Chapter 25: Spontaneous Order through Gravitational Collapse

How do orderly structures arise spontaneously given that the universe starts in a chaotic state? Why does this not violate the second law of thermodynamics?

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According to the Big Bang theory, the universe is supposed to have started in a highly dense and extremely hot state consisting of radiation and particles in random motion, devoid of structure. According to the second law of thermodynamics the entropy of a closed system cannot decrease. And yet here we are, 13.7 billion years later. We inhabit a planet which is replete with natural order, not least its flora and fauna, including ourselves. It is not immediately obvious how this apparent transition from disorder to order can be reconciled with the second law of thermodynamics. If you are not already acquainted with the answer it is worth pondering upon the question a little before reading on.

The spontaneous production of order is not a purely terrestrial phenomenon. Stars are also examples of order (consider, for example, the shell structure of AGB stars). More convincing still are the galaxies, whose inner workings as the nurseries and domiciles of stars is sufficiently complex to resemble living organisms (this is an analogy, not some cosmic Gaia hypothesis). Of course, it is clear what is responsible for the formation of galaxies, stars and planets: it is gravity. Could it be, therefore, that the process of gravitational collapse is responsible for this reduction of entropy of the collapsed matter? The answer, as we shall see, is "yes", though the second law continues to be respected. The gravitational collapse occurs only at the expense of jettisoning entropy into the interstellar environment, a greater amount of entropy than is lost by the collapsing material. Gravitational collapse is only possible if a mechanism exists for this entropy ejection. This is generally referred to as the "cooling mechanism" and is something of an obsession with astrophysicists (and rightly so).

As a simple model for star or galaxy formation, consider a cloud of gas collapsing under its own gravity such that it passes through a sequence of gravitationally bound states. Suppose the gas cloud is at temperature T and consists of N particles of mass m. The average kinetic energy per particle is, for non-relativistic motion in three spatial dimensions, $3k_B T/2$. The virial theorem tells us that the total kinetic energy, $K = 3Nk_B T/2$ equals $-P/2$, where P is the total potential energy. Strictly these should be time average energies, but for a large gas cloud we can ignore fluctuations. Hence,

$$P = -3Nk_B T \qquad (1)$$

But the gravitational potential energy of a spherically symmetric mass distribution of total mass $M = Nm$ and radius R is,

$$P = -\eta \frac{GM^2}{R} \qquad (2)$$

where η is a dimensionless constant of order unity depending on the distribution (e.g. $\eta = 3/5$ for uniform density). In terms of the volume of the cloud,

$$V = \frac{4\pi}{3} R^3 \qquad (3)$$

we thus have, $$k_B T = \frac{\eta}{3}\left(\frac{4\pi}{3}\right)^{1/3} Gm^2 \cdot \frac{N}{V^{1/3}} \tag{4}$$

Now the entropy of an ideal gas is given by the Sackur-Tetrode equation…

$$S_{gas} = Nk_B\left\{\frac{5}{2} + \log\left[\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} \cdot \frac{V}{N}\right]\right\} \tag{5}$$

This gives the absolute total entropy. For our purposes we only need the change in entropy between two states of different volume and temperature. Thus we can simplify (5) to,

$$S_{gas} = Nk_B \log\left[VT^{3/2}\right] + \text{constant} \tag{6}$$

Using (4) we can substitute for T in terms of V, and hence (6) simplifies further to,

$$S_{gas} = \frac{Nk_B}{2}\log[V] + \text{constant} \tag{7}$$

The total energy of the gas cloud is,

$$E_{gas} = K + P = \frac{P}{2} = -\frac{3}{2}NkT = -\frac{\eta}{2}\cdot\frac{GM^2}{R} \tag{8}$$

Note the following,

(a) From (8), the total energy of the gas is negative: inevitably because it is a gravitational bound system – we could not have employed the Virial Theorem otherwise;

(b) From (8), as the cloud collapses, R reduces and hence the *magnitude* of the energy increases, i.e., the absolute energy E_{gas} decreases;

(c) From (4), as the cloud collapses, so that V reduces, the temperature increases.

(d) From (b) and (c) it follows that the gas cloud has a negative specific heat (the temperature increases as energy is removed from it);

(e) Whilst from (7) it follows that as the cloud collapses (V reduces) the entropy of the gas decreases.

This latter observation is initially disconcerting due to the apparent violation of the second law of thermodynamics (apparent, not real). The resolution lies simply in the fact that the assumption that the gas cloud passes through a sequence of bound states, so that the Virial Theorem always applies, implies that energy must be lost, i.e., (b). There must be some entropy ejection which goes hand-in-hand with this energy ejection (cooling mechanism).

To evaluate the entropy which is ejected note firstly that we have assumed that the gas cloud does not loose mass (in the non-relativistic approximation) so that the energy and entropy losses must be due to radiation. The entropy of a given quantity of radiant energy dQ is found from,

$$dS_{radiation} = \frac{dQ}{T} \tag{9}$$

The radiated energy, dQ , is minus the change in the total energy of the gas, which from (8) is,

$$dQ = -dE_{gas} = \frac{3}{2} Nk_B dT \tag{10}$$

Using (4) we can write,

$$k_B T = \frac{B}{V^{1/3}} \Rightarrow k_B dT = -\frac{1}{3} \cdot \frac{B}{V^{4/3}} dV = -\frac{1}{3} \cdot \frac{k_B T}{V} dV \tag{11}$$

Hence, substituting (11) into (10) gives,

$$dQ = -\frac{Nk_B T}{2V} dV \tag{12}$$

and hence (9) gives,

$$dS_{radiation} = -\frac{Nk_B}{2V} dV \tag{13}$$

But from (7) the change of the entropy of the gas for a small change in volume is,

$$dS_{gas} = \frac{Nk_B}{2V} dV \tag{14}$$

Consequently the radiation carries off exactly the same quantity of entropy as the collapsing material looses. Entropy is conserved in this case because we have assumed conditions equivalent to thermodynamic reversibility. Real gravitational collapse will be a messier business and inevitably lead to overall entropy production, with more entropy being exported into the inter-stellar medium than is lost by the collapsing matter.

If the gas cloud were opaque, so that radiation could not escape from it, it could not collapse since there would be no mechanism for obeying the second law of thermodynamics. In practice it will always be possible to loose heat at some rate, if only from the surface of the gas cloud. The lesson we learn from the preceding analysis is that the rate at which the gas cloud collapses will be controlled by the rate at which heat (and entropy) can be radiated away from it. The latter will depend upon the opacity of the gas and upon whatever other heat transport mechanisms might be active (e.g., convection). In these circumstances, when the gas is not perfectly transparent to radiation, there will inevitably be variations in the temperature of the gas at different depths, and corresponding variations in its pressure and density. Here we have the beginnings of a more realistic model of star formation.