Chapter 3 – The Time-Temperature Relationship in The Early Universe (Radiation Dominated)

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1. Introduction

In Chapter 2 we made use of the proportionality $\rho \propto 1/R^n$ (n = 3 or 4 for matter and radiation respectively). We were able to derive explicit expressions for the mean density of the early universe as a function of time and G. The temperature of the universe can be found in terms of the density. Hence, the temperature can also be found as a function of time. This is the subject of this Chapter. However, first we say a little about the contents of the universe in the first seconds and minutes.

We shall shortly see that the temperature of the universe at 1 second is about 10^{10} K. The universe will therefore contain a high density of gamma photons. The typical energy of photons at this temperature (~3kT) is around 2.5 MeV. This means that any particle whose rest mass is less than 1.25 MeV can readily be created out of the ambient thermal energy, in conjunction with its antiparticle. The only particles whose rest masses are less than this are the electron (and its antiparticle, the positron) and the neutrinos. Consequently, at times before about 1 second we can expect the universe to contain a high density of electrons, positrons and all the species of neutrinos. In fact, there will be roughly similar numbers of these particles as photons.

The only other particles present will be those which have been produced in the complex processes occurring even earlier and that are stable[1]. This includes protons and neutrons[2]. However, protons and neutrons have masses of about 938 MeV. This is three orders of magnitude greater than the ambient thermal energy. Consequently, protons and neutrons cannot be created spontaneously as particle-antiparticle pairs. In fact there are essentially no antiprotons and antineutrons. The universe has already decided to be composed of matter rather than antimatter. The protons and neutrons are not present as a consequence of being created out of the ambient thermal energy, in contrast to the electrons and positrons. Hence, there is no reason to suppose that the numbers of protons and neutrons would be comparable with the numbers of photons, electrons and positrons. In fact there is just one nucleon for about every 2 x 10^9 photons. Amazingly, this was true at 1 second and remains true today. We give no derivation of this huge ratio here, but rather regard it as one of the 'freely chosen' universal parameters. Its value is probably determined by the complex interactions occurring at much earlier times[3] and mediated by the strong nuclear force.

At about one millisecond the typical thermal energy has dropped below that required to create particle-antiparticle pairs for any other particles. Before this time the next lightest particles, the muons, would be around in large numbers – and, a little earlier still, the pions. For this reason we start our story at about a millisecond. The

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[1] In this context 'stable' means having a half life longer than a few seconds. Thus we include neutrons, although they are actually unstable – with a half life of about 15 minutes.

[2] It may also include particles of dark matter, if these are distinct from both ordinary matter and neutrinos, as seems likely at present. We exclude them from consideration only because we currently have no idea of their properties.

[3] How can we talk about 'much earlier' than 1 second? Time is best regarded on a logarithmic scale. Thus, we mean at a value of log(t) much less than zero. The Big Bang is pushed back to minus infinity on this scale.

composition of the universe after this time was simpler than at earlier times. This simplicity of composition and structure was to persist for some hundreds of thousands of years, at which time the first neutral atoms formed. But arguably the universe only starts to become more complex in its structure when gravitational clumping of matter leads to the formation of the first stars, after the order of hundreds of millions of years. But we get ahead of ourselves. Let us first derive the temperature of the early universe.

2. Radiation Dominated Era

The energy density due to photons is simply the well known black body radiation density. The energy density is related to the radiated power flux, which in turn is given by the Stefan-Boltzmann law, as follows,

$$\frac{1}{4}\rho_{energy}^{radiation} c = J = \sigma T^4 \qquad (3.2.1)$$

The mass density equivalent is obtained by dividing by c^2, hence we have,

$$\rho_{mass}^{radiation} = \frac{4\sigma T^4}{c^3} \qquad \text{(photons only)} \qquad (3.2.2)$$

The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ $Wm^{-2}K^{-4}$.

Other forms of radiation will contribute also. The only other radiation of relevance after about the first few minutes is that due to neutrinos and anti-neutrinos. (Before ~14 seconds there are still electrons and positrons in comparable numbers, but 99% of the positrons have annihilated by 3 minutes – see Chapter 7). It can be shown that the three species of neutrinos plus antineutrinos contribute 68.1% as much energy as the photons (see Chapter 5).

Note that after ~1 sec the neutrinos decouple from the rest of the universe (see Chapter 6). After ~14 secs the neutrinos are therefore at a lower temperature than the photons, since the latter benefit from the additional thermal energy arising from the electron-positron annihilations whereas the former do not. The additional 68.1% of energy takes account of this difference in temperature. Hence,

$$\rho_{mass}^{radiation} = \frac{6.72\sigma T^4}{c^3} \qquad \text{(photons plus neutrinos)} \qquad (3.2.3)$$

Equating this density to that derived in Chapter 2, gives,

$$\frac{6.72\sigma T^4}{c^3} = \frac{3}{2\pi 4^2 G t^2} \quad \textit{hence} \quad T^4 = 0.00444\left(\frac{c^3}{G\sigma t^2}\right) \qquad (3.2.4)$$

hence, using $G = 6.67 \times 10^{-11}$ $kg^{-1}m^3sec^{-2}$ gives, Temperature of Everything Except Neutrinos after Positron Annihilation (i.e. after a few minutes):-

$$T = \frac{1.33 \times 10^{10}}{\sqrt{t}} \quad \text{(K, with t in sec.)} \qquad (3.2.5)$$

[Note that the inclusion of the third species of neutrino, the tau neutrino, rather than just two, changes the coefficient in Equ.(3.2.5) only marginally, from 1.38 to 1.33].

The derivation of Equs.(3.2.4, 5) has used the expression in Chapter 2 for the density $\rho \propto 1/t^2$, which is derived from the power-law expression for the size scale variation $R \propto t^{2/n}$, and these hold only at sufficiently early times (or for flat spacetime).

We will see in Chapter 5 that the annihilation of the electron-positron pairs increases the photon temperature by a factor of 1.40. Thus, prior to ~14 secs, both the photons and the neutrinos - and the electrons and positrons – would have been at a temperature of,

Temperature of Everything Before ~14 secs, and for Neutrinos at all Times

$$T = \frac{0.95 \times 10^{10}}{\sqrt{t}} \quad \text{(K, with t in sec.)} \qquad (3.2.6)$$

We shall see in Chapter 4 that Equ.(3.2.5) gives quite an impressive prediction of the temperature of the cosmic microwave background radiation. However, since COBE/WMAP have provided a very accurate measurement of this temperature, we shall use this measurement in Chapter 4 to fine-tune the values of the coefficients in Equs.(3.2.5,6).

3. Relation Between Temperature and Size – Radiation Era

Note that from (3.2.3) together with $\rho \propto 1/R^4$ (from Chapter 2) it follows that,

$$R \propto 1/T \qquad \text{(radiation dominated era)} \qquad (3.3.6)$$

Actually, this is merely a re-statement of the fact that the wavelength of the radiation varies in proportion to the size scale R. Hence the frequency, energy and temperature of the photon radiation, are inversely proportional to R.

4. Relation Between Temperature and Size - Matter Dominated Era

As far as the radiation is concerned, that is the photons and the neutrinos, Equ.(3.3.6) continues to hold irrespective of the fact that matter now accounts for more mass-energy than the radiation. Consequently, we may write simply,

$$R \propto 1/T \qquad \text{(radiation, any era)} \qquad (3.3.7)$$

As for the temperature of the, now dominant, matter – that is another question. As it happens (and it's not obvious why) matter forms neutral atoms, and hence stops interacting with the electromagnetic radiation, at about the same time that matter becomes dominant in terms of mass-energy. Consequently, matter ceases to be in thermal equilibrium once it becomes dominant. Prior to this time we could safely assume that the temperature of the matter and the temperature of the radiation were the same. We can no longer make this assumption in the era of matter dominance. And it is not true. Although we will not prove this here, following the decoupling of matter and radiation, the temperature of matter initially falls below that of the radiation. Ultimate the temperature of matter – at least some of it – is destined to become far greater than that of the cosmic microwave background radiation. This happens as soon as gravitational clumping starts to occur.