Chapter 16

Entangled Interference

How does entanglement affect interference and does it permit faster-than-light communication?

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1. The False Paradox

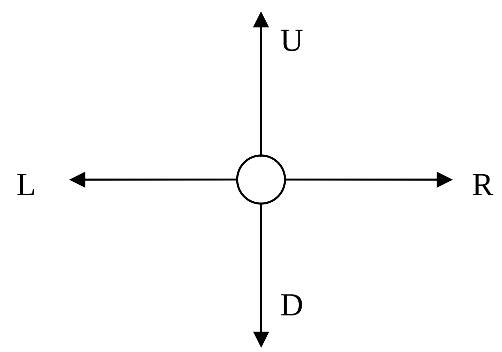
What happens when interference experiments are carried out on pairs of entangled particles? There is a greatly illuminating conundrum regarding whether a measurement on one of the entangled pair will destroy the interference observed for the other. Since the entanglement means that a measurement carried out on one of the pair is effectively also a measurement on the other, this should destroy the interference. But this appears to create the opportunity for faster-than-light (FTL) communication. This is because we could, it seems, observe the disappearance of an interference pattern here and now as a consequence of an action at a spacelike separation. Modulating such remote actions on a stream of entangled particle-pairs would therefore allow a signal to appear here – faster than light – in the form of the repeated appearance and disappearance of the interference. Of course this cannot be so, but the reason why is important and is elucidated here.

The lazy assertion that, "entangled particles cannot cause interference" clearly cannot always be true because, in the real world, particles will always be entangled with things you know nothing of - by virtue of their history. So, if it were true, no interference would ever be observed. More accurate guidance is that entanglement between specified degrees of freedom will prevent interference between these same degrees of freedom (as we shall see). But the fact that my particle happens to be entangled with another particle, currently perhaps in the vicinity of Alpha Centauri, will not prevent it creating interference fringes when incident upon a double slit screen here on Earth.

2. Entangled Photons Input to Mach-Zehnder Interferometers

Suppose we have a means of generating pairs of entangled photons. Further suppose that these photons can be emitted only along either the x-axis or the y-axis, and these occur with equal probability from the same precursor state (and hence are a coherent superposition). This can be represented diagrammatically by,

Figure 1



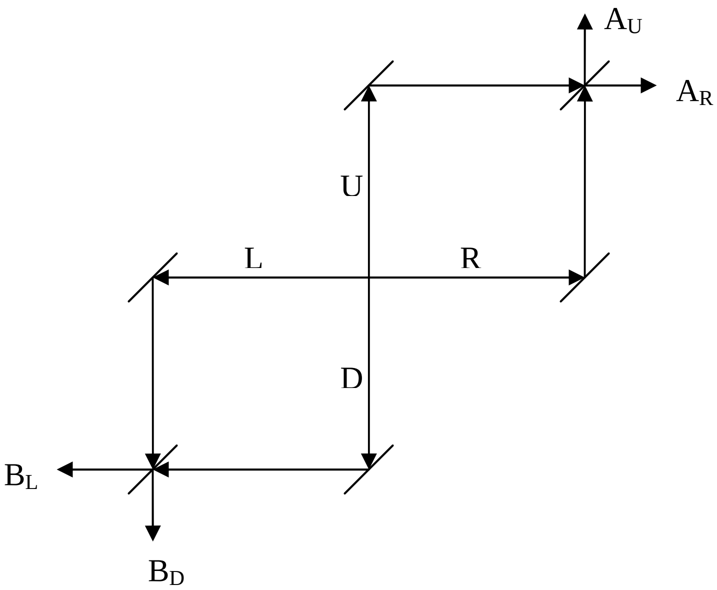
The corresponding quantum state is,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|R\rangle|L\rangle + |U\rangle|D\rangle\right) \qquad (1)$$

This means that either the two photons emerge in the left and right directions, or in the up and down directions. I don't know whether such an arrangement would be easy to achieve experimentally, but this does not matter to the principle being illustrated. The two photons are entangled since (1) is not a product state.

Now suppose we erect two Mach-Zehnder interferometers around these photons, thus,

Figure 2



The first mirror that each photon encounters is a full mirror. The second mirror is half-silvered (i.e., a beam splitter). We refer to the top right interferometer as the A device (or the A photon), and the bottom left as B. The labels A_U, A_R, B_L, B_D are four photon detectors.

The mirrors and beam splitters consist of a flat piece of glass onto one side of which has been deposited a thin layer of silver, or similar reflective metal. The terms "silvered" and "half-silvered" refer to different thickness of metal film, the first being sufficient to prevent any transmission of light through the mirror, whereas the latter is calibrated to allow about as much transmission as reflection. The phase shift caused by such mirrors differs according to whether the incoming beam is incident on the silvered surface (the "front" of the mirror) or the glass surface (the "back" of the mirror). Note that mirrors in domestic use usually have the "back" (the glass) facing forwards, and the "front" is generally covered with something opaque to protect the silver film. The phase changes caused by the mirrors and beam splitters are,

- Reflection off the front (silvered) face causes a factor of -1;
- Transmission causes a factor of $e^{i\Delta}$ where Δ depends upon the thickness and refractive index of the glass substrate;
- Reflection off the back of the mirror causes a factor $e^{2i\Delta}$ due to the passage twice through the glass substrate (the reflection itself causing no phase change).

For the interested reader, these phase change rules are derived in the Appendix. They differ from those for plane glass plates used in Chapter 8. In the analysis below the phase change Δ will be assumed the same for all four mirrors. This implies that the mirrors all have the same thickness to optical precision, i.e., to an accuracy much less than the wavelength of light. In practice this is improbable but this is not really important to the functioning of the interferometer. Some adjustment (calibration) of the interferometer is merely required prior to use to compensate for this practical limitation.

The four mirrors have their silvered surface (their "front") facing the beam, and hence all cause a phase change factor of -1. Since all beam paths are equally phase shifted by the mirrors, the mirrors are not significant to the interference. They are present simply to bring the beams together again.

The two beam splitters both have their silvered surface (their "front") on the lower surface. Different beam paths will be phase-shifted differently by the beam splitters, and this is the cause of the interference.

Note also that each of the pair of possible states after a beam splitter picks up a factor of $1/\sqrt{2}$ to maintain normalisation.

Using these rules we can work out what the quantum state will be for photons entering any of the four detectors. Before we do this, though, pause to consider what we might expect based on our knowledge of Mach-Zehnder interferometers, and in the context of the conundrum of §1.

Consider the A device. Since the initial state is a superposition of U and R, we would expect that all the photons would emerge into just one of the detectors A_U and A_R, and none in the other (providing that we have tuned the set-up appropriately). For an explanation see Chapter 8. This behaviour is the manifestation of interference for this device. Let's say all the photons would be expected in detector A_R and none in A_U (as would be deduced from the above rules if $e^{i\Delta} = -1$). Now suppose we make a "which path" measurement in device B. We can make the arms of the B interferometer very long so as to ensure that this measurement is at a space-like separation from the A-detectors. We now know which path contains the photon in the B device, and hence we also know which path contains the photon in the A device. But this must destroy the interference in the A device (as well as in the B device) and this will be apparent because the A_U detector will start registering photons. So we have achieved faster than light communication!

Of course we have not really achieved FTL communication. It turns out that where we went wrong in this analysis is in assuming that the entangled photons in this set-up behave in the same way as single, un-entangled photons in a Mach-Zehnder interferometer. They do not, as we will now show.

Consider firstly photons entering detector A_U. We call their state $|\psi : A_U\rangle$. This state can be arrived at via either of paths R or U. Following the phase factors at the mirrors and beam splitter we find that the input photon state becomes,

$$|\psi\rangle \to |\psi : A_U\rangle = \frac{1}{2}\left(-e^{i\Delta}|A_U\rangle|L\rangle - e^{2i\Delta}|A_U\rangle|D\rangle\right) = -\frac{e^{i\Delta}}{2}|A_U\rangle\left(|L\rangle + e^{i\Delta}|D\rangle\right) \quad (2)$$

Similarly the state entering detector A_R is,

$$\left|\psi\right\rangle \rightarrow \left|\psi : A_R\right\rangle = \frac{1}{2}\left(-1\times -1\left|A_R\right\rangle\left|L\right\rangle - e^{i\Delta}\left|A_R\right\rangle\left|D\right\rangle\right) = \frac{1}{2}\left|A_R\right\rangle\left(\left|L\right\rangle - e^{i\Delta}\left|D\right\rangle\right) \quad (3)$$

Assuming orthogonality, $\left\langle L\middle|D\right\rangle = 0$, we see that $\left|\psi : A_U\right\rangle$ and $\left|\psi : A_R\right\rangle$ are also orthogonal. The square moduli of both these states is ½. Hence half the photons enter each detector. There is no interference (which would have been characterised by all photons entering just one detector).

This exposes the error in the initial analysis: the entangled photons do not exhibit interference in this set-up. So there is no FTL communication.

But there is another sense in which interference *does* occur in this experimental arrangement – but without any possibility of FTL communication. This involves correlated behaviour between the photons in the A and B interferometers. To see this, consider how the state in (2) is modified by the other photon's passage through the B interferometer. The state for entry into both A_U and B_D is,

$$\left|\psi : A_U, B_D\right\rangle = -\frac{e^{i\Delta}}{2\sqrt{2}}\left|A_U\right\rangle\left(-e^{i\Delta}\left|B_D\right\rangle + e^{i\Delta}\times(-1)^2\left|B_D\right\rangle\right) = 0 \quad (4)$$

The state for entry into both A_U and B_L is,

$$\left|\psi : A_U, B_L\right\rangle = -\frac{e^{i\Delta}}{2\sqrt{2}}\left|A_U\right\rangle\left(-e^{2i\Delta}\left|B_L\right\rangle + e^{i\Delta}\times\left(-e^{i\Delta}\right)\left|B_L\right\rangle\right) = \frac{e^{3i\Delta}}{\sqrt{2}}\left|A_U\right\rangle\left|B_L\right\rangle \quad (5)$$

We conclude that if a photon registers in detector A_U then there will be no coincident photon in detector B_D, but rather there will always be a coincident photon in detector B_L. For photons entering A_R we find,

$$\left|\psi : A_R, B_D\right\rangle = \frac{1}{2\sqrt{2}}\left|A_R\right\rangle\left(-e^{i\Delta}\left|B_D\right\rangle - e^{i\Delta}\times(-1)^2\left|B_D\right\rangle\right) = -\frac{e^{i\Delta}}{\sqrt{2}}\left|A_R\right\rangle\left|B_D\right\rangle \quad (6)$$

$$\left|\psi : A_R, B_L\right\rangle = \frac{1}{2\sqrt{2}}\left|A_R\right\rangle\left(-e^{2i\Delta}\left|B_L\right\rangle - e^{i\Delta}\times\left(-e^{i\Delta}\right)\left|B_L\right\rangle\right) = 0 \quad (7)$$

So similarly we conclude that if a photon registers in detector A_R then there will be no coincident photon in detector B_L, but rather there will always be a coincident photon in detector B_D. Note that the four states in (4-7) maintain normalisation to unity.

Hence we see that interference *does* occur in a modified sense that depends upon coincident observations of photons in the A and B devices. Recall that the signature of interference in a Mach-Zehnder interferometer is that all the photons appear in one detector and none in the other. This is indeed found for the B device, provided that we filter out all the instances in which A_U records a photon and keep only cases when A_R detects a photon (or *vice-versa*). But this type of coincident, or correlated, interference does not provide any means of FTL communication. If the A detectors are at a space-like separation from the B detectors, we can only discover the 'interference' some time later, when a perfectly normal, sub-luminal, signal has communicated the results of the B detectors to the operator at A.

Caution is needed when reading the literature since this type of coincident, or correlated, interference is sometimes simply called "interference" without qualification. This is horribly confusing since true 'local' interference in such situations *would* violate causality.

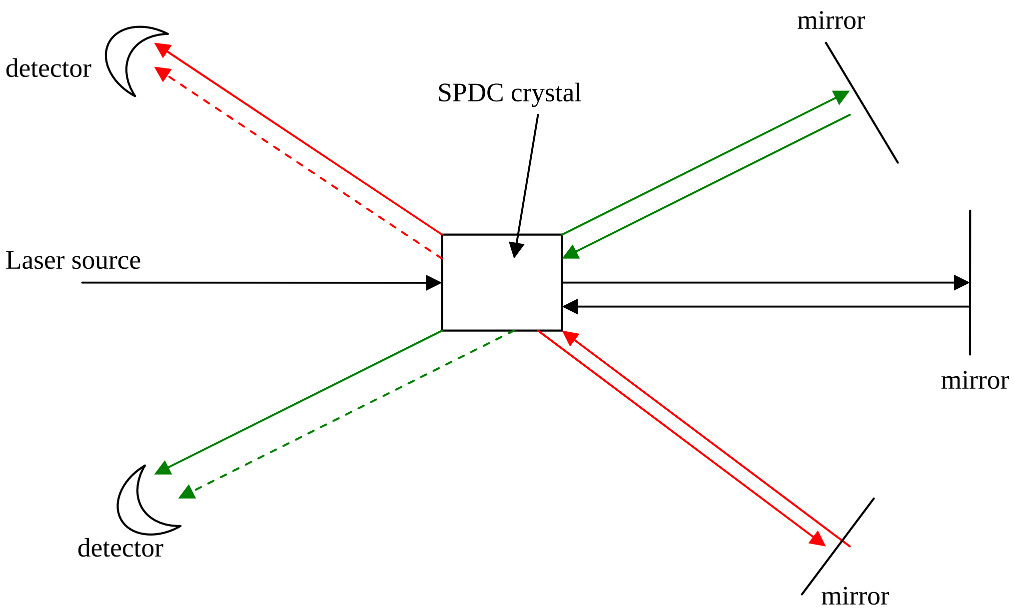
3. Entangled Interference with Crossed SPDC Beams

Here we illustrate exactly the same phenomena described in §2 but for what initially appears to be a very different experimental arrangement. The algebraic properties, though, turn out to be essentially the same. This apparatus uses spontaneous parametric down-conversion (SPDC) which splits an input photon into two identical photons of half the energy. This can be done using, for example, a lithium iodate crystal. The arrangement is shown in Figure 3.

The incoming beam from the laser is split by the SPDC crystal into two beams (shown green and red) which emerge at some characteristic angle. These beams are reflected off mirrors back through the crystal and into detectors (continuous green and red lines). These beams are not significantly affected by the crystal on their second passage (due to their reduced energy). However, the crystal acts as a beam splitter also in another sense. Not all the incoming beam is initially down-converted. What is not is transmitted through the crystal and gets reflected back to the crystal by another mirror. On this second passage there is therefore a second down-conversion which creates the green and red *dashed* beams. These are also directed into the same detectors.

Hence, a 'green' photon can reach the lower detector by either of two paths: by being down-converted at the first pass and following the continuous green beam, or by being down-converted at the second pass and following the dashed green beam. These two beam paths can (potentially) cause interference at the detectors. But do they? We are now alert to the possibility that they may not, by virtue of being one half of an entangled pair.

Figure 3



How is the state of the photon expressed algebraically? If the 'green photon' follows the continuous line, then so does the red photon. Conversely, if the green photon follows the dashed line, then so does the red photon. So the state of the entangled pair at the detectors is,

$$\left|\psi\right\rangle = \frac{1}{\sqrt{2}}\left(\left|G\right\rangle\left|R\right\rangle + \left|Gd\right\rangle\left|Rd\right\rangle\right) \tag{8}$$

Here $\left|G\right\rangle, \left|R\right\rangle$ represent the continuous green and red beams, and $\left|Gd\right\rangle, \left|Rd\right\rangle$ represent the dashed beams. We can express the Hilbert states in position representation, using the variable *x* for the green beams and *y* for the red beams. So (8) becomes,

$$\psi(x, y) = \frac{1}{\sqrt{2}}\left(\psi_G(x)\psi_R(y) + \psi_{Gd}(x)\psi_{Rd}(y)\right) \tag{9}$$

But the dashed beams differ from the continuous beams where they enter the detectors only by their phase (because they are coherent). So we can write,

$$\psi_{Gd}(x) = e^{i\theta_G}\psi_G(x) \quad \text{and} \quad \psi_{Rd}(y) = e^{i\theta_R}\psi_R(y) \tag{10}$$

Here the phase angles will be functions of position, and so properly written $\theta_G(x)$ and $\theta_R(y)$. Hence (9) becomes,

$$\psi(x, y) = \frac{1}{\sqrt{2}}\psi_G(x)\psi_R(y)\left(1 + e^{i(\theta_G + \theta_R)}\right) \tag{11}$$

Now for plane waves we can take $|\psi_G| = |\psi_R| = 1$ so that the square modulus of (11) is,

$$|\psi(x, y)|^2 = 1 + \cos(\theta_G + \theta_R) \tag{12}$$

If we ignore whereabouts on the y-screen the red photon is detected, do we see an interference pattern on the *x*-screen which is detecting the green photon? The answer is “no” since the intensity on the *x*-screen is then the average of (12) over all *y* values, i.e.,

$$\left\langle|\psi(x, y)|\right\rangle_y = \frac{1}{2\pi}\int_0^{2\pi} d\theta_R\left(1 + \cos(\theta_G + \theta_R)\right) = 1 \tag{13}$$

This retains no θ_G dependence (i.e., no *x* dependence), and hence is a uniform illumination without an interference pattern.

However, just as with the Mach-Zehnder interferometer example, we can resurrect an interference pattern by considering correlated measurements. Thus, we choose any position on the y-screen (i.e., any θ_R) and scan our detector slowly over the x-screen recording counts only if there is a coincident count at the fixed y-position detector. The pattern which emerges from the *x*-screen is just (12), for a constant value of θ_R (and noting that, for small angles, we generally get $\theta_G \propto x$). This is an interference pattern, varying from 0 to 2 as the cosine varies from -1 to +1.

The absence of a “local” interference pattern, i.e., one which does not require knowledge of the results from the other detector, saves us from a causality disaster. However, interference is occurring but can only be revealed when the results of both detectors are brought together – by some causal (sub-luminal) communication.

So this SPDC crossed-beam arrangement is essentially the same as the Mach-Zehnder example. The reason is that the algebraic structure of the quantum states, given by (8) and (1) respectively, are the same.

4. Double-Slit Interference with Spin-Entangled Particles

The preceding experimental arrangements may have given the impression that local interference cannot be observed using one of an entangled pair of particles. However this would be too loose a statement. Careful examination of these examples shows that the reason why local interference does not occur is due to the entangled degrees of freedom being the same as the degrees of freedom involved in the potential interference. However, it may be possible to obtain interference from one particle of an entangled pair provided that the interference involves different degrees of freedom from those which are entangled.

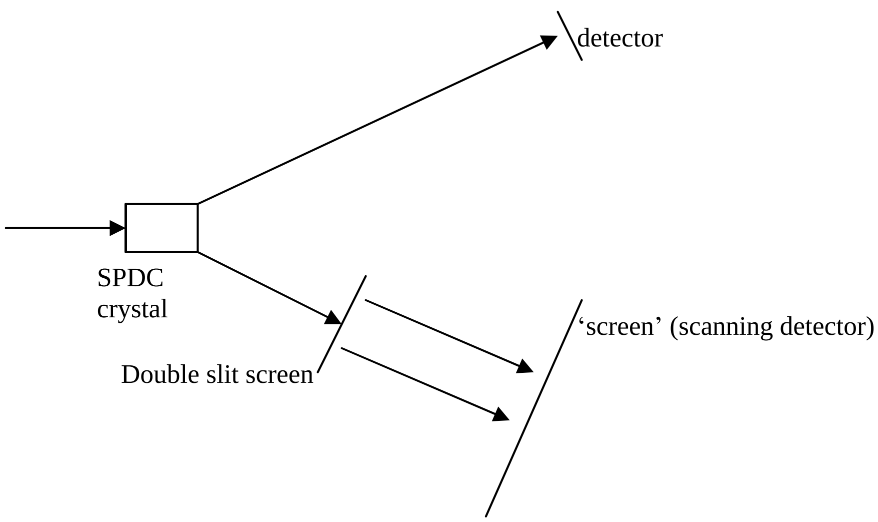
Consider the arrangement shown in Figure 4. Here just one of the beams output from an SPDC crystal (e.g., beta barium borate) is incident on a double slit screen. The other photon path plays no part initially, other than perhaps vetoing noise measurements via a coincidence counter. Do we see (purely local) interference from the lower beam? The answer is "yes" as can be seen algebraically as follows.

Suppose that either photon can emerge from the SPDC crystal in a vertical or a horizontal polarisation state, but that the tow must be polarised differently. We shall denote these states *v* or *h*. Also suppose we refer to the upper and lower beams emerging from the SPDC crystal U and L. The initial state can therefore be written,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|Uv\rangle|Lh\rangle + |Uh\rangle|Lv\rangle\right) \tag{14}$$

The two photons are thus entangled via their spin (polarisation) states in (14).

Figure 4



After the double slit screen the lower beam gets further split into beams which we label as 1 and 2. Hence, when the lower beam is split by the double slit the total state becomes,

$$|\psi\rangle = \frac{1}{2}\left(|Uv\rangle\left[|L1h\rangle + |L2h\rangle\right] + |Uh\rangle\left[|L1v\rangle + |L2v\rangle\right]\right) \tag{15}$$

Introducing a screen position variable x, the two horizontally polarised states emerging from the slits will differ by some phase factor $e^{i\theta}$ where θ is a function of the screen position, *x*. The same is true for the vertically polarised states. In the *x*-basis the overall state, (15), is therefore,

$$\begin{aligned}|\psi\rangle &= \frac{1}{2}\left(|Uv\rangle\psi_{L1}(x)\left[|h\rangle + e^{i\theta}|h\rangle\right] + |Uh\rangle\psi_{L1}(x)\left[|v\rangle + e^{i\theta}|v\rangle\right]\right)\\ &= \frac{1}{2}\left(|Uv\rangle|h\rangle + |Uh\rangle|v\rangle\right)\psi_{L1}(x)\left(1+e^{i\theta}\right)\end{aligned} \tag{16}$$

where $|\psi_{L1}(x)| = 1$ for a normalised plane wave state. Taking the absolute square of (16) we get the intensity on the *x*-screen to be,

$$|\psi(x)|^2 = 1 + \cos\theta(x) \tag{17}$$

so that there ***is*** an interference pattern in this case. How has this happened?

The two ket terms on the second line of (16) are, of course, orthogonal – so there is no cross-term to cause interference arising from them. This corresponds to our previous observations because this term involves the entangled degrees of freedom, i.e., the polarisation states. However, the interference does not arise from this term. The interference arises from the last term, the factor of $\left(1+e^{i\theta}\right)$. And this arises from the purely lower beam based cross-terms in (16), i.e., the cross-term arising from $\left[|L1h\rangle + |L2h\rangle\right]$ and equally that with the opposite polarisation. So the interference arises from the spatial degrees of freedom – the spatial separation of the two beams emerging from the double slits, beams 1 and 2. The entanglement of the polarisation degrees of freedom does not prejudice local interference arising from the spatial degrees of freedom. Nor does it provide an opportunity for FTL communication since any measurement of the polarisation in the upper beam has no effect upon the interference on the *x*-screen. This can be seen by collapsing the wavefunction of the first term in (16), to leave just one of the ket terms. It does not matter which is left since either will retain the factor of $\left(1+e^{i\theta}\right)$ which causes the interference.

So it would be quite wrong to claim that "entangled particles cannot produce local interference". In this example the entanglement with the upper beam is simply irrelevant.

Appendix: Derivation of Phase Change Rules for Mirrors

All the phase changes derived below are with respect to what the phase would have been at the same place if propagation had involved passage through air alone (i.e., with respect to the phase $e^{i\bar{k}\cdot\bar{r}}$ of a propagating wave). The phase change rules are the same for silvered and half-silvered mirrors.

Propagation through the glass of a mirror causes a phase change of $e^{ik'x}$ for a distance *x* of travel, compared with the phase change of e^{ikx} through air. The wave-numbers are related by $k' = nk$, where *n* is the refractive index of the glass, so the phase factor relative to air propagation is $e^{i(n-1)ka} = e^{i\Delta}$, as given in §2. For reflection from the rear of the mirror, the beam passes through the glass twice so the total phase factor is $e^{2i\Delta}$.

If a wave in air, e^{ikx}, meets a silvered, or half-silvered, glass surface, so that any wave entering the glass will be $Ce^{ik'x}$, the boundary conditions at the surface are that the wavefunction and its x-derivative be continuous. We must also account for a reflected wave, Be^{-ikx}. Hence we require, assuming the surface is at $x = 0$, and recalling that the incident wave has datum zero phase,

$$1 + B = C \qquad \text{and} \qquad k - Bk = k'C \tag{A.1}$$

These equations are readily solved to give,

$$B = -\left(\frac{n-1}{n+1}\right) \qquad \text{and} \qquad C = \frac{2}{n+1} \tag{A.2}$$

Since $n > 1$ it is clear that B is real and negative, corresponding to a phase change with respect to the incident wave of 180^{o}, a phase factor of -1. This confirms the phase change rule relating to reflection from the 'front' (silvered) face of a mirror, i.e., a phase factor of -1.

Now consider reflection from the "back" face. If a wave in glass meets the air boundary, the above analysis still applies except that the roles of k and k' are reversed. Hence $k' = k/n$ and hence n is replaced by $1/n$ throughout. So the reflection and transmission coefficients are now respectively,

$$B = +\left(\frac{n-1}{n+1}\right) \qquad \text{and} \qquad C = \frac{2n}{n+1} \tag{A.3}$$

The reflection coefficient is now real and positive, and so there is no phase change between the reflected and incident waves, consistent with the rule that the phase factor for reflection from the "back" face is just the factor $e^{2i\Delta}$ associated with two passages through the glass.