# Chapter 15
# Bounds On The Mass, Temperature and Density Of Stars

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## 1. Introduction

In Chapter 11 we derived representative pressure, temperature and density distributions for solar mass stars. In Chapter 18 we shall see how these scale for stars of different masses. However, key aspects of both these derivations rest on a knowledge of the actual mass, size or density appropriate for typical stars in this universe. In this Chapter we shall derive bounds within which the size, mass and temperature of stars must lie. This is done using simple approximations. The objective is to derive closed-form expressions which can be used to examine the scope for stars to exist in alternative universes in which the universal constants take different values.

To address this question at all requires a definition of what we mean by a "star". If any cloud of gas were counted as a star, then there could be no lower limit to its size – and perhaps no upper limit either, at least for limited periods of time. A star is defined by the onset of nuclear burning. The definition is completed by the requirement that the star be stable over astronomical time scales. Thus, the nuclear reactions, and the pressure, temperature and density distributions, are required to be sustained over very long periods. This precludes a sudden burst of nuclear activity within a body of gas, which then immediately blows itself apart, from being counted as a star. A star must be a recognisable individual entity with a substantial lifetime. Since it involves nuclear reactions, it will also be a power source.

It will be seen that the lower bound mass is determined by the onset of electron quantum degeneracy. Such stars are called white dwarfs. They also provide the upper bound on temperature. For this reason we start, in Section 2, with a brief review of degeneracy energy and pressure.

The upper bound mass is determined by stability. Larger stars become increasingly unstable due to radiation pressure. It will be seen that radiation pressure is unimportant for stars of stellar mass or less. However, the importance of radiation pressure increases as mass$^2$, and hence eventually dominates for stars of larger mass.

## 2. Fermion Degeneracy

A gas of fermions is said to be degenerate when the temperature (kT) is significantly less than the Fermi energy. The Fermi energy is defined as the highest occupied energy level if all the available quantum states were filled up, starting from the lowest energy level, until all the fermions were used up. Thus, consider N free fermions within a volume V:-

- The volume of k-space per spatial state is $V_k = \frac{\pi^3}{V}$; (2.1)
- Hence, the number of spatial states with k-numbers between k and k + dk is
  $$dN_k = \frac{1}{8} \times \frac{4\pi k^2 dk}{V_k} = \frac{Vk^2 dk}{2\pi^2}, \quad (2.2)$$
  where $N_k$ denotes the number of quantum states;

- Hence, the number of spatial states with k-numbers ≤ k is $N_k = \frac{Vk^3}{6\pi^2}$; (2.3)
- Since the fermions can have either of two spin states (assuming spin ½ fermions, e.g., electrons or nucleons), the Fermi surface is given by $\frac{Vk_F^3}{6\pi^2} = \frac{N}{2}$, where N is the number of fermions in the volume V. Thus, $k_F^3 = \frac{3\pi^2 N}{V} = 3\pi^2 \rho_N$, (2.4)
  where $\rho_N$ is the number density of the fermions.
- Assuming non-relativistic conditions, the Fermi energy is $E_F = (\hbar k_F)^2 / 2m_e$, where $m_e$ is the mass of the fermions in question (which will be electrons in this Chapter). Hence, $E_F = \frac{\hbar^2}{2m_e}\left(3\pi^2\rho_N\right)^{2/3}$. (2.5)

At any finite temperature, a degenerate fermion gas will have a small number of particles at energies above the Fermi energy. However, the vast majority will have energies less than or equal to the Fermi energy. In the case of an extreme degenerate Fermi gas ($kT << E_F$), we can approximate all the states with energies up to and including the Fermi energy to be occupied, but none of the higher energy levels. The mean energy per particle is then just,

$$< E >= \frac{\int_0^{k_F} E_k dN_k}{\int_0^{k_F} dN_k} = \frac{\int_0^{k_F} \frac{(\hbar k)^2}{2m_e} .k^2 dk}{\int_0^{k_F} k^2 dk} = \frac{\frac{\hbar^2}{2m_e} \cdot \frac{k_F^5}{5}}{\frac{k_F^3}{3}} = \frac{3}{5} E_F \qquad (2.6)$$

The degeneracy pressure follows from (2.6) and (2.5), noting that the pressure is given in terms of the total energy, U, by,

$$P = -\frac{\partial U}{\partial V} \qquad \text{where, } U = N < E >= \frac{3}{5} N E_F \qquad (2.7)$$

Thus, we find, $$P = \frac{\left(3\pi^2\right)^{2/3}}{5} \cdot \frac{\hbar^2}{m_e} \cdot \rho_N^{5/3} \qquad (2.8)$$

The same result follows if we consider the pressure arising from rate of change of momentum per unit area, using Equ.(39) of Appendix A0, i.e.,

$$P = \int dP_k = \int \frac{1}{3} d\rho_k^N p_k v_k = \int_o^{k_F} \frac{1}{3} \frac{k^2 dk}{\pi^2} \cdot \hbar k \cdot \frac{\hbar k}{m_e} = \frac{\hbar^2 k_F^5}{15\pi^2 m_e} = \frac{\left(3\pi^2\right)^{2/3}}{5} \cdot \frac{\hbar^2}{m_e} \cdot \rho_N^{5/3} \qquad (2.9)$$

This relation between pressure and density is reminiscent of the adiabatic relation for gases, $P \propto \rho^\gamma$, and also the polytropic relation for the stellar medium as discussed in Chapters 11 and 18. It can readily be seen that if the electrons were extremely relativistic, with $E_f >> m_e c^2$, then $E_f \approx \hbar kc \propto \rho_N^{1/3}$ and this leads via differentiation to an exponent of $^4/_3$ for the extreme relativistic, degenerate electron gas (rather than the exponent of $^5/_3$ which applies for the non-relativistic, but degenerate, electron gas). Curiously, the exponent γ in the polytropic relation is required to lie between just these values, $^5/_3 < \gamma < {}^4/_3$ , in order for a star to be stable.

**3. Equilibrium - The Three Pressures**

The inward force of gravity must be countered by pressure if the gas cloud is to be in equilibrium. There are three sources of pressure: gas pressure, degeneracy pressure (as derived above) and radiation pressure. Consider an element of mass bounded by a small solid angle and a small increment of radius. The inward gravitational force is,

$$\delta F_{gravity} = \frac{Gm(r)\rho(r)r^2\delta\Omega\delta r}{r^2} \tag{3.1}$$

where, as usual, m(r) is the mass enclosed within radius ‘r’. The outward force due to the (combined sources of) pressure is,

$$\delta F_{pressure} = -\frac{dP}{dr} r^2 \delta\Omega\delta r \tag{3.2}$$

Equating (3.1) and (3.2) reproduces the usual equation of hydrostatic equilibrium. Inserting explicitly our three different pressures we have,

$$\frac{dP_{gas}}{dr} + \frac{dP_{\deg eneracy}}{dr} + \frac{dP_{radiation}}{dr} = -\frac{Gm(r)\rho(r)}{r^2} \tag{3.3}$$

To explore approximate bounds in this Section, we will not be concerned with distributions of the field variables (pressure, density, temperature). We shall not attempt, therefore, to deal with the exact differential relationship, (3.3). Instead, and inspired by the distributions investigated in Chapters 11 and 18, we shall reduce (3.3) to a relation involving representative pressures. To do this systematically we assume the density to be far greater within a radius ‘a’ than between ‘a’ and the star’s radius ‘R’. Thus, most of the mass of the star is assumed to lie within r < a, i.e. $m(r) \sim M(r/a)^3$ . Note that the temperature may still be of the order of many hundreds of thousands of K for r > a, and the pressure and density will be many orders of magnitude bigger than interstellar space. Integrating (3.3) from 0 to ‘a’ thus gives,

$$\left[P_{gas}(a) - P_{gas}(0)\right] + \left[P_{\deg eneracy}(a) - P_{\deg eneracy}(0)\right] + \left[P_{radiation}(a) - P_{radiation}(0)\right] \sim -\frac{3GM^2}{8\pi a^4} \tag{3.4}$$

To progress, we assume that ‘a’ is also large enough to ensure that the pressures at r = a are small compared with the pressures at the centre. (3.4) then becomes,

$$P_{gas}(0) + P_{\deg eneracy}(0) + P_{radiation}(0) \sim \frac{3GM^2}{8\pi a^4} \tag{3.5}$$

Obviously, taking a = R would ensure the LHS to be accurate. However, in this case the assumption of uniform density which results in the RHS is grossly incorrect. The RHS is a reasonable approximation only if 'a' is sufficiently small that the density does not vary too much within r < a, and yet it is large enough to encompass nearly all the star's mass. There is no guarantee that an 'a' exists which makes (3.5) a particularly good approximation. For now we rest content that the distributions studied in Chapters 11 and 18 are not too far from meeting these conditions, for R / a ~ 3 to 5 or so. If R/a = 4, say, the difference between R and a is worth a factor of 256 in Equ.(3.5). This emphasises that the actual field distributions are important, and that Equ.(3.5) is therefore quite a crude approximation.

**4. Maximum Density and Temperature; Minimum Number of Particles**

In this Section we explore the consequences if the radiation pressure is negligible. We can substitute into (3.5) for the degeneracy pressure from (2.8) and use the gas pressure in the form,

$$P_{gas} = \frac{\rho}{M_p} kT = \rho_N kT \tag{3.7}$$

where we are using the convenient approximation that the gas is hydrogen only, so that the number of protons equals the number of electrons, and the density is their number density times $M_p$. Thus we get,

$$\rho_N kT + \frac{(3\pi^2)^{2/3}\hbar^2}{5m_e}\rho_N^{5/3} = \frac{3GM^2}{8\pi a^4} \tag{3.8}$$

noting, from (3.5), that the number density and temperature in (3.8) relate to the centre of the star. Since we have used the approximation that the density is constant within r < a and thereafter is very small, we have,

$$\rho_N \approx \frac{3N}{4\pi a^3} \equiv \frac{1}{\Delta^3} \tag{3.9}$$

where $\Delta$ is the average spacing between protons (and electrons). Thus, (3.8) becomes, on multiplying by $\Delta^3$,

$$kT = \left(\frac{\pi}{6}\right)^{1/3} \frac{GN^{2/3}M_p^2}{\Delta} - \frac{(3\pi^2)^{2/3}\hbar^2}{5m_e\Delta^2} \tag{3.10}$$

Starting from a large volume of low density gas, Equ.(3.10) shows how the temperature increases as the cloud collapses under its own gravity – the first term dominating when the gas is dilute (i.e. when $\Delta$ is large). However, as $\Delta$ decreases further, the second term (which originates from the electron degeneracy) becomes larger and eventually causes the temperature to reach a maximum at,

$$\Delta = \frac{3\pi}{5} 2^{4/3} \cdot \frac{\hbar^2}{G m_e N^{2/3} M_p^2} \quad \text{and} \quad kT_{max} = \frac{5}{2^{8/3} 3^{4/3} \pi^{2/3}} \cdot \frac{G^2 N^{4/3} M_p^4 m_e}{\hbar^2} \qquad (3.11)$$

The maximum temperature can also be written,

$$kT_{max} = \frac{5}{2^{8/3} 3^{4/3} \pi^{2/3}} \cdot \left(\alpha_G N^{2/3}\right)^2 m_e c^2 \quad \text{where,} \quad \alpha_G = \frac{G M_p^2}{\hbar c} \qquad (3.12)$$

i.e.

$$kT_{max} = 0.0848 \left(\alpha_G N^{2/3}\right)^2 m_e c^2 = 1.50 \text{x} 10^{-78} N^{4/3} \text{ MeV} \qquad (3.13)$$

For example, for N = $10^{57}$ we find a maximum temperature of 0.015 MeV or 174 x $10^6$ K. The average proton (and electron) spacing is then about 3 x $10^{-12}$ m, or 0.03 Angstroms. Since the latter is about 30 or more times smaller than the size of neutral atoms in ordinary matter, it follows that the density of the material at the centre of such a star is at least about $30^3$ ~ 30,000 times greater than that of ordinary matter.

In general, (3.12) gives the largest temperature which is achievable by gravitational collapse of a gravitationally bound gas cloud consisting of N electrons and N protons. The corresponding minimum spacing is given by (3.11). The maximum density is thus,

$$\rho_{max} = \left(\frac{5}{3\pi}\right)^3 \frac{1}{2^4} \left[\frac{G M_p^2 m_e}{\hbar^2}\right]^3 N^2 M_p = \left(\frac{5}{3\pi}\right)^3 \frac{1}{2^4} \left[\alpha_G \frac{m_e c}{\hbar}\right]^3 N^2 M_p \qquad (3.11b)$$

Thus, for N = $10^{57}$ we find a maximum density of 5.58 x $10^7$ kg/m$^3$ = 5.58 x $10^4$ g/cm$^3$ which is indeed about 30,000 times greater than ordinary matter.

We note that (3.11) gives the minimum spacing possible, i.e., explicitly,

$$\Delta > \Delta_{min} = \frac{3\pi}{5} 2^{4/3} \cdot \frac{\hbar^2}{G m_e N^{2/3} M_p^2} \qquad (3.11c)$$

Now we note that (3.13) can be written as a lower bound for the number of particles, and this lower bound can be evaluated if we assume some minimum temperature compatible with nuclear reactions (say, 6.6 x $10^6$ K, or 0.000569 MeV, see Section 6). Hence,

$$kT_{nuc} = 0.000569 \text{MeV} < kT < 0.0848 \left(\alpha_G N^{2/3}\right)^2 m_e c^2 \Rightarrow \alpha_G N^{2/3} > 0.115 \qquad (3.13b)$$

Hence,

$$N > \left(\frac{0.115}{\alpha_G}\right)^{3/2} = 8.6 \text{ x } 10^{55} \qquad (3.13c)$$

## 5. Minimum Density and Temperature; Maximum Number of Particles

In this Section we include radiation pressure and appeal to an upper bound on radiation pressure for which a star may be stable. The equation of equilibrium becomes, in place of (3.8),

$$\rho_N kT + \frac{(3\pi^2)^{2/3}\hbar^2}{5m_e}\rho_N^{5/3} + P_{radiation} = \frac{3GM^2}{8\pi a^4} \tag{3.14}$$

Hence, in place of (3.10) we have,

$$kT\left[1+\frac{P_{radiation}}{P_{gas}}\right] = \left(\frac{\pi}{6}\right)^{1/3}\frac{GN^{2/3}M_p^2}{\Delta} - \frac{(3\pi^2)^{2/3}\hbar^2}{5m_e\Delta^2} \tag{3.15}$$

The radiation pressure is given by (see Appendix A0),

$$P_{radiation} = 0.2193\left(\frac{kT}{\hbar c}\right)^3 kT \tag{3.16}$$

Hence, from (3.7) we get,

$$\frac{P_{radiation}}{P_{gas}} = \frac{0.2193}{\rho_N}\left(\frac{kT}{\hbar c}\right)^3 = 0.2193\Delta^3\left(\frac{kT}{\hbar c}\right)^3 \tag{3.17}$$

We are now interested in the *largest* possible spacing, Δ, giving rise to a stable star. We will state without proof that a star becomes unstable when the radiation pressure exceeds about twice the gas pressure. Thus, (3.17) gives the upper limit for the spacing with no further effort, i.e.,

$$\Delta < \Delta_{max} = 2.09\left(\frac{\hbar c}{kT}\right) \tag{3.18}$$

and,

$$\rho_{min} = 0.11M_p\left(\frac{kT}{\hbar c}\right)^3 \tag{3.19}$$

Substituting (3.18) into (3.15), together with $P_{radiation}/P_{gas} = 2$, gives,

$$\left(\frac{\pi}{6}\right)^{1/3}\frac{GN^{2/3}M_p^2}{\Delta} - \frac{(3\pi^2)^{2/3}\hbar^2}{5m_e\Delta^2} = 3kT < 3\left(2.09\frac{\hbar c}{\Delta}\right) \tag{3.20}$$

Hence,

$$\Delta_{max} = \frac{0.3053\left(\dfrac{\hbar}{m_e c}\right)}{0.1285\alpha_G N^{2/3} - 1} \tag{3.21}$$

Thus, the spacing can be divergently large if the number of protons (and electrons) approaches,

$$N \to \left( \frac{7.782}{\alpha_G} \right)^{3/2} = 4.8 \times 10^{58} \tag{3.22}$$

However, in this case, from (3.18), the temperature is vanishingly low. This is not consistent with nuclear reactions occurring, and hence does not comply with our definition of a star. Taking the lower bound temperature consistent with nuclear reactions at a sensible rate to be 6.6 x $10^6$ K (derived in Section 6 below), (3.18) gives the corresponding maximum spacing consistent with stellar stability to be 7.25 x $10^{-10}$ m = 7.25 Angstroms. This is greater than the size of neutral atomic hydrogen, and hence corresponds to normal densities. Re-arranging (3.20) gives,

$$0.806 G M_p^2 N^{2/3} < 6.27 \hbar c + 1.9142 \frac{\hbar^2}{m_e \Delta} \tag{3.23}$$

which gives,

$$N^{2/3} < \frac{7.78}{\alpha_G} \left\{ 1 + 0.3053 \left( \frac{\hbar c}{m_e c^2 \Delta} \right) \right\} \tag{3.24}$$

For the minimum spacing compatible with both nuclear reactions and stellar stability derived above, i.e., 7.25 x $10^{-10}$ m, the second term in {…} is 0.000163 and hence negligible. We therefore have the upper bound for the number of nucleons in a stable star to be,

$$N < \left( \frac{7.78}{\alpha_G} \right)^{3/2} = 4.8 \times 10^{58} \tag{3.25}$$

This is the Landau-Chandrasekhar limit. This gives the same result as (3.22) because we have found the numerator of (3.21) to be essentially zero (negligible), hence the denominator also must be virtually zero to be consistent with our finite required spacing for nuclear reactions.

Combining (3.25) with (3.13c) we find that stars are restricted to lie in the mass range,

$$\left( \frac{0.115}{\alpha_G} \right)^{3/2} = 8.6 \times 10^{55} < N < \left( \frac{7.78}{\alpha_G} \right)^{3/2} = 4.8 \times 10^{58} \tag{3.26}$$

Since the Sun has N = 1.2 x $10^{57}$, in terms of solar masses the possible mass range of stars is,

$$0.07 < \frac{M}{M_{Sun}} < 40 \tag{3.27}$$

Thus, the largest mass star is about 550 times as massive as the lowest mass star. The Sun is roughly in the middle of the possible range.

## 6. General Expressions for the Stellar Mass, Density and Temperature Limits

In the preceding Sections 4 and 5 we assumed a specific numerical value for the effective temperature at which nuclear reactions initiate (namely, 6.6 x $10^6$ K). This value is specific to this universe. In this Section we express the preceding results more generally in terms of the universal constants so that they may be interpreted for different values of these constants in alternative universes.

In Chapter 14 we have investigated the competition between the Coulomb barrier and the Maxwell distribution. This competition is manifest as an exponential factor in the rate of the reaction $p + p \to D + e^+ + \nu_e$. We found that the energy which maximises this exponential factor is $E_0 = (0.5bkT)^{2/3}$ where, for this reaction, $b = \pi\alpha\sqrt{M_p c^2}$ = 0.351 √MeV. Numerically we found in Chapter 14 that the reaction rate (which also involves other factors) is maximum for particles of energy roughly 6kT. Equating these two expressions gives,

$$kT_{Nuc} \approx \frac{\pi^2\alpha^2}{2^2 6^3} M_p c^2 = 0.000571\text{MeV} \Rightarrow T_{Nuc} \approx 6.6\text{x}10^6 \text{ K} \qquad (6.1)$$

This is the origin of our estimate of the lowest temperature, $T_{Nuc}$, at which nuclear reactions will occur, as employed in Sections 4 and 5. Equ.(6.1) also gives the general algebraic form in terms of the universal constants.

From Section 4, we find the general expression for the minimum number of particles consistent with nuclear reactions occurring to be,

$$kT_{nuc} = \frac{\pi^2\alpha^2}{2^2 6^3} M_p c^2 < kT < 0.0848\left(\alpha_G N^{2/3}\right)^2 m_e c^2 \Rightarrow \alpha_G N^{2/3} > 0.367\alpha\sqrt{\frac{M_p}{m_e}} \qquad (6.2)$$

Hence, $$N > N_{min} = \left(\frac{0.367\alpha}{\alpha_G}\right)^{3/2}\left(\frac{M_p}{m_e}\right)^{3/4} \quad \text{where,} \quad \alpha_G = \frac{GM_p^2}{\hbar c} \qquad (6.3)$$

The same arguments as in Section 4 lead to expressions for the minimum spacing and maximum density in terms of the number, N, of particles. Note that these conditions lead to the maximum temperature. The number, N, of particles does not have to be the minimum number to just achieve nuclear reaction temperatures, however. Thus, the inequalities unchanged from Section 4 are,

$$kT < kT_{max} = \frac{5}{2^{8/3}3^{4/3}\pi^{2/3}} \cdot \left(\alpha_G N^{2/3}\right)^2 m_e c^2 \qquad (3.12)$$

$$\Delta > \Delta_{min} = \frac{3\pi}{5} 2^{4/3} \cdot \frac{1}{\alpha_G N^{2/3}} \cdot \frac{\hbar}{m_e c} \qquad (3.11c)$$

$$\rho < \rho_{max} = \left(\frac{5}{3\pi}\right)^3 \frac{1}{2^4}\left[\alpha_G \frac{m_e c}{\hbar}\right]^3 N^2 M_p \tag{3.11b}$$

In Section 5 we have found the maximum number of particles consistent with pulsational stability under conditions of significant radiation pressure to be,

$$\Delta < \Delta_{max} = 2.09\left(\frac{\hbar c}{kT}\right) \tag{3.18}$$

As in Section 5, re-arranging (3.20) gives,

$$0.806 G M_p^2 N^{2/3} < 6.27\hbar c + 1.9142\frac{\hbar^2}{m_e \Delta} \tag{3.23}$$

which gives,

$$N^{2/3} < \frac{7.78}{\alpha_G}\left\{1 + 0.3053\left(\frac{\hbar c}{m_e c^2 \Delta}\right)\right\} \tag{3.24}$$

For the minimum spacing compatible with nuclear reactions as well as stability in the presence of radiation pressure, from (3.18) and (6.1) we have,

$$\Delta < \Delta_{max} = 2.09\left(\frac{\hbar c}{kT}\right) < 2.09\left(\frac{\hbar c}{kT_{Nuc}}\right) \approx 183.0\frac{\hbar}{\alpha^2 M_p c} \tag{6.4}$$

Thus, (3.24) becomes,

$$N^{2/3} < \frac{7.78}{\alpha_G}\left\{1 + 0.3053\left(\frac{\hbar c}{m_e c^2 \Delta}\right)\right\} < \frac{7.78}{\alpha_G}\left\{1 + 0.00167\alpha^2 \frac{M_p}{m_e}\right\} \tag{6.5}$$

Thus, in our universe, the second term in {…} is of order $10^{-4}$, as noted above, and hence is negligible. It is feasible, in an alternative universe with a much stronger electromagnetic force and an even larger nucleon-electron mass ratio, that the second term could become comparable or larger than unity. We therefore retain it in the general case, giving,

$$N < N_{max} \approx \left[\frac{7.78}{\alpha_G}\left\{1 + 0.00167\alpha^2 \frac{M_p}{m_e}\right\}\right]^{3/2} \tag{6.6}$$

In summary, the bounds we have derived for the temperature, particle spacing, density and numbers of protons in stars are (using Equs.3.11b, 3.11c, 3.12, 6.1, 6.3, 6.4, 6.6):-

$$0.0114\alpha^2 M_p c^2 < kT < 0.0848\left(\alpha_G N^{2/3}\right)^2 m_e c^2 \tag{6.7}$$

$$4.75\frac{1}{\alpha_G N^{2/3}}\cdot\frac{\hbar}{m_e c} < \Delta < 183.0\frac{\hbar}{\alpha^2 M_p c} \quad (6.8)$$

$$\left(\frac{\alpha^2 M_p c}{183.0\hbar}\right)^3 M_p < \rho < 0.00933\left[\alpha_G \frac{m_e c}{\hbar}\right]^3 N^2 M_p \quad (6.9)$$

$$\left(\frac{0.367\alpha}{\alpha_G}\right)^{3/2}\left(\frac{M_p}{m_e}\right)^{3/4} < N < \left[\frac{7.78}{\alpha_G}\left\{1+0.00167\alpha^2\frac{M_p}{m_e}\right\}\right]^{3/2} \quad (6.10)$$

Where the number of protons in the star is not known, the absolute bounds on the RHS of (6.7), the LHS of (6.8) and the RHS of (6.9) can be found by substituting the largest possible N from the RHS of (6.10), giving,

$$0.0114\alpha^2 M_p c^2 < kT < 5.13 m_e c^2 \quad (6.11)$$

$$0.611\frac{\hbar}{m_e c} < \Delta < 183.0\frac{\hbar}{\alpha^2 M_p c} \quad (6.12)$$

$$\left(\frac{\alpha^2 M_p c}{183.0\hbar}\right)^3 M_p < \rho < 4.39\left[\frac{m_e c}{\hbar}\right]^3 M_p \quad (6.13)$$

In this universe, (6.10-13) give,

$$8.6\times 10^{55} < N < 4.8\times 10^{58} \quad (6.14)$$

$$0.00057\text{ MeV} < kT < 2.62\text{ MeV} \quad (6.15)$$

$$6.6\times 10^{6}\text{ K} < T < 3.0\times 10^{10}\text{ K} \quad (6.16)$$

$$2.36\times 10^{-13}\text{ m} < \Delta < 0.72\times 10^{-9}\text{ m} \quad (6.17)$$

$$4.4\text{ kg/m}^3 < \rho < 1.27\times 10^{11}\text{ kg/m}^3 \quad (6.18)$$

The normal density of (say) iron is ~7800 kg/m$^3$, or atmospheric air ~0.64 kg/m$^3$, so the possible density ranges from roughly comparable to that of air, up to 7 orders of magnitude greater than that of ordinary matter.