Chapter 9 – When Does The Universe Become Transparent?

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1. Introduction

In Chapter 8 we saw that before 185,000 years the majority of the ordinary[1] material in the universe was in the form of free electrons and protons. In other words, the ordinary matter was mostly ionised gas, or plasma. After 363,000 years, however, only a fraction of a percent was still ionised. Before 185,000 years the charged electrons, protons and nuclei will interact with any 'light', or, more generally, with any electromagnetic radiation. Such interactions would disrupt and scatter any coherent beams of 'light'. In other words, the universe is opaque before 185,000 years. But at what time does the universe become transparent? Are there sufficiently many free charged particles at 363,000 years to render the universe still opaque?

Of course, there are degrees of transparency. The clearest glass will be opaque if thick enough. The standard of transparency we have in mind is an extreme one. Astronomers wish to be able to see the whole of the observable universe. This is not merely a tautology. In this context, the 'observable' universe is the region which can in principal be observed without violating the precepts of relativity (as discussed in Chapter 5C). On the other hand, by 'see' we mean the ability to actually form an image from the light, or other electromagnetic radiation, from these distant parts of the universe. To be able to form an image from light originating at such a distance, the mean free path of photons must be comparable with, or larger than, the size of the observable universe. This is feasible only because the mean density of matter in the universe is so exceedingly small.

What interactions between the photons and matter can occur? Once all matter is in the form of neutral atoms in their ground state, the photons can no longer interact with the electrons. This is because, to do so, the electrons would need to be raised to a higher energy level, there being no lower energy levels unoccupied (by definition of the ground state, and thanks to the exclusion principle). But we have seen in Chapter 8 that, by this time, the photon energies are far too small to excite the atomic electrons. This, of course, is exactly why the neutral, ground state, atoms are stable[2].

Hence, when all matter is in the form of neutral atoms, the only possibility for photon interactions is with the atomic nuclei. We will see below that the cross sections for photon-nucleus interactions are very small compared with photon-electron interactions (that is, interactions with <u>free</u> electrons).

We have seen in Chapter 8 that by 363,000 years, essentially all helium nuclei and 99% of hydrogen nuclei have become neutral atoms. For this reason, 363,000 years is often taken as the time at which the universe became transparent. However, even after this time there will be some free protons and electrons left (less than 1 %). The fraction of remaining free protons (and electrons) falls to $<10^{-4}$ by 2574^{o}K (516,000 years). However, even after that time, there are still lithium ions, and hence a

[1] i.e. excluding dark matter and dark energy

[2] It is interesting to note, however, that this is true only because the electron transition from the ground state (n = 1) to the first excited state (n = 2) requires a larger photon energy than subsequent transitions would, i.e. from n = 2 to n = 3, etc. If the reverse were the case it would be possible to absorb photons by internal transitions whilst the atom could not be ionised.

matching number of free electrons, i.e. about 1 electron for every 10^8 initial free electrons, which persist until around 3 million years. In view of the persistent non-zero density of ions and free electrons, is the claim that the universe became transparent at ~363,000 years valid? In other words, is the *absolute* density of free electrons and ions sufficiently low after 363,000 years to render the universe transparent?

It will readily be seen that the question hinges on the size of the primordial photon:baryon ratio, since, for a given remnant fraction of free protons and electrons, this is what determines their absolute density.

To address this we need to estimate the mean free path of photons at different epochs, comparing it with the corresponding size of the observable universe. Equivalently, we need to estimate the typical time between photon interactions, and find when this first exceeds the age of the universe.

2. Calculation of the Photon Interaction Time

We shall neglect photon interactions with atoms and ions, and concentrate upon the photon interactions with free electrons. This assumption will be justified at the end of this Section.

The photon-electron differential cross-section (Compton scattering) is given by the Klein-Nishina formula (see Bjorken & Drell Equ.7.74 or Mandl & Shaw, Equ.8.82). In the case of unpolarised particles, and in the non-relativistic limit, the total cross-section reduces to the Thomson formula, i.e.,

$$\sigma_{\text{lab}} = \frac{8\pi\alpha^2}{3}\left(\frac{\hbar}{mc}\right)^2 = \frac{8\pi e^4}{3(mc^2)^2} = 6.68 \times 10^{-29}\,\text{m}^2 \qquad (1)$$

where m is the electron mass (0.511MeV). Note that the dependence on $\hbar$ cancels out of Equ.(1). Note that we are safely in the non-relativistic regime here, since the typical photon energies are <1eV at the times of interest. Hence, we need not worry about the distinction between the laboratory frame and the centre-of-mass frame.

The photon number density at temperature T is given as usual by,

$$\rho_\gamma^N = 0.2436\left(\frac{k_B T}{\hbar c}\right)^3 \qquad (2)$$

The electron density before hydrogen recombination, but after helium recombination, is given by,

$$\rho_{e0}^N = 0.75\left(\frac{\rho_\gamma^N}{1.9 \times 10^9}\right) \qquad (3)$$

where 1.9×10^9 is the ratio of photons to baryons (see Chapter 4). The flux of photons is given as usual by,

$$J = \frac{1}{4}\rho_\gamma^N c \tag{4}$$

The number of photons interacting per second with a given electron is Jσ. Thus, the number of photon-electron interactions per second per unit volume is,

$$\text{Interactions per sec per m}^3 = \frac{1}{4}\rho_\gamma^N \rho_e^N \sigma c \tag{5}$$

Alternatively, the number of interactions which each photon undergoes per second is,

$$\text{Interactions per sec per photon} = \frac{1}{4}\rho_e^N \sigma c \tag{6}$$

Taking the reciprocal, the average time between interactions for any given photon is,

$$\text{Interaction Time for a given photon, } T_I = 4/\rho_e^N \sigma c \tag{7}$$

The electron number-density may be written as a fraction, y, of the starting density, i.e.,

$$\rho_e^N = y\rho_{e0}^N \tag{8}$$

The fraction, y, has been derived as a function of the temperature (time) in Chapter 8, and the starting density of electrons is also known in terms of the temperature from Equs.(2,3) above. Hence, using Equ.1 for the cross-section, σ, allows the interaction time, T_I, to be found at any temperature (time). The result is as follows:-

time, t years	**Temperature, ^{o}K**	**y**	**Interaction Time T_I, years**	**T_I / t**
10,000	18,200	1	113	0.006
30,000	10,500	1	580	0.019
100,000	5,750	1	3,570	0.036
180,000	4,290	0.99	8,650	0.048
220,000	3,880	0.75	15,500	0.070
238,000	3,730	0.50	26,000	0.11
260,000	3,570	0.25	60,000	0.23
286,000	3,400	0.10	173,000	0.60
300,000	3,320	0.057	325,000	1.08
314,000	3,246	0.033	600,000	1.92
350,000[(2)]	3,070	0.010	2,350,000	6.7
*585,000**	*2,200*	*10^{-6} (1)*	*6×10^{10}*	*$\sim 10^5$*
*1,760,000**	*1,055*	*10^{-8} (1)*	*6×10^{13}*	*$\sim 2 \times 10^7$*
*2,440,000**	*850*	*10^{-10} (1)*	*$\sim 10^{16}$*	*$\sim 2 \times 10^9$*

*using $Tt^{3/2} = 1.54\text{x}10^{12}$ for T<3000^{o}K. Earlier times use $T\sqrt{t} = 1.02\text{x}10^{10}$.

[(1)]These very low y values are actually erroneous because freeze-out of hydrogen recombination occurs at y ~ 7 x10^{-5} (as shown in Chapter 9b), forming a lower limit for y.

[(2)]Using $T\sqrt{t} = 1.038\text{x}10^{10}$ this would be 363,000 years.

The final column gives the interaction time as a fraction of the age of the universe. Hence, we see from the above Table that the universe does indeed become transparent over the same time period that hydrogen is recombining. The universe is opaque at the start of this period (say, 180,000 years) since the average photon will experience ~20 interactions during its passage across the universe. By the end of the period of hydrogen recombination, however (at, say, 363,000 years) most photons will circumnavigate the whole universe without interaction. The time at which half the photons interact in their circumnavigation is ~300,000 years (which is 310,000 years using $T\sqrt{t} = 1.038 \times 10^{10}$). This is the nearest we can judge to the middle of the transition between opaque and transparent conditions (translucent?). Hence, the usual statement is confirmed.

Note that we can consider the transparency condition as being another example of a reaction being frozen out by the universal expansion, this time the 'reaction' being elastic collisions between electrons and photons. In this case we would compare the reaction rate with H =1/(2t), or equivalently compare the reaction time with 2t. It can be seen from the above Table that this makes little difference to the conclusion, the transparency time being delayed from 300,000 years (y = 0.057) to about 314,000 years (y = 0.033).

Note that the interaction time is proportional to the inverse of the fraction, y, of electrons remaining (see Equs.7, 8). Examining the above Table we see that it is the reduction in y that causes the universe to become transparent. For example, between 180,000 years and 350,000 years, y decreases by a factor of 100 whereas T^3 decreases only by a factor of around 3. If y remained at unity, the T_I/t ratio would not increase above ~0.2 and the universe would remain opaque indefinitely. This is entirely reasonable since it is the reduction in the number of free electrons which eventually renders the universe transparent.

However, it is the absolute density of free electrons that determines whether the universe is opaque or transparent. As well as depending upon the fraction of the original number of free electrons remaining, the absolute density of free electrons is proportional to the number primordial baryons, i.e. inversely proportional to the photon:baryon ratio – which has been assumed to be 1.9×10^9 in the above calculation. This raises the interesting questions,

(a) If the photon:baryon ratio were different, might the onset of universal transparency not coincide with recombination after all?;
(b) If the photon:baryon ratio were different, might the universe have remained opaque indefinitely?

The answer to (a) is "yes", as demonstrated in the next Section. The answer to (b) is "no", the universe would become transparent even if the photon:baryon ratio were changed by up to 4 orders of magnitude. This will be demonstrated in Chapter 9b.

3. Recombination and Transparency Times' Dependence On Photon:Baryon Ratio

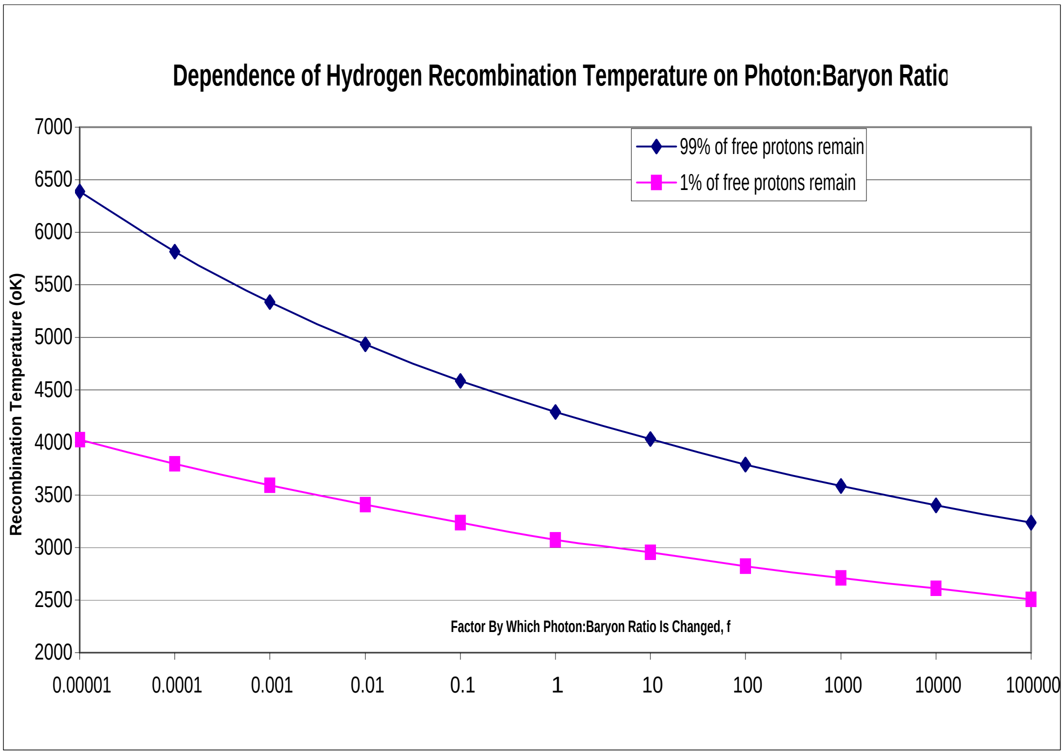
Firstly we re-calculate the recombination time for hydrogen for a range of different photon:baryon ratios. Equ.(39) of Chapter 8 may be written for an arbitrary photon:baryon ratio ξ as,

$$\frac{y^2}{1-y} = 0.3\xi\left(\frac{mc^2}{k_B T}\right)^{3/2} e^{-\chi/kT} \qquad (9)$$

where χ is the ionisation potential of hydrogen (13.6eV) and y is the fraction of free electrons remaining at temperature T, as per Chapter 8 and Section 2, above. We consider orders of magnitude variations in $\xi_{\gamma N}$, defining the factor of change, f, by,

$$\xi_{\gamma N} = f \; x 1.9 \, x 10^9 \qquad (10)$$

The temperatures at the onset of recombination (y = 0.99) and the near-completion of recombination (y = 0.01) are given in the graph and Table below;



f	Temperature (^{o}K) for y = 0.99	Temperature (^{o}K) for y = 0.01
0.00001	6388	4029
0.0001	5815	3797
0.001	5337	3589
0.01	4933	3405
0.1	4585	3239
1	4290	3070
10	4030	2957
100	3791	2823
1000	3588	2713
10,000	3403	2609
100,000	3235	2508

These temperatures may be converted to times as usual using $\sqrt{t(\sec s)} = 1.021 \, x \, 10^{10} / T$, or $Tt^{3/2} = 1.536x10^{12}$ for $T<3000^o$K.

We can now calculate the average photon-electron interaction time, Ti, in exactly the same way as in Section 2 except that the electron density in Equ.2 is factored down by f. We perform the calculation at the two temperatures corresponding to the onset and end of recombination (y = 0.99 and 0.01 respectively). Thus, for each different photon:baryon ratio we can deduce whether the onset of universal transparency coincides with the onset of hydrogen recombination (y = 99%) or with recombination being 99% complete (y = 1%). In general it does not, as we see from the following Table,

p:b ratio factor f	**temp ^oK**	**y**	**Ti years**	**universe age, yrs**	**Ti / universe age**	
0.00001	6388	0.99	2.63E-02	81006	3.25E-07	opaque
0.00001	4029	0.01	1.04E+01	203634	5.10E-05	opaque
0.0001	5815	0.99	3.49E-01	97756	3.57E-06	opaque
0.0001	3797	0.01	1.24E+02	229279	5.41E-04	opaque
0.001	5337	0.99	4.51E+00	116051	3.89E-05	opaque
0.001	3589	0.01	1.47E+03	256624	5.72E-03	opaque
0.01	4933	0.99	5.71E+01	135838	4.20E-04	opaque
0.01	3405	0.01	1.72E+04	285109	6.03E-02	opaque
0.1	4585	0.99	7.11E+02	157241	4.52E-03	opaque
0.1	3239	0.01	2.00E+05	315082	6.34E-01	borderline
1	4290	0.99	8.68E+03	179610	4.83E-02	opaque
1	3070	0.01	2.35E+06	350726	6.69E+00	transparent
10	4030	0.99	1.05E+05	203533	5.15E-01	borderline
10	2957	0.01	2.62E+07	375000	6.98E+01	transparent
100	3791	0.99	1.26E+06	230005	5.47E+00	transparent
100	2823	0.01	3.02E+08	402500	7.50E+02	transparent
1000	3588	0.99	1.48E+07	256767	5.78E+01	transparent
1000	2713	0.01	3.40E+09	427000	7.96E+03	transparent
10000	3403	0.99	1.74E+08	285444	6.09E+02	transparent
10000	2609	0.01	3.82E+10	453000	8.40E+04	transparent
100000	3235	0.99	2.02E+09	315861	6.41E+03	transparent
100000	2508	0.01	4.30E+11	480600	8.90E+05	transparent

We see that changing the photon:baryon ratio by a factor of ten either way just preserves the coincidence between transparency and recombination, although the actual value ($\sim 1.9 \times 10^9$) makes the coincidence most accurate. However, changing the photon:baryon ratio by more than a factor of 10 either way would lead to different times (temperatures) for transparency and recombination. In short, the coincidence of transparency and recombination is an 'accident' which depends upon the particular (apparently contingent) value for the photon:baryon ratio adopted by our universe.

Turning this around, we could postulate that an equality of the recombination and transparency times is required - which would allow the correct photon:baryon ratio to be derived, albeit only within about an order of magnitude. Quite why the transparency and recombination times should be required to be equal is not clear.

4. Algebraic Expressions and the Origin of the Coincidence

It is instructive to re-examine the coincidence between the hydrogen recombination time and the transparency time using algebraic expressions. This clarifies how the coincidence arises, and what physical parameters it depends upon. Thus, we combine Equs.2, 3, 7, 8 above to give the interaction time as,

$$T_I = 1.96\left(\frac{\xi}{yf_p}\right)\frac{1}{\alpha^2}\cdot\frac{\hbar\lambda^2}{kT} \tag{11}$$

where the following are all dimensionless,

- f_p = the proton:baryon ratio (0.875 from Chapter 6);
- ξ = the photon:baryon ratio (1.9 x 10^9 from Chapter 4);
- α = the fine structure constant ($e^2/\hbar c$ in cgs units $= 1/137$);
- $\lambda = mc^2/kT$;
- y = the fraction of the original number of free electrons left (that is, as a fraction of those remaining after helium recombination);

Both y and λ vary with time. For completeness we note that the coefficient of 1.96 derives from the black body equation and the Thomson cross section (Equ.1) and other factors, giving,

$$1.96 = \frac{3}{2\pi \text{ x } 0.2436} \quad \text{and} \quad 0.2436 = \frac{1}{\pi^2}\int_0^\infty \frac{x^2 dx}{e^x - 1} = \frac{2.4041...}{\pi^2} \tag{12}$$

and 0.2436 is the coefficient that appears in the expression for the number density of black body photons.

The age of the universe can be written, from Chapter 3 Equ.3,

$$T\sqrt{t_{univ}} = \frac{0.405}{k}\left(\frac{c^5\hbar^3}{G}\right)^{1/4} \tag{13}$$

where the numerical coefficient is,

$$0.405 = \left[\frac{1}{1.681}\cdot\frac{45}{32\pi^3}\right]^{1/4} \tag{14}$$

and the term 1.681 is the factor by which the three neutrino species increase the radiation energy density with respect to that due to the photons alone. The latter is derived in Chapter 5 as,

$$1.681 = 1 + \frac{21}{8}\left(\frac{4}{11}\right)^{4/3}$$

We display all these exact expressions explicitly as a reminder of how far we can go in calculating these features of the universe from first principles. However, at some point reality must impose itself. We have already noted in Chapter 4 that Equ.(13) gives a remarkably good prediction of the cosmic microwave background temperature for a 'first principles' derivation (3.5 – 4°K). However, it is better to use the accurately known value of the CMB temperature (2.728°K) to tune the coefficient in Equ.13, i.e. we put instead,

$$T\sqrt{t_{univ}} = \frac{0.31}{k}\left(\frac{c^5\hbar^3}{G}\right)^{1/4} = 1.021 \text{ x } 10^{10} \ {}^{o}K\sqrt{sec} \qquad (13b)$$

Hence, equating the age of the universe from Equ.13b to the photon interaction time from Equ.11, our definition of transparency, we can simplify the resulting expression to…

$$\frac{\xi}{yf_p} = \frac{0.049\alpha^2}{\lambda}\left(\frac{M_{Planck}}{m}\right), \quad \text{where} \quad M_{Planck} = \left(\frac{\hbar c}{G}\right)^{1/2} \qquad (14b)$$

Hence, given the ratio of the Planck mass to the electron mass (2.4 x 10^{22}), and given the ratio of the densities of the remaining free electrons to photons prevailing at the time (the latter being yf_p/ξ), we can derive the value of λ, which yields the prevailing temperature, and hence time, of transparency. Thus, we can check consistency with the results of Section 2, e.g. noting that by interpolation the Table in Section 2 gives equality of T_I and t_{univ} for y = 0.064. Substituting this value for y into Equ.14 together with the usual values of the other parameters, as summarised at the start of this Section, we get λ = 1.85 x 10^6 and hence a transparency temperature of 3,200°K, in agreement with Section 2.

The recombination temperature from Chapter 8 or Equ.9 above may be written,

$$\frac{y^2}{1-y} = 0.261\frac{\xi}{f_p}\left(\frac{mc^2}{kT}\right)^{3/2} e^{-\chi/kT} \qquad (15)$$

where the numerical coefficient derives from 0.261 = 1/[0.2436 x $(2\pi)^{3/2}$] and 0.2436 derives from Equ.12. Now the ionisation potential of hydrogen is just the ground state energy, and hence can be written,

$$\chi = \alpha^2\frac{mc^2}{2} \quad \text{Hence} \quad \frac{\chi}{kT} = \frac{\alpha^2}{2}\lambda \qquad (16)$$

So Equ.15 becomes,

$$\frac{y^2}{1-y} = 0.261\frac{\xi}{f_p}\lambda^{3/2}e^{-\alpha^2\lambda/2} \qquad (17)$$

which can be re-arranged as,

$$\frac{\xi}{yf_p} = 3.84\left(\frac{y}{1-y}\right)\lambda^{-3/2}e^{\alpha^2\lambda/2} \qquad (18)$$

Again we may check that this is consistent with the numerical results in the Table in Section 3 (but noting that the RHS is extremely sensitive to small changes in λ).

We may now explore the consequences of assuming that the recombination time and the transparency time are the same. In other words, we equate Equs.14b and 18 for the same value of the parameter λ:-

$$\frac{0.049\alpha^2}{\lambda}\left(\frac{M_{Planck}}{m}\right) = 3.84\left(\frac{y}{1-y}\right)\lambda^{-3/2}e^{\alpha^2\lambda/2} \quad (19)$$

The value for y which we insert into Equ.19 depends upon our assumption as to when recombination can be said to have happened. If the transparency time is supposed to align with a time when recombination is largely complete, then y will be a smallish fraction. However, it is meaningless to insert zero since this is clearly only going to result in an infinite time (zero temperature). An assumption y = 0.064 is as reasonable as any. Solving Equ.19 for λ with y = 0.064 gives $\lambda = 1.772 \times 10^6$, and hence T = 3343^{o}K at a time 296,000 years. Not surprisingly, this is compatible with our previous results in Sections 2 and 3.

However, in this case we have made no assumption for the photon:baryon ratio – since this does not appear in Equ.19. Rather, having found the value of λ, we can use Equ.14b or Equ.18 to predict the photon:baryon ratio that results from the hypothesis that the recombination time and the transparency time are the same. The result, assuming the usual proton:baryon ratio $f_p = 0.875$, is $\xi_{\gamma N} \sim 2 \times 10^9$.

In conclusion, the hypothesis that the recombination time and the transparency time are the same provides a means of calculating the required photon:baryon ratio. The result is in remarkable agreement with the observed value, $\xi_{\gamma N} \sim 2 \times 10^9$, although we note that there is an arbitrariness in the definition of the recombination time (defined here as y = 0.064, which effectively tunes the result for $\xi_{\gamma N}$).