Chapter 5
The Cosmic Background Neutrino Temperature - Neutrino Numerology
What Tau Neutrino Mass Would Be Needed To Account For Dark Matter?

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1. Introduction

In Chapter 3 we derived, from very simple theoretical considerations, an estimate for the temperature of the electromagnetic microwave background radiation (CMB). It was encouragingly close to the accurately measured value of 2.728K (Chapter 4). However, neutrinos are also relics of the very early universe. At one time, namely before 1 second, they were in thermal equilibrium with everything else. So, we expect that there should also be a cosmic neutrino background with a thermal energy spectrum. Unfortunately, such neutrinos are of such low energy that they cannot be detected at present. This is because neutrinos interact only via the weak nuclear force. The weakness of this interaction is exacerbated by the very low neutrino energy, since cross sections for weak-force interactions decrease typically as energy-squared. Consequently there is no observational evidence that the cosmic neutrino background (CNB) truly exists, nor is there likely to be in the foreseeable future[1]. Confirmation of the existence of the CNB, together with a thermal spectrum at the expected temperature, would be further impressive confirmation of the Big Bang hypothesis. Conversely, evidence against the existence of the CNB would be difficult to understand.

Do we expect the CNB to have the same temperature, i.e. 2.728K, as the electromagnetic background?

2. Cosmic Neutrino Background Temperature

The answer is "no", as we have already anticipated in Chapters 3 and 4. The reason is that the neutrinos cease to interact significantly with the rest of the universe at only ~1 second after the big bang (see Chapter 6 for a demonstration of this). The neutrinos are no longer in thermal equilibrium after this time. Whilst this does not cause any immediate difference in temperature between the neutrinos and everything else, it will do so only ~13 seconds later. After 14 seconds the electrons and positrons have mostly annihilated, producing large numbers of very energetic gamma rays (photons). These gamma rays quickly become 'thermalised' in the very dense conditions that prevail. In other words, the energy released by annihilation of the electron/positron pairs heats up the remaining contents of the universe – at least, those constituents which interact sufficiently to enable the energy transfer to take place. Thus, the temperature of the photons, and of the remnant electrons, protons and neutrons, are almost instantaneously increased. The neutrinos, however, do not partake of this free hand-out of energy. They cannot do so because they are no longer interacting with anything. Hence, from ~14 seconds until the present day, the neutrino background has been cooler than the microwave background. But by how much?

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[1] An indication of just how difficult it would be to detect the cosmic neutrino background is provided by the experience of detecting solar, and supernova, neutrinos. These require underground detectors consisting of many thousands of tons of fluid, and even so the count rate is very slow. The solar neutrinos detected have an energy $\sim 10^{10}$ times greater than cosmic background neutrinos. The count rate for cosmic neutrinos (if the detectors could detect them at all) would be $\sim 10^{-20}$ times slower. So it is difficult to envisage a direct detection of the cosmic neutrinos ever being feasible.

It is essentially book-keeping to evaluate this ‘discontinuous’ temperature increase of the thermal photons. We are concerned with the time after the annihilation of the bulk of the electrons and positrons. We know that the number of remnant electrons, protons and neutrons is very small compared with the number of photons (about a factor of 10^{-9}, see Chapter 4). Consequently, these ‘matter’ particles tend to follow the temperature of the photons in this era. Virtually all the thermal energy resides in the photons immediately after positron annihilation, and hence we need only consider the photons in estimating the associated temperature increase. Just before the annihilation of the e^- / e^+, however, these particles were comparable to the photons in their abundance – and they were also thermal. The trick to the book-keeping exercise is to consider the entropy of the photon and e^- / e^+ radiations.

The entropy of a thermalised (equilibrium) particle species with N_s spin states and N_a distinct antiparticles is, per unit volume,

$$S = fN_a N_s \frac{2\pi^2 k_B}{45}\left(\frac{k_B T}{\hbar c}\right)^3 \qquad (5.2.1)$$

where f = 1 for bosons (e.g. photons) but f = 7/8 for fermions (e.g. electrons). [See my General Physics web pages for a derivation]. Consequently, the leading factor fN_aN_s before the e^- / e^+ annihilation is,

$$fN_aN_s = \frac{7}{8}x2x2 + 1x1x2 = \frac{7}{2} + 2 = \frac{11}{2} \qquad (5.2.1b)$$

where the first term accounts for the electrons and positrons, and the second term for the photons. After the annihilation of the electrons and positrons, this factor is just 2, i.e. the photons alone. Hence, if the temperature remained constant, Equ.(5.2.1) would imply that the entropy had decreased by a factor of 11/4. However, we know that the temperature must increase, in order to conserve energy. Moreover, we know that entropy will not decrease. Thus, the temperature must increase sufficiently to counteract the decrease in the fN_aN_s factor in (5.2.1).

Thus, if we assume the entropy to be constant, Equ.(5.2.1) implies that the temperature increases by a factor of,

$$\text{photon temperature increases by a factor of } \left(\frac{11}{4}\right)^{1/3} = 1.401 \qquad (5.2.2)$$

Hence, the neutrino temperature today will be the photon temperature / 1.401, i.e. 2.728^oK / 1.401 = **1.947^oK**. Note that the neutrinos themselves have played no part in the derivation of this temperature. It has been based solely on the measured photon temperature, and on the numerology of the photons and electrons/positrons.

If evidence of a neutrino background were obtained it would be further confirmation of the big bang hypothesis, or at least of a hot dense origin of the universe. If the temperature of 1.947^oK could be accurately confirmed, the evidence would be compelling.

3. Contribution of Neutrinos to the Universal Density (Energy)?

The energy (and hence mass) density of a blackbody field is proportional to the absolute temperature to the power 4. However, the factor of fN_aN_s from Equ.(5.2.1) also occurs in the energy density. Neutrinos occur in just one spin state, despite being spin ½ (see footnote[2]). Hence, for a given type of neutrino (say, the electron neutrino), fN_aN_s = 7/8 x 2 x 1 = 7/4. But there are three types of neutrino, the electron neutrino, the mu neutrino, and the tau neutrino. Hence, fN_aN_s = 21/4 in total. Compared to the photon field (fN_aN_s = 2), this factor is therefore 21/8 larger. But, each of the neutrino fields is at a temperature which is only $(4/11)^{1/3}$ times that of the photons, from Equ.(5.2.2). Hence, the three neutrino fields contribute to the total energy density of the universe as follows,

3 neutrinos: $$\frac{\textit{energy density of all neutrinos}}{\textit{energy density of photons}} = \frac{21}{8}\left(\frac{4}{11}\right)^{4/3} = 0.681 \qquad (5.3.3)$$

NB: If there were just two species of neutrino (e.g. if the tau neutrino did not exist) then this factor would have been,

2 neutrinos: $$\frac{\textit{energy density of all neutrinos}}{\textit{energy density of photons}} = \frac{7}{4}\left(\frac{4}{11}\right)^{4/3} = 0.454 \qquad (5.3.4)$$

However, with 3 neutrinos, the total energy density of the photon and neutrino fields is 1.681 times that for the photons alone. We have used this factor in Chapter 3 to derive theoretically a time-temperature relationship in the early universe.

4. What Neutrino Mass Would Account For Dark Matter?

For sake of argument we assume that the electron and muon neutrinos are too light to contribute much to the dark matter density. The following Section is therefore phrased as if only the tau neutrino contributes significantly, i.e. that the mass of the tau neutrino is far bigger than that of the electron and muon neutrinos. More generally, the term 'tau neutrino mass' can be reinterpreted as 'the sum of the three neutrino masses'.

If the neutrinos and the photons were at the same temperature, the number density of each neutrino type would be ¾ of that of photons of a specified polarisation, counting antineutrinos as distinct from neutrinos [i.e. in the number-density expression, fermions have a factor of ¾, rather than the factor of f = $^7/_8$ which appears in the entropy expression, (5.2.1)]. Remembering that there are two photon polarisations, the number density of each neutrino type would be $^3/_8$ of that of all photons, again counting antineutrinos as distinct from neutrinos. However, the neutrinos are at a temperature 1/1.401 lower than the photons, and number density increases as

[2] In the standard theory of weak interactions, only negative helicity neutrinos and positive helicity antineutrinos take part in weak processes. Whether this means that positive helicity neutrinos and negative helicity antineutrinos do not exist is not certain, but because they do not interact at all, except by gravity, we do not know of them. If they do exist then the neutrino density will be double what we derive here.

temperature to the power 3. Hence, the number density of tau neutrinos plus tau antineutrinos is 2 x $^3/_8$ x $(1/1.401)^3$ = 0.2727 times that of the photons.

Now in Chapter 4 we have found that the photon:baryon ratio is ~1.9 x 10^9. Hence, we conclude that the ratio of tau neutrinos (+antineutrinos) to baryons is 0.52 x 10^9. The ratio of the total baryon density to the critical density is Ω_b ~ 3.8% (using a mean baryon density of 0.22 m^{-3}, as in Chapter 4, and hence consistent with the photon:baryon ratio of 1.9 x 10^9).

The total normalised matter density, Ω_{matter}, is about 29%, which includes dark matter but not dark energy (derived from WMAP measurements). Hence, dark matter accounts for about Ω_{dark} ~ 25%, and the dark matter to baryonic matter mass ratio is ~6.6 (but with a large error bar). Since the baryonic matter consists of nucleons with masses of ~0.939GeV, dark matter has a total mass of about 6.2GeV per nucleon. Since there are 0.52 x 10^9 tau neutrinos (including antineutrinos) per nucleon, it follows that the whole of the dark matter can be accounted for if tau neutrinos have a mass of 6.2GeV / 0.52 x 10^9 ~ 12eV (see footnote[3]).

Rather lamentably, the most recent bounds on the sum of the neutrino masses, derived from WMAP, is <0.6eV[4]. Consequently, it would appear that neutrinos cannot account for more than 5% of dark matter, and probably less.

[3] Note that the mass equivalent of the neutrino thermal energy is negligible in the present epoch, i.e. less than 10^{-3} eV.
[4] A.Goobar et al, "The Neutrino Mass Bound for WMAP-3….", arXiv:astro-ph 0602155, May 2006.