Chapter 13 – Stellar Nuclear Processes

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1. Introduction

In this Chapter we take a detailed look at the nuclear reactions powering stars. Attention will be confined in this Chapter to the hydrogen burning phase (main sequence stars). The burning of helium is considered in Chapter 21. Hydrogen does not, of course, fuse into helium in a single step. Various different reaction pathways are possible. Above a temperature of about 18×10^6 K, the dominant reaction pathways are referred to as the CN and CNO sequences. In these, carbon, nitrogen and oxygen act as catalysts in a sequence of reactions whose overall effect is to fuse hydrogen to helium. In the CN cycle it is necessary that some carbon should already be present in the star. In the CNO cycle, it is necessary that some nitrogen should already be present in the star. Consequently, these reaction pathways would not be applicable immediately in the first stars. We will not consider the CN and CNO sequences further here.

Below 18×10^6 K the dominant reaction pathways are the so-called pp sequences. Hence, the pp sequences are the relevant reactions for solar mass stars (central temperature 13.7×10^6 K). These comprise three distinct reaction sequences which occur in parallel. These are described in Section 2 below. The rest of this Chapter examines the dominant pp sequence (ppI) in detail, with some brief remarks regarding ppII. We are interested in several things:-

- What are the rates of each contributing reaction, and hence the overall rate of helium-4 production and hence power density?
- Why are the particular reactions of the ppI sequence the relevant ones, rather than the long list of other potentially possible reactions?
- What are the concentrations of intermediate nuclei, such as deuterium, helium-3 and tritium, and why is the product of the reaction sequences essentially just helium-4?

In this Chapter we take the basic reaction rates from the literature. However, in Chapter 14 we shall derive the rate of the first reaction in the pp sequences from quantum mechanical first principles. The same is done for the second reaction in Appendix A3. Consequently, since the first reaction is the overall rate determining step, we will be able to trace the power density of stars back to the universal constants.

2. The Nuclear Sequences – Qualitative Summary

The three pp sequences of nuclear fusion reactions are known as ppI, ppII, and ppIII, in order of decreasing contribution to helium-4 production. These sequences are defined in the following Table,

<u>The Three pp Nuclear Reaction Sequences (fusing H-to-He)</u>

<table>
<tr><td colspan="3">(1) $p + p \to {}^2_1 D + e^+ + \nu_e$</td></tr>
<tr><td colspan="3">(2) $p + {}^2_1 D \to {}^3_2 He + \gamma$</td></tr>
<tr><td rowspan="4">(3)
${}^3_2 He + {}^3_2 He \to {}^4_2 He + 2p$</td><td colspan="2">(4) ${}^3_2 He + {}^4_2 He \to {}^7_4 Be + \gamma$</td></tr>
<tr><td>(5) ${}^7_4 Be + e^- \to {}^7_3 Li + \nu_e$</td><td>(7) ${}^7_4 Be + p \to {}^8_5 B + \gamma$</td></tr>
<tr><td rowspan="2">(6) ${}^7_3 Li + p \to {}^4_2 He + {}^4_2 He$</td><td>(8) ${}^8_5 B \to {}^8_4 Be^* + e^+ + \nu_e$</td></tr>
<tr><td>(9) ${}^8_4 Be^* \to {}^4_2 He + {}^4_2 He$</td></tr>
<tr><td>85%</td><td>15%</td><td>0.02%</td></tr>
<tr><td>Heat 26.2 MeV</td><td>Heat 25.7 MeV</td><td>Heat 19.27 MeV</td></tr>
<tr><td>ν – loss 1.9%</td><td>3.9%</td><td>27.8%</td></tr>
<tr><td>ppI</td><td>ppII</td><td>ppIII</td></tr>
</table>

NB: Some references imply that reaction (5) is ${}^7_4 Be + e^- \to {}^7_3 Li + \nu_e + \gamma$. Similarly, some references replace (6) with ${}^7_3 Li + p \to \gamma + {}^8_4 B_e \to {}^4_2 He + {}^4_2 He$.

NB: $\frac{26.2}{1-0.019} = \frac{25.7}{1-0.039} = \frac{19.27}{1-0.278} = 26.7 MeV$, i.e. the heat plus the neutrino energy equals the same total energy released in each case.

The numerical lines in the Table give the following data. The first indicates the percentage contribution of the sequence to overall helium-4 production. Since ppII and ppIII involve weak-force reactions [i.e. in addition to the common reaction (1)], as evidenced by the neutrino production, it is not surprising that these are the minority sequences.

The second numerical row gives the total heat produced per helium-4 nucleus formed. The difference between the sequences in this respect is due to the differing losses due to the neutrinos. The above figures are consistent with a total energy production of 26.7 MeV. This can be deduced from a consideration of the difference of the rest masses involved. The binding energy of helium-4 is 28.3 MeV. Hence, the difference between the helium-4 rest mass and that of the four protons which are involved in creating it equals this binding energy less twice the mass difference between the neutron and proton, i.e. 28.3 - 2 x 1.2933 = 25.7 MeV. However, whichever sequence we follow there are two electrons also needed as ingredients. This is obvious because two protons must be converted to neutrons, but it can also been seen from detailed scrutiny of the above sequences [i.e. for ppI, two reactions (1) are need to produce one helium-4, and each requires an electron to annihilate with the positron produced. On the other hand, for ppII and ppIII, only one reaction (1) is needed, but an additional electron is needed for ppII in reaction (5), whereas for ppIII and additional electron is needed to annihilate the positron produced by reaction (8)]. Hence there is an additional conversion of rest mass of 2 x 0.5 MeV = 1 MeV per helium-4 produced, making 25.7 + 1 = 26.7 MeV in total overall, in agreement with the above Table.

The last numerical row indicates the percentage of the energy which is lost from the star in the form of neutrinos. The neutrinos interact too weakly to remain within the star. All the remaining energy (i.e. the difference between the rest masses of the products and the initial particles) appears as heat. Either the kinetic energy of the products becomes thermalised through further interactions (and this includes the photons), or, in the case of the positrons from reaction (1), annihilation with electrons produces gamma rays which subsequently thermalise. The neutrinos from reaction (1) cause some energy loss from all three sequences, but this is relatively slight. The sequence ppIII produces the most energetic neutrinos via the decay of boron-8 to beryllium-8. It is the neutrinos from this sequence which are detected on earth and which gave rise to the 'solar neutrino controversy' (now resolved). It is interesting to note that this controversy relates to a minority sequence which accounts for a mere 0.02% of the helium produced. The neutrinos are produced in three-body product reactions (for ppI and ppIII), and hence they have a range of possible energies. The above figures are averages.

The above Table relates to stars of roughly solar mass, i.e. for which the central temperature is around 13.7 million K. We shall see in Chapter 18 that for slightly more massive stars, with central temperatures of around 18 million K or higher, the burning of hydrogen is dominated by a different sequence of reactions, the so-called CN and CNO sequences. However, these sequences are catalysed by ^{12}C and ^{14}N, so they would only be available to second generation stars, i.e. the ^{12}C and ^{14}N would need to have been made in earlier generations of stars[1].

Note that the hydrogen burning sequence ceases with the production of ^{4}He. The reason why a further capture of a proton, to form a nucleus of atomic mass 5, does not occur is that no such nuclei are stable. There are four such nuclei, ^{5}H, ^{5}He, ^{5}Li and ^{5}Be, but they are all extremely unstable, i.e. mere resonances, decaying on strong interaction timescales of around 10^{-21} secs. The other reactions which might be considered feasible in taking fusion beyond ^{4}He are discussed in later Chapters. For now, suffice it to say that none can occur at these temperatures. With further gravitational collapse, the core temperature can be increased by another order of magnitude. This becomes sufficient to initiate the binary fusion of two ^{4}He nuclei. What follows is a long and interesting story, taken up in Chapter 21.

All the pp sequences have reactions (1) and (2) in common. Reaction (1) is the slowest of all the reactions. In a sense it is reaction (1) which determines the lifetime of stars[2] for which ppI, ppII and ppIII are the available sequences. The reaction time for reaction (1) is $\sim 16 \times 10^9$ years, which is indeed the same order as the lifetime of solar mass stars.

In stark contrast, the reaction time for reaction (2) is just a couple of seconds. The reason for the dramatic difference of timescale is easy to see qualitatively. Reaction (1) is a weak-force driven reaction, whereas reaction (2) is electro-strong-force driven. Actually, this is only part of the story. Another reason why all the nuclear reactions are far slower than they might have been is due to the Coulomb barrier between the two nuclei or protons. The typical thermal energies at temperatures of 10

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[1] Or later in the life of a first generation star when sufficient carbon and nitrogen has been produced – assuming sufficient mixing has taken place for these 'metals' to be present in the region of hydrogen shell burning.

[2] This is both true and also grossly misleading. The rates of nuclear reactions are extremely sensitive to temperature. It is better to say that the rate of Reaction (1) is *consistent* with the lifetime of the star for the assumed conditions of temperature and density. It is also true that the assumed conditions of temperature and density are consistent with the available heat transport mechanisms and heat production and loss rates. This latter condition is what really determines the stellar lifetime. Roughly speaking, if the nuclear reaction rate were faster, say, then the temperature would automatically reduce to compensate and keep the power density similar and the heat flows in balance.

to 20 million K are insufficient to drive the protons close enough to react. All reactions involving reactants with like charges are subject to this retarding mechanism. However, reactions mediated by the weak nuclear force, like reaction (1), are particularly susceptible. This is because the weak nuclear forces are even shorter range than the strong nuclear force. Indeed, it is the very short range nature of the weak interactions which is responsible for their weakness. Whereas the range of the strong force is determined by the mass of the pion (~140 MeV, and hence a range of ~1.4 fm), the range of the weak interaction is determined by the far greater mass of the W and Z bosons (~80-91MeV, and hence ~0.0025 fm).

In either case, though, penetration of the Coulomb barrier requires quantum tunnelling. Each proton essentially has a small probability of being found within the Coulomb barrier of another proton, despite this being energetically impossible from the classical point of view.

In Chapter 14 we carry out an explicit, if very crude, calculation of the rate of reaction (1) from first principles. The same program is carried out for reaction (2) in Appendix A3. We find that, despite the Coulomb barrier being operative in both cases [in fact somewhat stronger for reaction (2)], the huge 17 orders of magnitude difference in the rates of the two reactions can be explained – and the explanation lies in the weakness of the weak interaction.

An anthropic argument which appears to follow from this, but which I believe to be false, is: "Since the slowness of reaction (1) is what determines the lifetime of stars, and since stars burning for billions of years appears to be a prerequisite for the spontaneous emergence of life, it follows that the weakness of the weak interaction is a crucial requirement for life in the universe." The reason why it is false is outlined in footnote[1] and will be discussed in detail in the Cosmic Coincidences.

The reaction time for reaction (3) is around 10^6 years. Thus, whilst reaction (3) is extremely slow by normal standards, it is not the overall rate determining step, being around 10,000 times faster than reaction (1).

3. Why Are Other Sequences Producing Helium-4 Not Significant?

We shall accept in this Section that the initial reaction is the p + p reaction (1). The question is how we arrive at helium-4 from protons and deuterons, and what the reaction rates are. Leaving aside the sequences ppII and ppIII, which proceed via nuclei more massive than helium, it is not immediately obvious why the only low-mass sequence is the ppI sequence, i.e. $p + {}^2_1D \rightarrow {}^3_2He + \gamma$; ${}^3_2He + {}^3_2He \rightarrow {}^4_2He + 2p$. One might think that there are a number of different sequences possible involving the following reactions,

a)	${}_1^2\mathrm{D} + {}_1^2\mathrm{D} \rightarrow {}_2^4\mathrm{He} + \gamma$	direct helium-4 production
b)	${}_1^2\mathrm{D} + {}_1^2\mathrm{D} \rightarrow {}_1^3\mathrm{H} + {}_1^1\mathrm{p}$	tritium production
c)	${}_1^2\mathrm{D} + {}_1^2\mathrm{D} \rightarrow {}_2^3\mathrm{He} + {}_0^1\mathrm{n}$	helium-3 production + neutrons
d)	${}_1^3\mathrm{H} + {}_1^2\mathrm{D} \rightarrow {}_2^4\mathrm{He} + {}_0^1\mathrm{n}$	helium-4 production + neutrons
e)	${}_2^3\mathrm{He} + {}_0^1\mathrm{n} \rightarrow {}_1^3\mathrm{H} + {}_1^1\mathrm{p}$	tritium production
f)	${}_0^1\mathrm{n} + {}_1^2\mathrm{D} \rightarrow {}_1^3\mathrm{H} + \gamma$	tritium production
g)	${}_1^3\mathrm{H} + {}_1^1\mathrm{p} \rightarrow {}_2^4\mathrm{He} + \gamma$	helium-4 production
h)	${}_2^3\mathrm{He} + {}_0^1\mathrm{n} \rightarrow {}_2^4\mathrm{He} + \gamma$	helium-4 production

Of course, not all these reactions can proceed immediately because there are initially only protons, helium-4 nuclei and some deuterons formed by reaction (1). At this point only reactions (a), (b) and (c) can proceed. However, these reactions produce the tritium, helium-3 and neutrons required by reactions (d) to (h). So, why shouldn't these reactions also contribute to helium-4 production, in addition to reactions (2) and (3)?

The answer lies in the relative rates of the reactions. Assuming that the reaction cross sections are all of the same order, the overall reaction rates are proportional to the product of the number densities of the reacting particles. Providing that reaction (2) is very rapid compared with reaction (1), which it is, the former will mop up the deuterons as soon as they are formed by the latter and maintain a very low density of deuterons. In particular, the deuteron density is very small compared with the proton density. Consequently, it is clear that the rate of reactions (a), (b) and (c) will be very slow compared with reaction (2). The same will be true of reactions (d) and (f), the other two reactions which can remove deuterons, since the densities of tritium and neutrons will also be low, being the product of the slow reactions (b) and (c). We will illustrate these contentions below by explicitly calculating the rate of reaction (a) and comparing it with that of reaction (2).

So much for the reactions which remove deuterons. The situation for reactions which create helium-4, in competition with reaction (3), is not so obvious. We can readily believe that reaction (h) is dominated by reaction (3), since the former depends upon the neutron concentration, and the neutrons are formed only by the second order reactions (c) and (d). However, it is not so clear why reaction (g) does not dominate helium-4 production in favour of reaction (3). This concern is genuine because the reactants in (g) have only single charges, in contrast to reaction (3) which involves a pair of doubly charged reactants. Hence, reaction (g) would be expected to be considerably faster than (3). We will see below that this is indeed the case. Reaction (g) *would be* about 9 orders of magnitude faster than reaction (3) at solar temperatures, *if* the concentration of the reactants were the same. However, we shall explicitly calculate the contribution of both reactions below. We shall see that reaction (g) actually does not contribute significantly because the tritium density is vanishingly low.

3.1 Rates of Reactions (1), (2) and (a), and Deuteron Density

For illustration we shall assume a star similar to the sun, with a central density of 90,000 kg/m^3 and a central temperature of 14 x 10^6 ^{o}K. The nucleon mass is about

1.67×10^{-27} kg, and 75% of the star's mass consists of free protons. The number density of protons at the centre of our star is thus ~4×10^{31} /m^3 = 4×10^{25} /cm^3 ~ 67 moles per cm^3.

The rates of reactions (1), (2) and (a) are given in the Table below. The rates are given in units "reactions per second per unit particle density". The "particle density" is measured in moles/cm^3. For example, for a reaction a + b → c + d, multiplying the reaction rate by the density of particle 'a' in moles/cm^3 gives the number of reactions per second per particle b. The roles of particles a and b may be reversed. Thus, multiplying by the molar density of particle b gives the number of reactions per second per particle a.

Reaction Rate (cm^{3}mole^{-1}s^{-1})

Temperature (^{o}K)	**Reaction (1)**	**Reaction (2)**	**Reaction (a)**
	$p + p \to {}^2_1D + e^+ + \nu_e$	$p + {}^2_1D \to {}^3_2He + \gamma$	${}^2_1D + {}^2_1D \to {}^4_2He + \gamma$
10×10^6	7.2×10^{-21}	-	1.4×10^{-6}
14×10^6	2.99×10^{-20}	0.00727	7.7×10^{-6}
15×10^6	3.95×10^{-20}	0.00997	1.1×10^{-5}
30×10^6	4.6×10^{-19}	0.21	2.8×10^{-4}
50×10^6	1.9×10^{-18}	1.10	1.7×10^{-3}

Black data are from Rob Hoffman et al (UCRL, web site www-phys.llul.gov).
Green data are from the NACRE web site. Note that, where identical reactants are involved, e.g. reaction (1), we have multiplied the NACRE data by 0.5. This means that *all* reaction rates are found in the same way, by multiplying the above data by the product of the reactant densities.

Reaction (1): $p + p \to {}^2_1D + e^+ + \nu_e$

The reaction rate per proton is the proton density (in moles/cm^3) times the above reaction rate, i.e. 67 x 2.99×10^{-20} = 2.0×10^{-18} per second. Hence, the reaction time per proton for reaction (1) is 5×10^{17} sec = 16×10^9 years. This is consistent with the contention that reaction (1) is the overall rate determining step as regards the fusion of H into He, i.e. it is broadly consistent with the accepted lifetime of a solar mass, main sequence star. However, we must confirm that the subsequent reactions are faster, and hence not rate limiting.

The rate of production of deuterons is evaluated from the above reaction rate by multiplying (for a second time) by the proton density – but this time in units of protons per cm^3 – giving 4×10^{25} x 2.0×10^{-18} = 8×10^7 deuterons per second per cm^3. In steady state, this rate of deuteron production must be balanced by the rate of deuteron consumption by reaction (2)…

Reaction (2): $p + {}^2_1D \to {}^3_2He + \gamma$

The reaction rate per deuteron is obtained by multiplying the reaction rate from the Table by the proton density (67 moles/cm^3), i.e. 67 x 0.00727 = 0.49 per second. The reaction time is thus ~2 seconds. This confirms the huge disparity in timescale between reaction (1) – roughly ten billion years – and reaction (2) – roughly a few seconds.

If reaction (2) is the dominant reaction consuming deuterons, then multiplying this reaction rate by the deuteron number density must balance with the rate of deuteron production (assuming steady conditions). Hence we conclude that the deuteron density is 8 x 10^7 / 0.49 = 1.64 x 10^8 deuterons per cm^3. This is a tiny 4 x 10^{-18} times the proton number density, confirming the preceding qualitative arguments.

Reaction (a): ${}^2_1D + {}^2_1D \rightarrow {}^4_2He + \gamma$
Multiplying the above deuteron density (converted to moles/cm^3 by dividing by Avogadro's number) by the reaction rate from the Table gives the reaction rate per deuteron as (1.64 x 10^8/6 x 10^{23}) x 7.7 x 10^{-6} = 2.1 x 10^{-21} per second. Thus we see that this reaction rate per deuteron is 20 orders of magnitude smaller than that for reaction (2), confirming that the 'direct production' of helium-4 can be ignored. Strictly we need to examine the rate of production of helium-4 via reaction (3) before concluding this, but at least we have confirmed that reaction (a) does not influence the deuteron density.

3.2 Rate of Reaction (3) and Helium-3 Density
To evaluate the rate of reaction (3) we need the density of helium-3. This can be found by balancing the rate of helium-3 production against its rate of loss. We have argued that the rate of helium-3 production is dominated by reaction (2). The consumption of helium-3 occurs via reactions (3), (e) and (h). We shall now, for simplicity, make the assumption that the neutron density is very low – and hence that reactions (e) and (h) do not significantly increase the rate of consumption of helium-3. Hence we will find the helium-3 density by balancing reaction (2) against reaction (3). For the former, the deuteron number density of 1.64 x 10^8 /cm^3 times the rate of reaction (2) per deuteron (0.49 per sec) gives a rate of production of helium-3 nuclei of 8 x 10^7 per sec per cm^3. Obviously, this is the same as the rate of production of deuterons in steady state.

By the same token, the rate of production of helium-4 via reaction (3) must be half the rate of helium-3 production (since it takes two helium-3 nuclei to make one helium-4 by this reaction). Thus, <u>the rate of helium-4 production is ~4 x 10^7 per sec per cm^3</u>.

As a check on the reasonableness of this result we may calculate the size of the region which, producing helium-4 at this rate, will produce a thermal power equal to the sun's luminosity of 3.86 x 10^{26} W. Since each helium-4 created produces 26.2 MeV = 4.2 x 10^{-12} J of heat, the number of helium-4 nuclei produced per second by the sun must be 3.86 x 10^{26} W / 4.2 x 10^{-12} J = 9 x 10^{37} per sec. At a production density of 3.5 x 10^7 per sec per cm^3 this requires a volume of 9 x 10^{37} / 4 x 10^7 = 2.3 x 10^{30} cm^3, or a radius of 8.1 x 10^9 cm = 8.1 x 10^7 m. This is just 12% of the sun's radius of 7 x 10^8 m, and hence seems reasonable as an estimate of the effective size of the core within which nuclear fusion is occurring. (Recall from Chapter 11 that the temperature falls off fairly quickly away from the centre, so fusion will only occur in the central region. For a more precise result it would be necessary to evaluate the reaction rate at each radial location corresponding to the prevailing temperature, and then integrate the total thermal power).

To evaluate the actual density of helium-3 nuclei we need to balance the rate of reaction (2) against (3) + (4). The rates of reactions (3), (b) and (g) are, Reaction Rate ($cm^3 mole^{-1} s^{-1}$)

Temperature (^oK)	Reaction (3)	Reaction (b)	Reaction (g)
	${}^3_2He + {}^3_2He \rightarrow {}^4_2He + 2p$	${}^2_1D + {}^2_1D \rightarrow {}^3_1H + {}^1_1p$	${}^3_1H + {}^1_1p \rightarrow {}^4_2He + \gamma$
10 x 10^6	1.15 x 10^{-13}	11.9	
14 x 10^6	3.62 x 10^{-11}	86	0.0462
15 x 10^6	1.10 x 10^{-10}	123	0.0640
30 x 10^6	2.09 x 10^{-6}	2,500	1.1
50 x 10^6	7.09 x 10^{-4}	15,400	6.0

Black data are from Rob Hoffman et al (UCRL, web site www-phys.llul.gov).
Green data are from the NACRE web site. See also the Note under the previous Table.
Blue data are from the CF88 web site. See also the Note under the previous Table.

Similarly, the rates of reactions (4), (5) and (6) are,

Reaction Rate ($cm^3 mole^{-1} s^{-1}$)

Temperature (^oK)	Reaction (4)	Reaction (5)	Reaction (6)
	${}^3_2He + {}^4_2He \rightarrow {}^7_4Be + \gamma$	${}^7_4Be + e^- \rightarrow {}^7_3Li + \nu_e$	${}^7_3Li + p \rightarrow {}^4_2He + {}^4_2He$
10 x 10^6	1.63 x 10^{-18}	1.89 x 10^{-9}	1.30 x 10^{-7}
14 x 10^6	7.21 x 10^{-16}	1.50 x 10^{-9}	6.79 x 10^{-6}
15 x 10^6	2.31 x 10^{-15}	1.44 x 10^{-9}	1.44 x 10^{-5}
30 x 10^6	6.61 x 10^{-11}	1.01 x 10^{-9}	0.0110
50 x 10^6	3.0 x 10^{-8}	8.29 x 10^{-10}	0.573

Green data are from the NACRE web site. Blue data are from the CF88 web site.

The rate of reaction (3) per helium-3 nucleus is obtained by multiplying the rate from the above Table by the helium-3 density in moles/cm^3. By multiplying this again by the helium-3 density in particles/cm^3 we obtain the rate at which reaction (3) consumes helium-3 nuclei in particles per sec per cm^3. To this we must add the rate of helium-3 consumption by reaction (4), which is proportional to the product of the helium-3 and helium-4 densities. The helium-4 number density is just $^1/_3$ x ¼ times the free proton number density, and hence is 3.37 x 10^{24} /cm^3. Equating the total consumption rate to the rate at which helium-3 nuclei are being created gives a quadratic equation in the helium-3 density,

$$3.62x10^{-11}\frac{\rho^N_{He3}}{6x10^{23}}\rho^N_{He3} + 7.21x10^{-16}\frac{\rho^N_{He3}}{6x10^{23}}3.37x10^{24} = 8x10^7 \qquad (1)$$
$$\Rightarrow \rho^N_{He3} = 1.12x10^{21}\ cm^{-3}$$

i.e. there is about one helium-3 to every 36,000 protons. There are thus far more helium-3 nuclei than deuterons, by about 13 orders of magnitude. This is because the reaction (3) which consumes helium-3 is very slow compared with the reaction (2) which creates them.

3.3 Rate of Reactions (b)-(f) and Tritium Density

Recall that we found it difficult to conclude qualitatively, above, that the reaction (g), i.e. ${}^3_1H + {}^1_1p \rightarrow {}^4_2He + \gamma$ was of no importance in the overall helium-4 production. Hence, we calculate here the rate of helium-4 production by this process. To evaluate the rate of reaction (g) we require the tritium density. Strictly this should be obtained by balancing the tritium production via reactions (b), (e) and (f) against its rate of consumption via reactions (d) and (g). However, we have already assumed that the neutron density is very low. Consistent with this we may assume that tritium production reactions (e) and (f) can be neglected, as can tritium consumption reaction (d). Thus, we estimate the tritium density by balancing production reaction (b) against consumption reaction (g):-

$$\rho_d^N \frac{\rho_d^N}{A} \mathrm{Rate(b)} = \rho_t^N \frac{\rho_p^N}{A} \mathrm{Rate(g)} \qquad (2)$$

where the subscripts d and t refer to deuterium and tritium respectively, and p to protons, and A is Avogadro's number (which cancels). Thus, the LHS of (2) is the rate of production of tritium by reaction (b), whereas the RHS of (2) is the rate at which tritium is consumed. Since we know, above, that the deuteron and proton concentrations are 1.64 x 10^8 and 4 x 10^{25} per cm^3 respectively, we can solve for the tritium density using the reaction rates from the above Table, giving $\rho_t^N \sim 10^{-6}$ per cm^3. This is an exceedingly small concentration, even by the sparse standards of the deuterons.

The rate of production of helium-4 by reaction (g) equals the rate of consumption of tritium given by Equ.(2) and hence is ~10^{-5} per sec per cm^3. Clearly, this is entirely negligible compared with the rate of helium-4 production from the ppI sequence, i.e. reaction (3), which is 4 x 10^7 per sec per cm^3.

3.4 The Neutron Reactions and Neutron Density

We have now very nearly demonstrated that all the reactions (a) to (h) are negligible compared with the ppI sequence. The only thing that remains is to check the assumption that the reactions involving neutrons are negligible. To do this it suffices to show that the neutron density is very low. The reactions which produce neutrons are (c) and (d). The latter can be neglected because we have already shown that the tritium density is virtually zero. The rate of reaction (c) is estimated from Hoffman's data to be 33.4 per sec per mole/cm^3 at 14 M^oK. Multiplying by the square of the deuteron density (1.64 x 10^8 /cm^3, see above), and dividing by Avogaro's number for compatibility of units, gives the rate of neutron production to be 1.5 x 10^{-6} per sec per cm^3. Thus, the rate of neutron production is very slow, some 13 orders of magnitude slower than the rate of production of deuterons, helium-3 and helium-4.

Neutrons are consumed by reactions (e), (f) and (h). Using the above estimates for the deuteron and helium-3 densities we can therefore find the neutron density by equating the sum of the neutron consumption rates via these three reactions to the above rate of neutron production, i.e.,

$$\left[\rho_d^N \mathrm{Rate(f)} + \rho_{He3}^N \mathrm{Rate(e)} + \rho_{He3}^N \mathrm{Rate(h)}\right] \frac{\rho_n^N}{A} = 1.5 \mathrm{x} 10^{-6} \ \mathrm{s^{-1} cm^{-3}} \qquad (3)$$

Hoffman gives the reaction rates as 69.0, 0.0 and 4.0 per sec per mole/cm^3 respectively for (f), (e) and (h) at below 50 M^oK. [NB: It is not immediately obvious to me why the rate of reaction (e) is zero – it <u>is</u> energetically possible since the binding energy of tritium, 8.482 MeV, exceeds that of helium-3, 7.718 MeV. Is some selection rule at work?]. Since the deuteron and helium-3 densities are 1.64 x 10^8 /cm^3 and 1.12 x 10^{21} /cm^3 respectively we find that the contribution of reaction (f) to the neutron consumption is also negligible and the neutron density is ~2 x 10^{-4} per cm^3. As expected, the neutron density is exceedingly low.

The rate of production of helium-4 via reaction (h) is thus ~1.5 x 10^{-6} per second per cm^3. This is 13 orders of magnitude smaller than the rate of helium-4 production via reaction (3).

Hence we have now completed the demonstration that the ppI sequence, reactions (1), (2), (3), is dominant over all the reaction pathways (a) through (h), above. It remains only to consider the sequences ppII and ppIII. Here we shall confine attention to ppII.

4. The Sequence ppII

This job is almost done already. Knowing the densities of both helium-4 and helium-3 we find the rate of reaction (4) to be a production rate of ${}^7_4\mathrm{Be}$ of,

$$\frac{3.37\mathrm{x}10^{24}}{6\mathrm{x}10^{23}}\times 1.12\mathrm{x}10^{21}\times 7.21\mathrm{x}10^{-16} = 4.5\mathrm{x}10^{6}\ \mathrm{s}^{-1}\mathrm{cm}^{-3}$$

Assuming reactions (4), (5) and (6) to be in equilibrium, 4.5 x 10^6 $s^{-1}cm^{-3}$ is also the production rate of ${}^7_3\mathrm{Li}$. Moreover, the production rate of ${}^4_2\mathrm{He}$ via ppII is just twice this, i.e. 9 x 10^6 $s^{-1}cm^{-3}$. This compares with the production rate of ${}^4_2\mathrm{He}$ via ppI, namely 4 x 10^7 $s^{-1}cm^{-3}$. Thus the ratio of the ${}^4_2\mathrm{He}$ production by ppII to that via ppI is ~9/40 ~ 0.2. This is in reasonable agreement with the Table on Page 1, i.e. a branching ratio of 15%/85% ~ 0.2. [The agreement is not terribly precise – but may be temperature dependent. It would be worth repeating the estimate at 15 x 10^6 K. However, the balance of very sensitive reaction rates is going to be tricky to do better than the first place of decimals].

Reactions (5) and (6) may be used to find the equilibrium densities of beryllium-7 and lithium-7, namely 3.8 x 10^{13} $s^{-1}cm^{-3}$ and 9.9 x 10^9 $s^{-1}cm^{-3}$ respectively. Thus, these nuclei are far more numerous than deuterons, but are still only trace quantities, being only around 10^{-10} and 10^{-16} times the proton density respectively. Note that beryllium-7 has a half-life of 53 days, whereas lithium-7 is stable.

Thus, this mechanism potentially produces some permanent lithium-7 in the universe – although the quantity is entirely negligible. Lithium-7 is one of the nuclei which is formed in the Big Bang. The *proportion* of lithium-7 formed in the Big Bang is about six orders of magnitude larger than that identified above (i.e. around 10^{-10} per proton, compared with ~10^{-16} per proton).

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