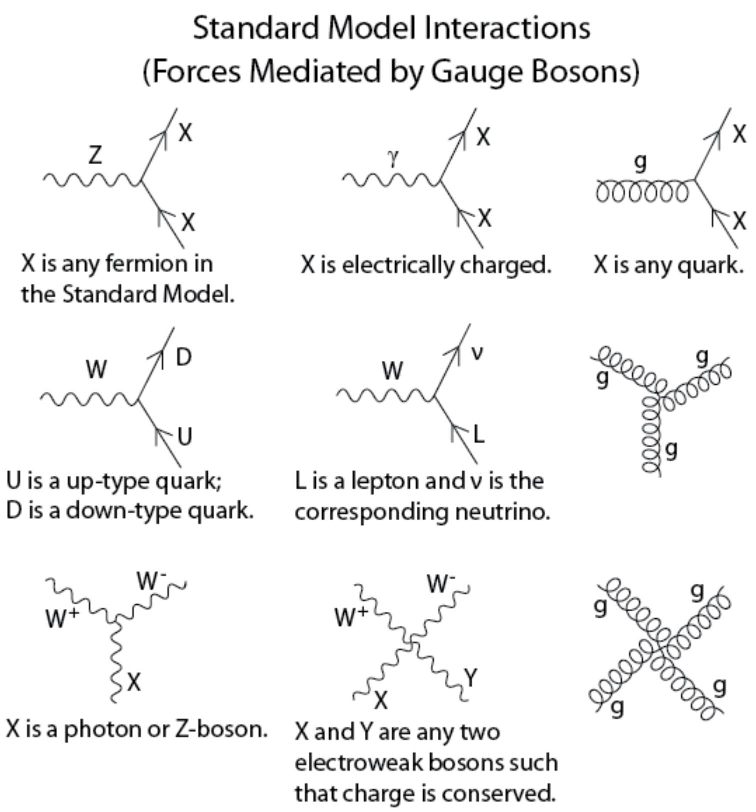
Figure 7 shows what vertices are possible in the Feynman diagrams corresponding to the total standard model Lagrangian,

$$L_{\text{standard model}} = L_{QCD} + L_{em-weak} = L_g + L_{qg} + L_{WZ} + L_L + L_{qew} \quad (8.15)$$

Note in particular that (8.2) shows that there are both cubic and quartic gluon terms in the free gluon Lagrangian, and hence there are gluon self-interactions involving both three and four gluon lines. Similarly there is a three-line intermediate vector boson self interaction involving W^+, W^-, Z and one involving W^+, W^-, γ, as well as four-line vertices such as W^+, W^-, W^+, W^- and W^+, W^-, Z, Z. However, there are no self interactions involving the neutral Z or γ only.

Purists will (rightly) scream that my representation of the standard model Lagrangian is wrong. I hope I confessed enough above. My intention was merely to indicate the essentials of the true beast, in particular how the vertices of Fig.7 occur. I addition to other sins, I have not included the Higgs field in (8.15) - see Chapter 48. The correct Lagrangian is shown in Fig.8 - enjoy.

Figure 7



9. Why?

Well, quite. Why does physics take on such an arcane, seemingly arbitrary, complicated structure at the sub-nuclear scale? Why is the standard model the way it is? Can it be derived from something which ***is*** simple?

There are 26 free parameters in the standard model which must be determined empirically. And yet only a far smaller number (five? six?) are required to give a good account of all physics other than particle physics, from atoms to things of cosmological scale, see for example http://rickbradford.co.uk/Coincidences_Abstract_AppH.html. All the complicated structure of particle physics seems quite irrelevant to the universe after the first second.

Fig.8: So you don't like my simplifying things and you really do want to see the true complete standard model Lagrangian? Ok.....

$$\mathcal{L}_{SM} = -\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu -$$
$$M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu\partial_\nu Z^0_\mu - \frac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)) -$$
$$igs_w(\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu -$$
$$W^-_\nu\partial_\nu W^+_\mu)) - \frac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu -$$
$$Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w(A_\mu Z^0_\nu(W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \frac{1}{2}\partial_\mu H\partial_\mu H - 2M^2\alpha_h H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - \frac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 -$$
$$\beta_h\left(\frac{2M^2}{g^2} + \frac{2M}{g}H + \frac{1}{2}(H^2+\phi^0\phi^0+2\phi^+\phi^-)\right) + \frac{2M^4}{g^2}\alpha_h -$$
$$g\alpha_h M\left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) -$$
$$\frac{1}{8}g^2\alpha_h\left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2\right) -$$
$$gMW^+_\mu W^-_\mu H - \frac{1}{2}g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H -$$
$$\frac{1}{2}ig\left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) +$$
$$\frac{1}{2}g\left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \frac{1}{2}g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) +$$
$$M\left(\frac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+\right) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_wMA_\mu(W^+_\mu\phi^- -$$
$$W^-_\mu\phi^+) - ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) -$$
$$\frac{1}{4}g^2W^+_\mu W^-_\mu\left(H^2+(\phi^0)^2+2\phi^+\phi^-\right) - \frac{1}{8}g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu\left(H^2+(\phi^0)^2+2(2s_w^2-1)^2\phi^+\phi^-\right) -$$
$$\frac{1}{2}g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \frac{1}{2}ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- +$$
$$W^-_\mu\phi^+) + \frac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\frac{s_w}{c_w}(2c_w^2-1)Z^0_\mu A_\mu\phi^+\phi^- -$$
$$g^2 s_w^2 A_\mu A_\mu\phi^+\phi^- + \frac{1}{2}ig_s\,\lambda^a_{ij}(\bar q^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar e^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar\nu^\lambda(\gamma\partial+m^\lambda_\nu)\nu^\lambda - \bar u^\lambda_j(\gamma\partial +$$
$$m^\lambda_u)u^\lambda_j - \bar d^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + igs_wA_\mu\left(-(\bar e^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar u^\lambda_j\gamma^\mu u^\lambda_j) - \frac{1}{3}(\bar d^\lambda_j\gamma^\mu d^\lambda_j)\right) +$$
$$\frac{ig}{4c_w}Z^0_\mu\{(\bar\nu^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar e^\lambda\gamma^\mu(4s_w^2-1-\gamma^5)e^\lambda) + (\bar d^\lambda_j\gamma^\mu(\tfrac{4}{3}s_w^2-1-\gamma^5)d^\lambda_j) +$$
$$(\bar u^\lambda_j\gamma^\mu(1-\tfrac{8}{3}s_w^2+\gamma^5)u^\lambda_j)\} + \frac{ig}{2\sqrt{2}}W^+_\mu\left((\bar\nu^\lambda\gamma^\mu(1+\gamma^5)U^{lep}{}_{\lambda\kappa}e^\kappa) + (\bar u^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) +$$
$$\frac{ig}{2\sqrt{2}}W^-_\mu\left((\bar e^\kappa U^{lep\dagger}{}_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar d^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) +$$
$$\frac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_e(\bar\nu^\lambda U^{lep}{}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar\nu^\lambda U^{lep}{}_{\lambda\kappa}(1+\gamma^5)e^\kappa\right) +$$
$$\frac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_e(\bar e^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar e^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1-\gamma^5)\nu^\kappa\right) - \frac{g}{2}\frac{m^\lambda_\nu}{M}H(\bar\nu^\lambda\nu^\lambda) -$$
$$\frac{g}{2}\frac{m^\lambda_e}{M}H(\bar e^\lambda e^\lambda) + \frac{ig}{2}\frac{m^\lambda_\nu}{M}\phi^0(\bar\nu^\lambda\gamma^5\nu^\lambda) - \frac{ig}{2}\frac{m^\lambda_e}{M}\phi^0(\bar e^\lambda\gamma^5 e^\lambda) - \frac{1}{4}\,\bar\nu_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat\nu_\kappa -$$
$$\frac{1}{4}\,\bar\nu_\lambda\,M^R_{\lambda\kappa}\,(1-\gamma_5)\hat\nu_\kappa + \frac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_d(\bar u^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar u^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j\right) +$$
$$\frac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_d(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j\right) - \frac{g}{2}\frac{m^\lambda_u}{M}H(\bar u^\lambda_j u^\lambda_j) -$$
$$\frac{g}{2}\frac{m^\lambda_d}{M}H(\bar d^\lambda_j d^\lambda_j) + \frac{ig}{2}\frac{m^\lambda_u}{M}\phi^0(\bar u^\lambda_j\gamma^5u^\lambda_j) - \frac{ig}{2}\frac{m^\lambda_d}{M}\phi^0(\bar d^\lambda_j\gamma^5 d^\lambda_j) + \bar G^a\partial^2G^a + g_sf^{abc}\partial_\mu\bar G^a G^b g^c_\mu +$$
$$\bar X^+(\partial^2-M^2)X^+ + \bar X^-(\partial^2-M^2)X^- + \bar X^0(\partial^2 - \frac{M^2}{c_w^2})X^0 + \bar Y\partial^2 Y + igc_wW^+_\mu(\partial_\mu\bar X^0X^- -$$
$$\partial_\mu\bar X^+X^0) + igs_wW^+_\mu(\partial_\mu\bar YX^- - \partial_\mu\bar X^+Y) + igc_wW^-_\mu(\partial_\mu\bar X^-X^0 -$$
$$\partial_\mu\bar X^0X^+) + igs_wW^-_\mu(\partial_\mu\bar X^-Y - \partial_\mu\bar YX^+) + igc_wZ^0_\mu(\partial_\mu\bar X^+X^+ -$$
$$\partial_\mu\bar X^-X^-) + igs_wA_\mu(\partial_\mu\bar X^+X^+ -$$
$$\partial_\mu\bar X^-X^-) - \frac{1}{2}gM\left(\bar X^+X^+H + \bar X^-X^-H + \frac{1}{c_w^2}\bar X^0X^0H\right) + \frac{1-2c_w^2}{2c_w}igM\left(\bar X^+X^0\phi^+ - \bar X^-X^0\phi^-\right) +$$
$$\frac{1}{2c_w}igM\left(\bar X^0X^-\phi^+ - \bar X^0X^+\phi^-\right) + igMs_w\left(\bar X^0X^-\phi^+ - \bar X^0X^+\phi^-\right) +$$
$$\frac{1}{2}igM\left(\bar X^+X^+\phi^0 - \bar X^-X^-\phi^0\right).$$