Chapter 6
The Shifting Neutron:Proton Ratio In The Early Universe: The First Five Minutes
With Some Remarks About Neutrino Decoupling

Last Update: 21 June 2006

1. Introduction

As explained in Chpater 3, the "easy" story of the early universe starts at around ~0.001 to ~0.01 second. The physics of the universe is relatively easy after this time because there are few particle species present. This is because the temperature is low enough to ensure that the only particles that can be created out of the available thermal energy are electrons/positrons and neutrinos (and, of course, photons). We say nothing of dark matter purely out of ignorance about what it is. At slightly earlier times the next lightest particles, muons, would have been abundant (mass ~105 MeV). At still earlier times, more and more of the heavier particles would have existed in large numbers. Proceed backwards in time. At times early enough for the strongly interacting particles to be present in large numbers, starting with pions of mass ~135-140 MeV, things become extremely complicated. This is because huge numbers of different hadrons can then be formed by strong interactions. For this reason, we pick up the story at around 0.001-0.01 seconds.

At this time, the typical thermal energy (~3kT) is much larger than the rest mass of the electron. Hence, the electrons, positrons and all the neutrino species are present in numbers comparable with the photons (i.e. as determined by the relativistic Fermion black-body spectrum, and hence the same as the photon density apart from the "numerology" factors – see the General Physics part of the web site for a derivation). However, there are also neutrons and protons present. These are the survivors from the earlier, "difficult", period, before 0.001 seconds. They are far fewer in number, about 1 nucleon for every 2 x 10^9 photons. This baryon:photon ratio cannot be derived from consideration of the "easy" period, after 0.001 seconds. To derive it theoretically would require analysis of the difficult earlier period in which the strong nuclear interactions were dominant. The nucleons now extant are the few survivors from the very early annihilations in which nucleons outnumbered antinucleons only very marginally. This misbalance in nucleon and antinucleon numbers implies a break down of time reversal symmetry. Thus, a theoretical prediction of the photon:baryon ratio requires a theory in which time-reversal symmetry is not respected, or is broken. We will not stray into these deep and murky waters. Instead we shall be content that, at present, our knowledge of the baryon:photon ratio comes from observational measurements rather than theory. We shall simply assume that this ratio is one of the 'God given' universal parameters.

At sufficiently early times the typical thermal energy, ~3kT, would have been great enough to create nucleon/antinucleon pairs. At these times, the nucleons, and many other hadrons, would have been roughly as abundant as photons, neutrinos, etc. Consequently, only 1-in-2x10^9 nucleons have survived from this period. It is not surprising that this is a very small fraction. If charge conjugation symmetry (or, equivalently, PT symmetry) were exact, the number of survivors would be precisely zero, because the number of nucleons and antinucleons would match exactly. Thus, the entire inventory of 'normal' matter in the universe today is solely the result of the deviation of the laws of physics from exact charge conjugation symmetry. By ‘normal’ matter we mean baryonic matter, plus electrons, i.e. excluding ‘dark’ matter[1] and ‘dark energy’. Hence, it should really be no surprise to find that normal matter accounts for only about 2% to 4% of the average density of the universe. At the time of writing, dark matter is thought to account for 25-27%, and dark energy for the rest (roughly 71%).

Hence, an unbiased perspective might be that the nucleons formed a quite unimportant minority constituent of the early universe, when photons and neutrinos were dominant, and that the same remains true today, since dark matter and dark energy are now dominant. However, we have a particular interest in the baryonic minority constituent. This is what we and our world are made of. Moreover, it is what everything we can see is made of, even the most distant of astronomical objects which are actually visible. However, there is an important note of caution here. We currently have very little idea at all of the nature of 96%-98% of the universe. It is not clear whether the dark matter and dark energy that seem to be present in the universe can arise out of the standard big bang model. Until this is resolved, the standard model must be regarded as tentative. Nevertheless, the standard model has some very impressive successes. The prediction of the existence, and approximate temperature, of the microwave background is the most obvious. Perhaps more impressive, though, is the correct prediction of the primordial helium abundance. This derivation is the subject of this Chapter.

2. Evolution of the Neutron:Proton Ratio in Outline

We briefly preview in this Section what will be covered in more detail in the subsequent sections.

- When we start our story, at ~0.001 seconds, the neutrons and protons are nearly equal in number. This applies when kT is large compared with the mass difference $M_n – M_p$.

- At ~0.25 seconds kT becomes comparable with $M_n – M_p$. Consequently, it is now significantly more difficult to make neutrons out of protons than vice-versa, and the neutron:proton ratio has fallen significantly below 50:50.

- In the first second or so, neutrons and protons can interconvert via the following reactions (mediated by the weak nuclear force):-

$$n + \nu_e \Leftrightarrow p + e\,, \qquad n + \overline{e} \Leftrightarrow p + \overline{\nu}_e$$

These are the reactions which provide the mechanism for maintaining the neutron:proton ratio in thermal equilibrium. If these reactions are prevented from occurring at some time, as indeed they are at around 1 sec (see below), the neutron:proton ratio will “freeze” at its value at that time – except for the natural beta decay of the neutrons[2]. Because the temperature at which the above reactions ‘freeze out’ determines the end of the fast phase of neutron destruction, it is the key feature which determines the cosmic helium abundance.

[1] If indeed this *is* distinct from ordinary matter.
[2] Free neutrons are unstable with a half-life of about 15 minutes. They are stable only when combined with protons within a nucleus.

- Aside: the neutrino reactions with the nucleons, together with the purely leptonic interactions like $e + \nu_e \Leftrightarrow e + \nu_e$ and $\mu + \nu_e \Leftrightarrow e + \nu_\mu$, are the mechanism by which the neutrinos maintain their thermal equilibrium with the rest of the universe (the neutrinos do not interact directly with photons). If the purely leptonic interactions cease at the same time as the neutrino-nucleonic interactions, then the neutrinos will decouple from the rest of the universe at this time. We shall show that this is true below. Virtually none of the 'cosmic background' neutrinos will ever interact with anything again in the life of the universe.

- After the neutrino-nucleon reactions have ceased, the neutron:proton ratio continues to fall, but now far more slowly due only to the beta decay of the free neutrons.

- If nothing intervened to save them, neutrons would therefore be destined to disappear from the universe. If nothing intervenes within a period of the order of several of their half-lives, it will already be too late to prevent the number of neutrons being far smaller than the number of protons for the rest of the life of the universe. However, something does intervene – and within the required timescale (namely within 5 minutes, the half-life of a free neutron being about 15 minutes). This something is the formation of stable nuclei. Neutrons become stable when bound with a proton to form a deuterium nucleus, provided that the temperature is not so high as to cause photodisintegration. Thus, the formation of stable deuterons constitutes a second crucial "freeze out" in the neutron abundance. We shall show below that deuterons become stable at ~2-5 minutes. This is the crucial time which determines the universal neutron abundance ever after. Since virtually all cosmic neutrons are in the form of helium nuclei (other elements being a tiny proportion) this also determines the cosmic helium abundance in comparison with that of hydrogen. Hence the time of this second "freeze out" is the second determining factor in the prediction of the cosmic helium abundance.

- The occurrence of both "freeze outs" is crucial to the helium (neutron) abundance. If either were substantially delayed, the cosmic helium abundance could be drastically reduced, potentially to virtually nothing. However, this would probably have no great implications for the subsequent evolution of the universe (see "Cosmic Coincidences" Part 1). On the other hand, if the first freeze out were much earlier, the abundance of neutrons and protons would be virtually equal. The primordial universe would then be virtually pure helium with very little hydrogen. This would have profound effects on the chemistry of the universe for ever after. Containing no hydrogen, the universe would contain no water, no hydrocarbons and no proteins as we know them. Note that the first freeze-out involves the weak nuclear force, whereas the second involves the strong nuclear force.

3. Details: Phase 1 – Equilibrium Ratio

In this first phase, the nucleon-neutrino reactions $n + \nu_e \Leftrightarrow p + e$, $n + \bar{e} \Leftrightarrow p + \bar{\nu}_e$ are active. They provide the mechanism by which thermal equilibrium between the density of neutrons and the density of protons can be achieved. Note that these reactions occur in both directions. One means of finding the neutron:proton ratio (in principle) is to accurately calculate the rate of each of these four reactions. These will depend on the particle densities and the cross sections for each reaction. The cross sections will depend upon the weak interaction coupling strength (e.g. Fermi's constant), the particle energies, and the form factors for the nucleons. The latter requires specialist input to be calculated accurately. The rate of change of the neutron density, which is equal and opposite to the rate of change of the proton density, is then found by subtracting the rate of neutron loss in the forward reactions from the rate of neutron production in the backward reactions.

It is clear that, as the temperature falls, the neutron density will reduce, and the proton density will increase. This is because in $n + \nu_e \Leftrightarrow p + e$, $n + \bar{e} \Leftrightarrow p + \bar{\nu}_e$ the forward reactions can occur even at vanishingly small neutrino or positron kinetic energies. In contrast, the reverse reactions can occur only if the combined kinetic energies of the particles on the RHS exceeds the mass difference, $M_n - (M_p + M_e)$ [or, for the second reaction, $M_n - M_p + M_e$]. If the reactions are in equilibrium at a given temperature, then to compensate for this kinematic disadvantage, the reverse reactions must have an advantage in terms of greater particle densities of the reacting particles, p + e and p + $\bar{\nu}_e$ respectively. Since the lepton densities are fixed by the Fermion black body spectrum, and the number of positrons is essentially equal to the number of electrons at these times, it follows that the proton density must exceed the neutron density in order that the forward and backward reactions balance. [This implicitly assumes that, apart from the kinematic factors discussed, the 'matrix element' part of the cross sections for the forward and reverse reactions are the same. This follows from time reversal symmetry, e.g. by expressing the matrix elements in terms of Feynman diagrams – in which the forward and backward reaction matrix elements are simply conjugates].

Whilst the above sketch of a possible calculational route is correct in principle, it has two drawbacks. The first is that it is complicated and requires specialist form factors for the nucleons to carry out the calculations of the reaction rates. The second is that it involves the delicate cancellation of large quantities, i.e. the rate of change of the neutron density is the difference in rates of the backward and forward reactions. Since it is likely that the reaction rates can only be estimated approximately, it is likely the difference in the (nearly matched) backward and forward reaction rate estimates will be grossly in error. Fortunately there is a solution to this problem. Instead of trying to calculate the small rate difference, we assume it is zero! In other words, we assume that an equilibrium state prevails, and the forward and backward reactions are in balance. A different equilibrium configuration will exist at different times, i.e. at different temperatures. We are thus assuming that equilibrium can be established sufficiently quickly (i.e. in extremely small fractions of a second) so that the universe passes from one equilibrium configuration to another continuously.

[NB: The total number of nucleons is fixed. Hence the nature of this thermal equilibrium is very different from that which prevails at temperatures ~100-1000 times higher, when nucleons can be created out of the thermal energy. The total number of nucleons plus antinucleons is then vastly greater, and can be deduced from the temperature. The baryon number is the same though, but not deducible simply from the temperature (at least, not without input from some Grand Unified Theory or string theory)].

For thermal equilibrium we can appeal to the Boltzmann distribution to give immediately the ratio of the neutron and proton densities, i.e.,

$$\frac{N_n}{N_p} = e^{-\Delta M / k_B T} \text{ where, } \Delta M = M_n - M_P = 1.293\text{MeV} \qquad (1)$$

From Chapter 4, the time-temperature relationship for the radiation era of the universe after annihilation of the electron/positron pairs is,

$$T(^0K) = \frac{1.021 \times 10^{10}}{\sqrt{t(\text{secs})}} \qquad (2a)$$

But after annihilation of the electron/positron pairs the temperature became a factor of 1.40 times greater than before. So our best estimate of the temperature before the annihilation of the electron/positron pairs is,

$$T(^0K) = \frac{0.73 \times 10^{10}}{\sqrt{t(\text{secs})}} \qquad (2b)$$

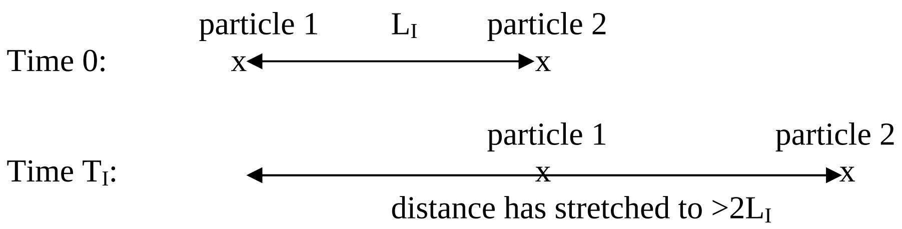
Hence, using (1) and (2) we can deduce the neutron:proton ratio at any time as long as equilibrium prevails. Thus we find,

t (sec)	**T (^{o}K)**	**N:P**	**N (%)**	**P (%)**
0.001	2.31E+11	0.94	48.38	51.62
0.01	7.30E+10	0.81	44.88	55.12
0.025	4.62E+10	0.72	41.95	58.05
0.05	3.26E+10	0.63	38.71	61.29
0.075	2.67E+10	0.57	36.29	63.71
0.1	2.31E+10	0.52	34.30	65.70
0.15	1.88E+10	0.45	31.09	68.91
0.2	1.63E+10	0.40	28.52	71.48
0.3	1.33E+10	0.32	24.50	75.50
0.4	1.15E+10	0.27	21.42	78.58
0.5	1.03E+10	0.23	18.95	81.05
0.65	9.0×10^9	0.19	15.89	84.11
0.75	8.43E+09	0.17	14.44	85.56
1	*7.30E+09*	*0.13*	*11.36*	*88.64*
1.5	*5.96E+09*	*0.08*	*7.47*	*92.53*
2	*5.16E+09*	*0.05*	*5.19*	*94.81*
2.5	*4.62E+09*	*0.04*	*3.74*	*96.26*
3	*4.21E+09*	*0.03*	*2.77*	*97.23*

NB: The figures in italics are hypothetical since 'freeze out' occurs at >1 sec.

4. Details: Phase 2 – The End of the Equilibrium

The first of the two 'freeze outs' occurs when the reactions $n + \nu_e \Leftrightarrow p + e$, $n + \bar{e} \Leftrightarrow p + \bar{\nu}_e$ cease. Why should this happen? The reason is that these reactions are occurring in an expanding universe. Providing that the universe does not expand very much in the time between nucleon/lepton interactions the expansion makes little difference. However, suppose that, for a given particle, the typical time between interactions is T_I and that a particle typically travels a distance L_I in this time. What if the universe expands sufficiently in this time interval that the distance L_I stretches to more than $2L_I$? In other words, what if the fractional expansion, $\Delta L/L$ of the universe in time T_I is greater than one? This would mean that, at the end of the period T_I, the particle has a greater distance still to travel before it interacts than it had at the start! The situation is illustrated as follows, where particle 1 is to be imagined as travelling towards particle 2 with which it would potentially interact,



We conclude that if the 'strain' of the universe exceeds unity in the time period T_I between interactions then the reactions will cease. Now the strain rate of the universe is just the Hubble parameter because,

$$\frac{dR}{dt} = HR \qquad \text{implies} \qquad H = \frac{\left(\frac{dR}{R}\right)}{dt} = \frac{d\varepsilon}{dt} \tag{3}$$

Hence, our requirement for switching off the interactions is,

$$HT_I \geq 1 \tag{4}$$

But since T_I is the mean time between interactions for a given particle, its reciprocal is just the reaction rate per particle. Hence, the interactions switch off if,

$$H > \text{reaction rate per particle} \tag{5}$$

We now need to find the reaction rate. It is unavoidable that the actual strength of the weak nuclear force should enter our calculation at this point. However, we will not attempt an accurate calculation involving nucleon form factors, etc. Instead we shall be guided by the cross-section for so-called pseudo-elastic muon-neutrino scattering $\mu + \nu_e \to e + \nu_\mu$. In the centre of mass frame the total cross section of this purely leptonic reaction is,

$$\sigma(\mu + \nu_e \to e + \nu_\mu) = \frac{4}{\pi} G_F^2 E_\nu^2 \tag{6}$$

where $G_F = \frac{1.03 \times 10^{-5}}{M_P^2}$ is Fermi's constant for the weak nuclear force, and E_ν is the energy of the incoming electron-neutrino. M_P is the proton mass. In Equ.(6) and in the value for Fermi's constant we are taking the usual particle physicists' liberties with units, i.e. G_F can be thought of as having dimensions 1/mass or 1/energy or length – according to taste. Results in physical units are obtained by factoring using c and $\hbar$ as required. Thus, to get a cross section in m^{-2} from Equ.(6), and for a neutrino energy expressed in J, we would use $G_F = 1.03 x 10^{-5} \hbar c / (M_p c^2)^2 = 1.44 x 10^{-11}$ m/J.

In applying Equ.(6) to a nucleon target, we shall assume that the three quarks of which the nucleon is composed each behave like the massive lepton, and each behave independently. Now a neutron is composed of the quarks udd, and a proton of uud. Thus, the reactions $n + \nu_e \Leftrightarrow p + e$, $n + \bar{e} \Leftrightarrow p + \bar{\nu}_e$ involve the conversion of a d quark into an u quark. Hence, just two of the three quarks in the neutron will do as a suitable target for the reactions, that is the d quarks. Thus, we shall assume the nucleon cross-section is just twice Equ.(6), and we assume the cross section is the same for both possible reactions, i.e.,

$$\sigma(n + \nu_e \to p + e) = \sigma(n + \bar{e} \to p + \bar{\nu}_e) = \frac{8}{\pi} G_F^2 E_\nu^2 \qquad (6b)$$

[<u>Aside</u>: Equs.6,6b apply in the centre-of-mass frame, E_ν being the incoming neutrino energy in that frame. In the frame in which the target particle of mass M is stationary, and the neutrino energy is E_L, the cross section is, for example,

$$\sigma(\mu + \nu_e \to e + \nu_\mu) = \frac{2}{\pi} G_F^2 M^2 . \frac{2E_L^2 / M^2}{1 + 2E_L / M} \qquad \textit{(6c)}$$

This is the form which is useful for laboratory data in which the target is stationary. For neutrino energies which are very large compared with M (e.g. for nuclear targets and neutrino energies of many GeV), this becomes,

$$\sigma(\mu + \nu_e \to e + \nu_\mu) = \frac{2}{\pi} G_F^2 M E_L \qquad \textit{(6d)}$$

Thus, in the laboratory frame the cross-section depends only linearly upon the neutrino energy, rather than quadratically as in Equ.6. For the case of present interest, in which both particles have thermal distributions of energy, neither the centre of mass nor the laboratory systems apply. The reader may be concerned that the significant difference between the cross section expressions in the lab and CoM systems implies that the actual thermal distribution may cause either to be seriously in error. This is not the case because our application, for times >0.001 sec, involves typical thermal energies of <60MeV at most. This is small compared with the nucleon mass (nearly 1GeV). In this limit we see that Equ.(6c) reduces to Equ.(6), because the nucleon is virtually stationary even in the centre of mass frame.

Note also that the significant feature of Equ.(6) is that the cross section for neutrino interactions increases according to the neutrino energy squared. This is partly why the cosmic neutrino background is so hard to detect – the energy of the neutrinos today is so very low, even lower than that of the microwave background photons (as we have seen in Chapter 5). This is, of course, in addition to the fact that the very small size of Fermi's constant, G_F, makes neutrinos interact very weakly.]

To use Equ.(6b) we shall take the neutrino energy to be the average of their (Fermion) spectrum, i.e.,

$$\langle E_\nu \rangle = 3.15 k_B T \tag{7}$$

The objective is to estimate the typical time, T_I, between nucleon/neutrino interactions for a given nucleon. Now the relative velocity between the nucleon and the neutrino with which we are supposing it will interact can be taken to be the velocity of light, c (to a very good approximation, i.e. neutrino masses are certainly very small compared with the energies in question, which are of order $k_B T$ and hence greater than around 1 MeV over the period of interest). Hence, for an effective cross-section σ, our chosen nucleon sweeps out a volume σc per unit time. If the number density of neutrinos is ρ_ν the number of neutrinos with which our nucleon will interact per unit time is thus $\sigma c \rho_\nu$, i.e. this is the reaction rate per nucleon,

$$\text{reaction rate per nucleon} = \frac{1}{T_I} = \sigma c \rho_\nu \tag{8}$$

Now the number density of electron neutrinos is given by the Fermion black body distribution,

$$\rho_\nu = 0.0913 \left(\frac{k_B T}{\hbar c} \right)^3 \tag{9}$$

(see the General Physics section for a derivation). Thus, Equs.6b, 8 and 9 will give the reaction rate for $n + \nu_e \to p + e$ per neutron. But there is also the reaction $n + \overline{e} \to p + \overline{\nu}_e$ to consider. The argument is exactly the same, as is the cross-section, in our simple approximation. The density of positrons per spin state is also the same as that of the electron neutrinos (noting that the positrons are relativistic at these times), but accounting for the two spin states gives an extra factor of two. Overall, therefore, the positron reaction causes the reaction rate to be three times faster. (Note that we shall find that the reactions freeze out well before the electron/positron annihilations take place at ~14 seconds. This is important since otherwise the density of positrons would not be given by an expression like (9). Rather, there would be virtually no positrons left and the second reaction, $n + \overline{e} \to p + \overline{\nu}_e$, would contribute negligibly to the reaction rate per neutron.).

Putting Equs.(6b), (7), (8) and (9) together, including the factor of three for the positron reaction, and converting the cross section to physical units, gives,

$$\frac{1}{T_I} = 3\frac{8}{\pi}\left(1.03\text{x}10^{-5}\right)^2 \frac{c(\hbar c)^2}{(M_P c^2)^4} 3.15^2 (kT)^2 0.0913 \left(\frac{kT}{\hbar c}\right)^3 = 7.342\text{x}10^{-10} \frac{(kT)^5}{\hbar \left(M_P c^2\right)^4} \tag{10}$$

Having found the reaction rate per particle, (Equ.10), we now need the Hubble parameter. (NB: It is the value of the Hubble parameter at the time of interest that is needed, not its current value!). This is given in terms of the density of the universe as discussed in Chapter 2, and the density of the universe is simply that of blackbody radiation, i.e.,

$$H^2 = \frac{8\pi G}{3}\rho \quad \text{and} \quad \rho = \chi.\frac{4\sigma T^4}{c^3} \text{ and } \textit{Stefan's constant,} \sigma = \frac{\pi^2 k_B^4}{60 c^2 \hbar^3} \qquad (11)$$

The factor χ accounts for the electrons/positrons and neutrinos as well as the photons. With the help of the usual numerology (see Chapter 5), this factor is seen to be,

$$\chi = \frac{\frac{7}{8}(2x2 + 3x1x2) + 2}{2} = \frac{43}{8} \qquad (12)$$

where the terms in the numerator represent the electrons (and positrons), the three neutrino species, and the photons respectively. (The denominator of 2 is required because the two photon modes are included in the definition of Stefan’s constant). Hence, using (10, 11) and equating the resulting H to $1/T_I$, as given by (10), gives,

$$H = \sqrt{\frac{8\pi G}{3}\frac{43}{8}\frac{\pi^2 k_B^4}{60c^2\hbar^3}\frac{4T^4}{c^3}} = \frac{1}{T_I} = 7.342x10^{-10}\frac{(kT)^5}{\hbar (M_p c^2)^4} \qquad (13)$$

which reduces to,

$$(k_B T)^3 = 0.742 \text{ x } 10^{10} \varsigma (M_p c^2)^3 \text{ where, } \varsigma = M_p \sqrt{\frac{G}{\hbar c}} = 7.76 \text{ x } 10^{-20} \qquad (14)$$

i.e. $$(k_B T) = 0.832 \text{ x } 10^{-3}(M_p c^2) \text{ hence } T = 0.9 \text{ x } 10^{10} \ {}^{o}K \qquad (15)$$

Consulting the above Table we see that this temperature occurs at about 0.65 seconds when the neutron:proton ratio is about **15.9% : 84.1%**.

Note that, contrary to the impression given by Equs.(13-15), the freeze out temperature does not depend upon the nucleon mass (which appears in these equations simply because the Fermi constant has been expressed as $\sim 10^{-5}/M_p^2$). The explicit expression for the freeze out temperature in terms of the constants on which it truly depends is,

$$kT_{fo1} = 0.923 G_F^{-2/3} G^{1/6} \hbar^{1/2} c^{-1/6} \qquad (15b)$$

5. Details: Phase 3 – Beta Decay of the Neutrons

Once the nucleon/lepton reactions $n + \nu_e \Leftrightarrow p + e$ and $n + \bar{e} \Leftrightarrow p + \bar{\nu}_e$ have ceased, the only means by which the number of neutrons can reduce is through beta decay, $n \to p + e + \bar{\nu}_e$. Thus, after ~0.65 seconds, the neutron number density reduces exponentially according to,

$$N_n(t) = N_n(0.65)e^{-(t-0.65)/1013} \qquad (16)$$

where t is the time in seconds, and 1013 secs is the half-life of a free neutron (NB: this is from Rowan-Robinson, though other sources use half-lives of ~886 secs. The latter is given by the 2004 edition of the Particle Data Booklet).

Because the total number of nucleons is fixed, N_n in Equ.16 may be interpreted either as either the absolute number of neutrons, or the as the percentage or fraction of the total nucleons which are neutrons.

Equ.16 holds from the time of 'freeze out' of the nucleon/lepton reactions (~0.65 seconds) until the time when nuclei become stable. Once the neutrons are safely within a nucleus they are stable.

6. Details: Phase 4 – The Formation of the First Nuclei

The lightest compound nucleus is the deuteron, np. The deuteron is quite lightly bound, the binding energy being only 2.2MeV. Equating the average photon energy (2.7kT) to this binding energy, we might expect that the deuterons would be stable against thermal fission at temperatures below ~2.5 x 10^{10} ^oK. Since the temperature is less than this after ~0.1 sec, it might be thought that the deuterons would be stable from this time onwards. However, this pays insufficient respect to the huge abundance of photons compared to neutrons.

Even at temperatures, say, only one-tenth of 2.5 x 10^{10} ^oK, a small percentage of the photons will have energies in excess of the 2.2MeV required to fission a deuteron into its p and n constituents. Since there are so many photons, only a very small proportion need have energies above 2.2MeV in order to fission all the deuterons. In fact, we need only as many photons with energies >2.2MeV as there are deuterons. And the number of deuterons is not greater than the number of neutrons.

At 0.65 seconds, ~16% of the nucleons are neutrons, and there are 1.9 x 10^9 photons per nucleon (see Chapter 4), hence the number density of neutrons is,

$$N_n = 0.16 \text{ x } \frac{N_\gamma}{1.9 \text{ x } 10^9} = 0.84 \text{ x } 10^{-10} N_\gamma \qquad (17)$$

and the number density of photons is,

$$N_\gamma = 0.2436\left(\frac{kT}{\hbar c}\right)^3 \qquad (18)$$

The blackbody formula for the number of photons per unit volume with energy between E and E + dE is,

$$dN_\gamma = \frac{1}{\pi^2(\hbar c)^3} \cdot \frac{E^2 dE}{e^{E/kT} - 1} \qquad (19)$$

Hence, from (18) and (19), the fraction of all photons which have energy greater than E_1 is,

$$\frac{1}{N_\gamma}\int dN_\gamma = \frac{1}{0.2436\pi^2 (kT)^3} \cdot \int_{E_1}^{\infty} \frac{E^2 dE}{e^{E/kT} - 1} = 0.416. \int_{x_1}^{\infty} \frac{x^2 dx}{e^x - 1} \qquad (20)$$

where $x_1 = E_1/kT$. Comparing with Equ.(17), we see that the fraction given by the RHS of (20) must be less than 0.84 x 10^{-10} if the deuterons are to be stable against photodisintegration. Hence, the condition for stable deuteron formation is,

$$\int_{x_1}^{\infty} \frac{x^2 dx}{e^x - 1} < 2.0 \times 10^{-10} \qquad (21)$$

with x_1 = 2.2MeV/kT. Since this clearly corresponds to $x_1 \gg 1$, a very good approximation for the integral is,

$$\int_{x_1}^{\infty} \frac{x^2 dx}{e^x - 1} \approx \int_{x_1}^{\infty} e^{-x} x^2 dx = \left(2 + 2x_1 + x_1^2\right) e^{-x_1} \qquad (22)$$

from which we find that x_1 = 29.145, and hence T = 8.75 x 10^8 ^{o}K. Recalling from Chapter 4 that, after ~14 secs, the time-temperature relation is $T\sqrt{t} = 1.02\text{x}10^{10}$, this temperature occurs at time ~136.2 secs.

Finally, substituting this time into Equ.(16) gives the neutron fraction when nuclei are first stable as,

$$N_n = 15.9\% \text{ x } e^{-135.6/1013} = 15.9\% \text{ x } 0.875 = 13.9\% \qquad (23)$$

Now, in Equs.(17,21) we assumed that the neutrons were 16% of the nucleons. Strictly, we should now repeat the calculation replacing 16% in Equs.(17,21) with 13.9%. However, carrying this out we find that it makes only ~1 sec difference to the time at which the deuterons become stable (specifically, 137.4 sec), and the predicted neutron fraction is changed negligibly (to 13.88%). A further sensitivity study on this result is to use a neutron half-life of 886 sec in Equ.(23). This gives 13.6% neutrons left when the deuterons become stable at ~137 secs, and this is probably the more accurate estimate.

Another possible refinement to the calculation is based on recognising that equality of the number of neutrons and high-energy photons only allows the formation of nuclei to start. To occur 'in volume' we presumably want to wait a little longer until the deuterons will have a degree of numerical advantage. For illustration, if we set our criterion to be that there are only half as many photons with energy >2.2MeV as neutrons, then we find the time at which this occurs is 143.2 secs, only ~6 secs later. The resulting neutron fraction is therefore negligibly different (namely 13.5%).

Similarly, requiring that there is only 1 sufficiently high energy photon per 100 deuterons, we find that the time becomes 183 sec. This implies a remaining neutron fraction of 12.9%.

More accurate calculations can be carried out taking into account the rate of the reactions forming deuterons, and their subsequent conversion to helium nuclei. However, taking it on trust that these reactions are rapid once the deuterons are stable, it is clear from the above scoping estimates that about 13% of the nucleons are neutrons at this stage.

7. Details: Phase 5 – The Neutron Sanctuary – The Cosmic Helium Abundance

The prediction of the cosmic helium abundance requires no extra work. We need only note that once deuterons become stable, all the neutrons then present quickly end up in the form of helium-4 ($^4_2\mathrm{He}$). Essentially this is because the nuclei heavier than the deuteron have larger binding energies, with helium-4 being the largest (and hence the tightest bound) of those that can be formed during this early stage of the universe. The binding energies are;

name	nucleus	total binding energy, MeV	binding energy per nucleon, MeV
deuterium	$^2_1\mathrm{D}$	2.2	1.1
tritium	$^3_1\mathrm{H}$	8.5	2.8
helium-3	$^3_2\mathrm{He}$	7.7	2.6
helium-4	$^4_2\mathrm{He}$	28.3	7.1
lithium-7	$^7_3\mathrm{Li}$	39.2	5.6

Since helium-4 involves 2 protons as well as 2 neutrons, and given the result that a negligible proportion of other nuclei are formed, it follows that the cosmic helium abundance <u>by mass</u> should be double that predicted for the neutrons when the formation of nuclei starts, i.e. 2 x 13% = 26%.

The best theoretical estimates, and indeed the measured value, are 24%-25%. This agreement is one of the greatest triumphs of the Big Bang theory.

Incidentally, different sources seem to differ as regards what sequence of nuclear reactions is most significant in leading to the production of helium-4. All begin with the production of deuterium. Some then list,

$^1_0\mathrm{n} + ^2_1\mathrm{D} \rightarrow ^3_1\mathrm{H} + \gamma$ tritium production

$^2_1\mathrm{D} + ^2_1\mathrm{D} \rightarrow ^3_1\mathrm{H} + ^1_1\mathrm{H}$ tritium production

$^2_1\mathrm{D} + ^1_1\mathrm{H} \rightarrow ^3_2\mathrm{He} + \gamma$ helium-3 production

$^2_1\mathrm{D} + ^2_1\mathrm{D} \rightarrow ^3_2\mathrm{He} + ^1_0\mathrm{n}$ helium-3 production

$^3_1\mathrm{H} + ^1_1\mathrm{H} \rightarrow ^4_2\mathrm{He} + \gamma$ helium-4 production

$^3_2\mathrm{He} + ^1_0\mathrm{n} \rightarrow ^4_2\mathrm{He} + \gamma$ helium-4 production

Others state that, after the deuterons absorb a proton to yield helium-3, the next reaction is,

$${}^3_2\text{He} + {}^3_2\text{He} \rightarrow {}^4_2\text{He} + {}^1_1\text{H} + {}^1_1\text{H} \qquad \text{helium-4 production}$$

And Rowan-Robinson, after production of helium-3, lists the reaction sequence as being,

$${}^3_2\text{He} + {}^1_0\text{n} \rightarrow {}^3_1\text{H} + {}^1_1\text{H} \qquad \text{tritium production}$$
$${}^3_1\text{H} + {}^2_1\text{H} \rightarrow {}^4_2\text{He} + {}^1_0\text{n} \qquad \text{helium-4 production}$$

Presumably all these reactions occur at various rates. It really does not matter if we are interested only in estimating the helium-4 abundance. All that matters is that a pathway to helium-4 exists and is rapid once deuterons are stable.

8. Summary

The correct prediction of the cosmic abundance of helium is a great triumph. There is no a priori reason why the amount of helium should be ~25% rather than 99% or 0.0001%, so the prediction is impressive. Moreover, carrying out the nuclear balance calculations for the other nuclei (helium-3, deuterium and lithium-7) all produce results in agreement with observation. Again this is impressive, since, if the origin of these elements in a Big Bang fireball were not correct, the abundances of these elements would have no *a priori* reason to all be consistent with the same time-temperature history. These minority nuclei are present only in very small proportions (the order of 0.01% for deuterium and helium-3, and 1 part in 10^9 for lithium-7). We shall derive these abundances very roughly in Chapter 6B.

Reviewing the above derivation of the helium-4 abundance, we see that the most sensitive feature is the temperature at which 'freeze out' of the nucleon/lepton reactions occur. Up until that time the neutron:proton ratio is changing rapidly due to the exponential dependence upon the temperature and the neutron / proton mass difference. Hence, the most significant parameter in the calculation is the strength of the weak nuclear interaction, the Fermi constant (G_F), since this determines the 'freeze out' temperature. The neutron:proton ratio at this freeze-out temperature is primarily what leads to the predicted helium-4 abundance.

The strength of gravity also plays a part in the calculation. The 'freeze out' temperature is proportional to $G^{1/6}$(see Equs.14,15b). Thus, if G were a factor of ten bigger, the cosmic helium abundance would be roughly doubled (i.e. close to 50%).

Other parameters which play a part in determining the helium (or neutron) abundance are: the neutron/proton mass difference; the deuteron binding energy; the neutron lifetime; the photon:baryon ratio; and finally the time-temperature relationship. The later is essentially given in terms of G (see Chapter 3) and so is not an independent quantity. Similarly, the neutron lifetime can be expressed (arguably) in terms of G_F and the neutron/proton mass difference (see Appendix A1), and so is also not an independent quantity.

The deuteron binding energy depends upon the strength of the strong nuclear force, as does (presumably) the neutron/proton mass difference. It is therefore not entirely obvious that these are independent quantities, though this is implied by theories such as SU(5) GUTs. (Whether this is also true in string theories I do not know). Thus, ignoring those constants which merely define our dimensions ($\hbar, c, k_B$), the independent parameters which may be varied in considering different universes are:-

$$G,\ G_F,\ M_n\text{-}M_p,\ B_D,\ \xi_{\gamma N}$$

We may expect other parameters to arise in the following Chapters which depend upon the strong nuclear force. We will need to be cautious as to whether these are dependent on, or independent of, M_n-M_p and B_D.