Chapter 4
The Microwave Background Temperature and the Photon:Baryon Ratio
Refinement of the Predicted Time-Temperature Relations

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1. Predicted and Actual Microwave Background Temperatures Compared

Weinberg ("The First Three Minutes") suggests that the important thing about the measurement of the cosmic microwave background (CMB) temperature is that it permits the ratio of the number of photons to baryons to be calculated. This seems an overstatement since the temperature of the microwave background can be calculated approximately from very simple considerations. The measured value of the CMB temperature is therefore not a surprise (though the *precision* of the measurements is very impressive). The principal benefit of detecting the CMB is the confirmation that it is there at all. This provides powerful support to the Big Bang hypothesis. In Chapter 3 we derived, from very simple considerations, the variation of temperature during the radiation dominated era. This gave,

$$T(^{o}K) = \frac{1.33x10^{10}}{\sqrt{t}} \quad \text{(after positron annihilation, ~a few minutes)} \qquad (4.1.1)$$

where time, t, is in seconds. The radiation era comes to an end when the universe becomes transparent, that is when atomic hydrogen forms from the previously separate electrons and protons. Thus, Equ.(4.1.1) has been derived for the period from around a few minutes (see Chapter 7) to around 362,800 years[1] (see Chapter 8). However, the time of 362,800 years will be derived in Chapter 8 from results derived later in this Chapter, so we cannot assume this value as yet. Fortunately we do know that the decoupling temperature is ~3070^{o}K[2].

In the matter dominated era, i.e. after about 362,800 years or more, the temperature of the radiation[3] continues to be dictated by proportionality with the universe's expansion, i.e.,

$$T(\text{radiation}) \propto 1/R \qquad (4.1.2)$$

We have also seen previously (Chapter 2) that, in the matter dominated era, the size scale of the universe increases as,

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[1] The end of the radiation era can be defined alternatively as when the radiation energy density equals the matter density. This may be earlier or later than 362,800 years depending upon exactly what is included as "radiation" and what as "matter", the debatable items being neutrinos and dark matter. This is discussed further in Chapter 10. The earliest the radiation era can end, on any reasonable definition, is in the order of tens of thousands of years.

[2] See Chapter 8, where it is shown that 99% of the initially free protons have combined with electrons to form hydrogen atoms by ~3070^{o}K. This is often referred to as the "recombination temperature", rather inappropriately in view of the fact that the electrons and protons have never previously been combined.

[3] …but not necessarily the neutral matter, which, at least initially, cools much faster – but which later on gets much hotter.

$$R \propto t^{2/3} \qquad \text{(matter dominated era)} \qquad (4.1.3)$$

Equs (4.1.2) and (4.1.3) show how the radiation temperature varies with time in the matter dominated era. Hence, since we know at what temperature the radiation era gives way to the matter dominated era, the constant of proportionality can be found by matching to (4.1.1).

Substituting 3070°K into (4.1.1), our provisional estimate of the time of decoupling (when 99% of neutral hydrogen has been formed) is found to be 595,000 years (1.88 x 10^{13} sec). (2) and (3) then give,

$$T(radiation) = 3070^o K\left(\frac{1.88x10^{13}}{t}\right)^{2/3} \qquad \text{(for t > 1.88 x } 10^{13}\text{sec)} \qquad (4.1.4)$$

Taking the present age of the universe to be 13.7 billion years (4.32 x 10^{17} sec) we estimate from (4.1.4) that the expected temperature of the microwave background should be 3.8°K. This is not at all bad for a very simple, first-principles, estimate. It compares with the best current value from COBE measurements of 2.728°K.

Alternatively, we can refine our time-temperature relations using the measured CMB temperature. Thus, if we suggest that a better time at which the decoupling temperature of 3070°K is attained is 362,800 years (1.144 x 10^{13} sec), and use this in place of the estimate from (4.1.1), we get,

$$T(radiation) = 3070^o K\left(\frac{1.144x10^{13}}{t}\right)^{2/3} \qquad \text{(for t > 1.144 x } 10^{13}\text{sec)} \qquad (4.1.5)$$

and this time-temperature relation reproduces the observed microwave background temperature of 2.728°K. We make no comment here as to whether the estimate, Equ.(4.1.1), of the temperature during the radiation dominated era is likely to be in error by the required amount. Our purpose here was merely to demonstrate that a simple, first-principles, estimate does give a good result for the background temperature at the present day.

We may use Equ.(4.1.5) to adjust the coefficient in Equ.(4.1.1) for the temperature of the photons during the radiation era, i.e. matching a temperature of 3070K at 362,800 years gives,

$$T(^o K) = \frac{1.038x10^{10}}{\sqrt{t}} \qquad \text{(for 14 sec < t < 1.144 x } 10^{13} \text{ sec)} \qquad (4.1.1a)$$

Recalling that prior to electron-positron annihilation the temperature was a factor of 1.40 lower (see Chapters 3 and 5), the temperature of the photons, neutrinos and electrons and positrons then would have been,

$$T(^{o}K) = \frac{0.74x10^{10}}{\sqrt{t}} \quad \text{(for t < 14 sec)} \qquad (4.1.1b)$$

Equ.(4.1.1b) will hold for the neutrinos until 1.144 x 10^{13} sec. Thereafter the neutrino temperature is simply a factor 1.4 smaller than Equ.(4.1.5).

2. Photon:Baryon Ratio

Whilst the term 'baryon' has a different meaning in particle physics, in this context it can be taken to mean either neutrons or protons or nuclei (which are composed of neutrons and protons). A better term would be 'nucleons'. It turns out that a parameter which occurs time and again in the evolution of the universe is the ratio of the number photons (from the CMB) to the number of baryons (which we shall denote ξ).

The photon density is obtained from the black body distributions for photons (see my General Physics page for a derivation). It is,

$$\rho_N^{\gamma} = 0.2436\left(\frac{k_B T}{\hbar c}\right)^3 \qquad (4.2.6)$$

(the subscript $_N$ denotes the particle number density, as opposed to an energy or mass density). At 2.728°K this evaluates to 4.1 x 10^8 photons per m^3.

Since the baryons (nucleons) constitute 99,95% of the mass of ordinary matter, determining their number is equivalent to finding the mean density of ordinary matter. Currently the best estimate of the mean baryon number density is ~0.22 per m^3 [Rowan-Robinson, "Cosmology", Equ.(5.21)]. Note that there is substantial uncertainty in this value[4]. We note that a mean baryon number density of ~0.22 per m^3 is only about 4% of the critical density (see Chapter 1).

Nevertheless, our best estimate of the photon:baryon ratio is ξ ~1.9 x 10^9.

However, we would have derived a result within a factor of 3 of this value on the basis of our simple estimate of the background radiation temperature (i.e. our 'first principles' estimate of the microwave background temperature was 3.8K and this implies 11 x 10^8 photons per m^3, hence a photon:baryon ratio of ~5 x 10^9) . Hence, Weinberg's view is justified if one is concerned about knowing this ratio to a better accuracy than a factor of 3. At the present time, however, the uncertainty in the photon-baryon ratio lies almost entirely on the baryon side. The photon density is known with great precision, thanks to the very accurate measurements of the CMB temperature. The mean baryon density, however, is uncertain by at least a factor of 2.

[4] The figure of 0.22 m^{-3} is derived from measurements of cosmic deuterium abundance compared with nucleosynthesis calculations. However, optimising the agreement of nucleosynthesis calculations with the observed helium abundance gives a baryon density only about 63% of the above. Nevertheless, we shall use the larger figure for definiteness. This is believed to be in better agreement with the fluctuations in the CMB.